

Duality between controllability and observability for target control and estimation in networks

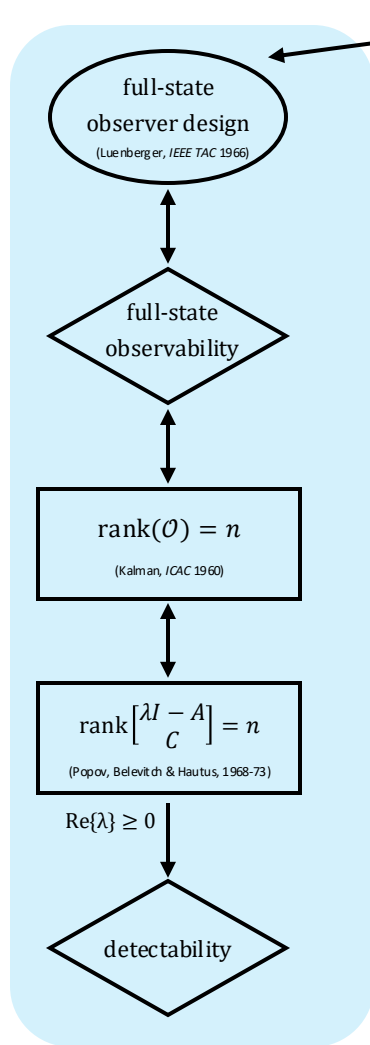
Arthur N. Montanari



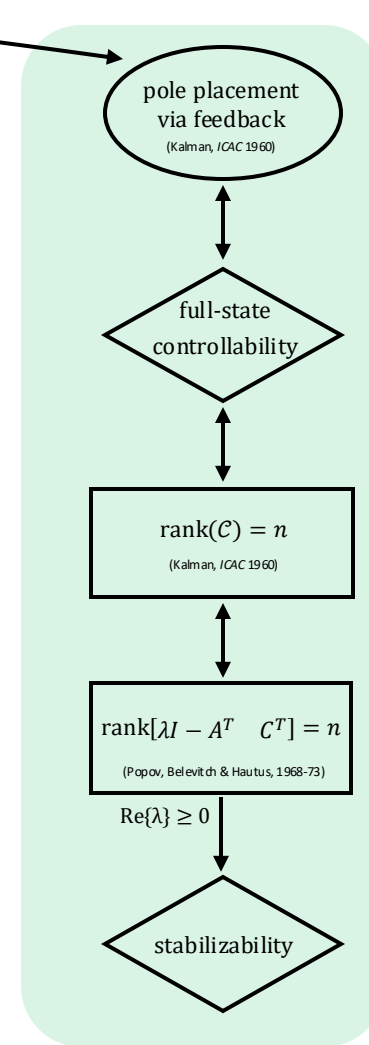
Northwestern University

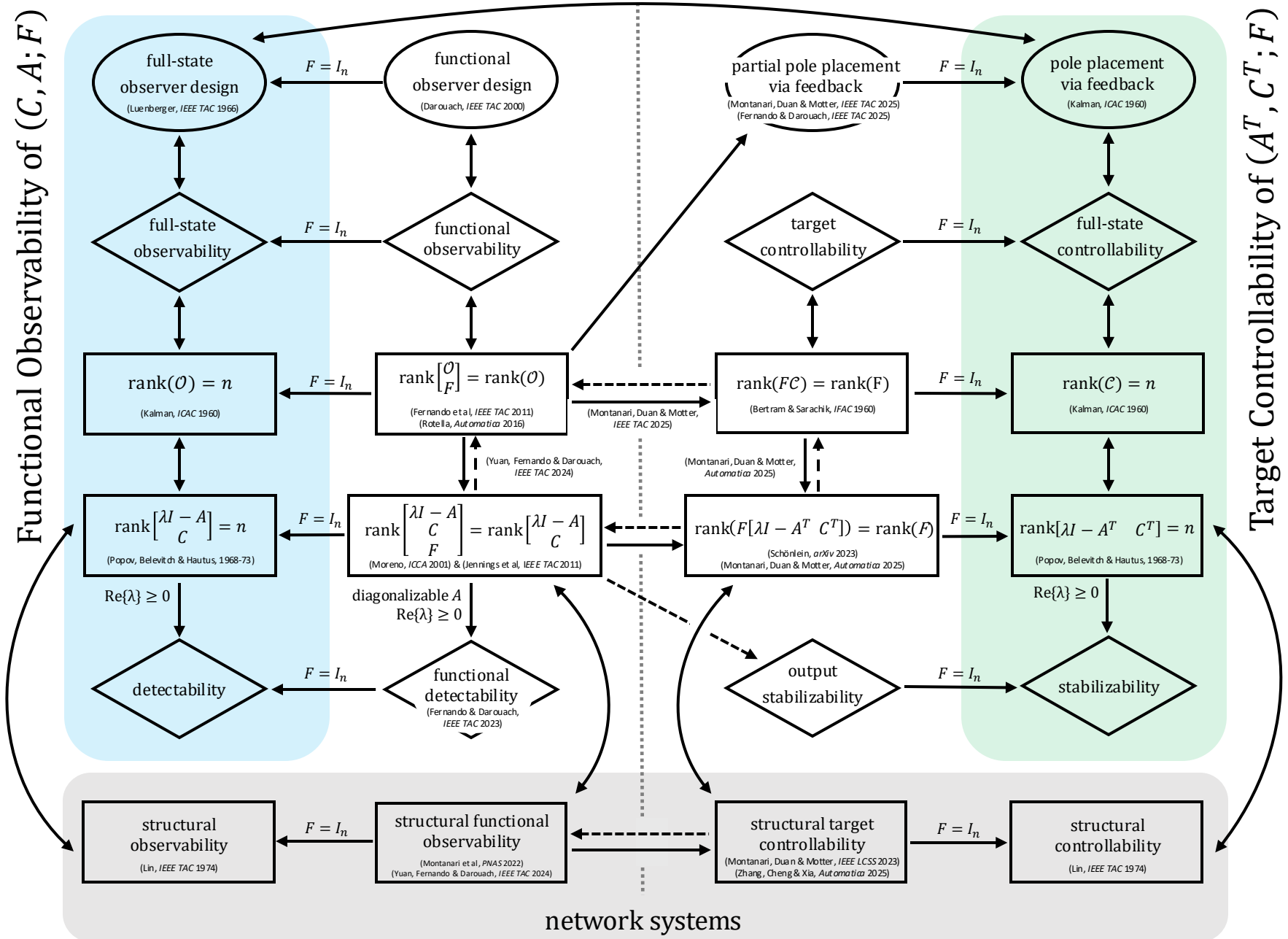
December 11, 2025

Observability of (C, A)

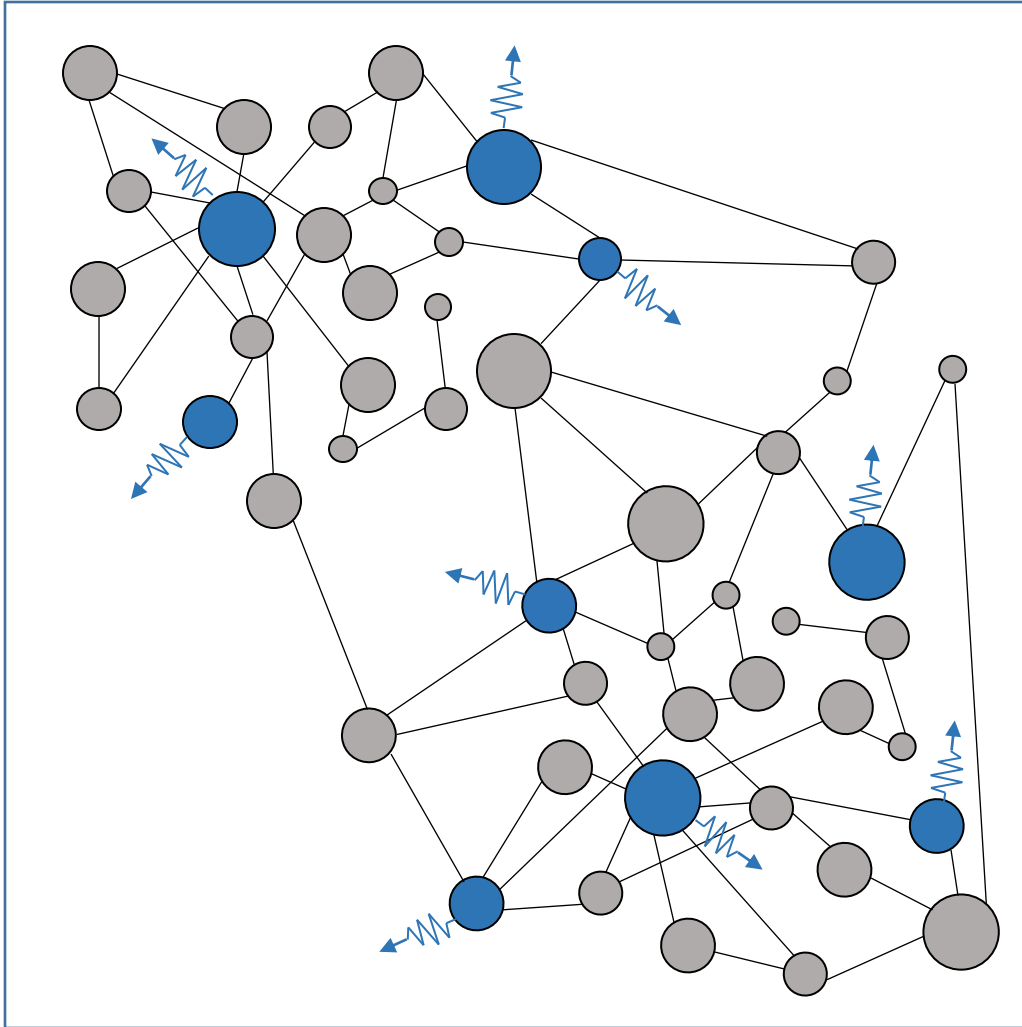


Controllability of (A^T, C^T)





Network system



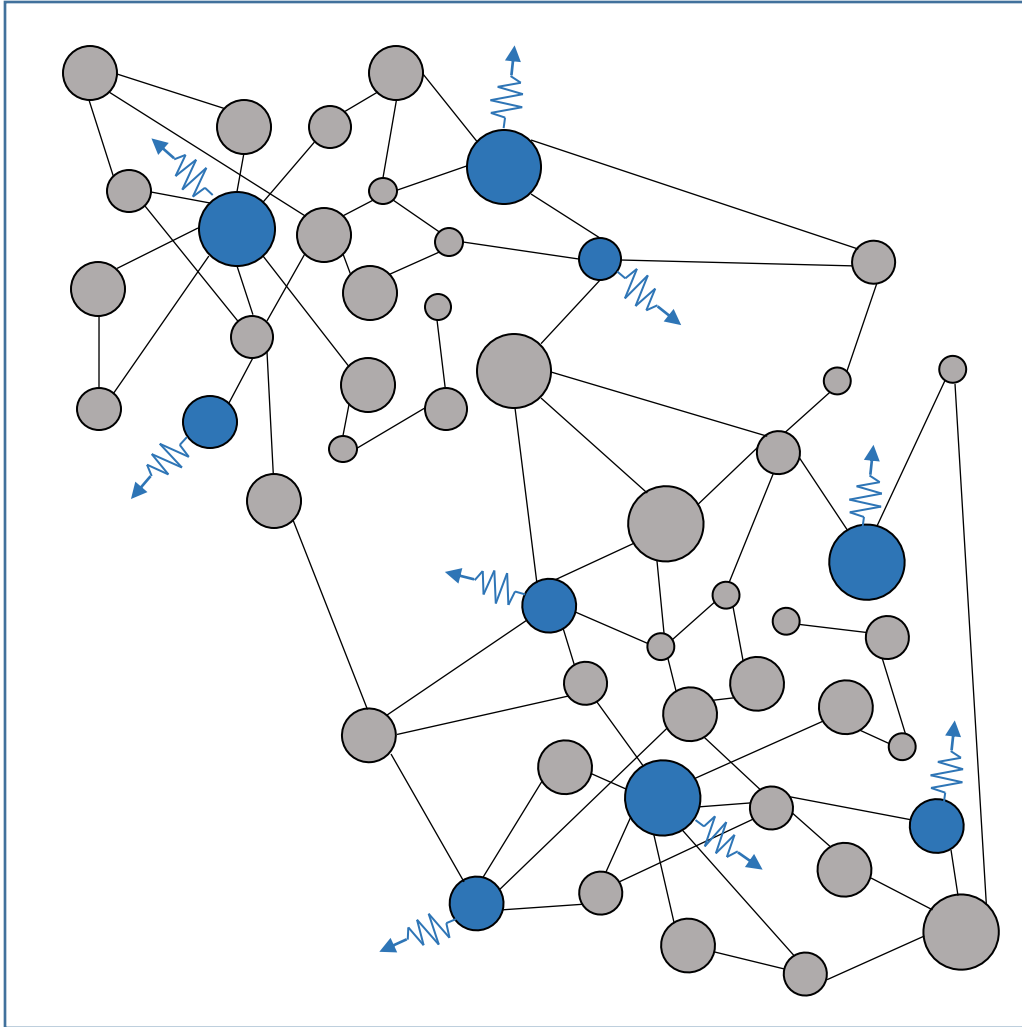
$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} = C\mathbf{x} \end{cases}$$

inference graph $\mathcal{G}(A)$
set of state variables $\mathcal{X}$
set of **sensor nodes** $\mathcal{S}$

$A_{ij} \neq 0 \rightarrow$ draw an edge from x_j to x_i

$C_{ij} \neq 0 \rightarrow$ node x_j is the i th sensor node

Network system



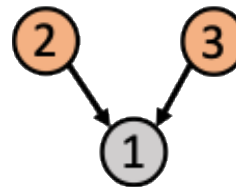
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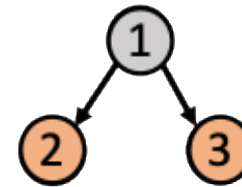
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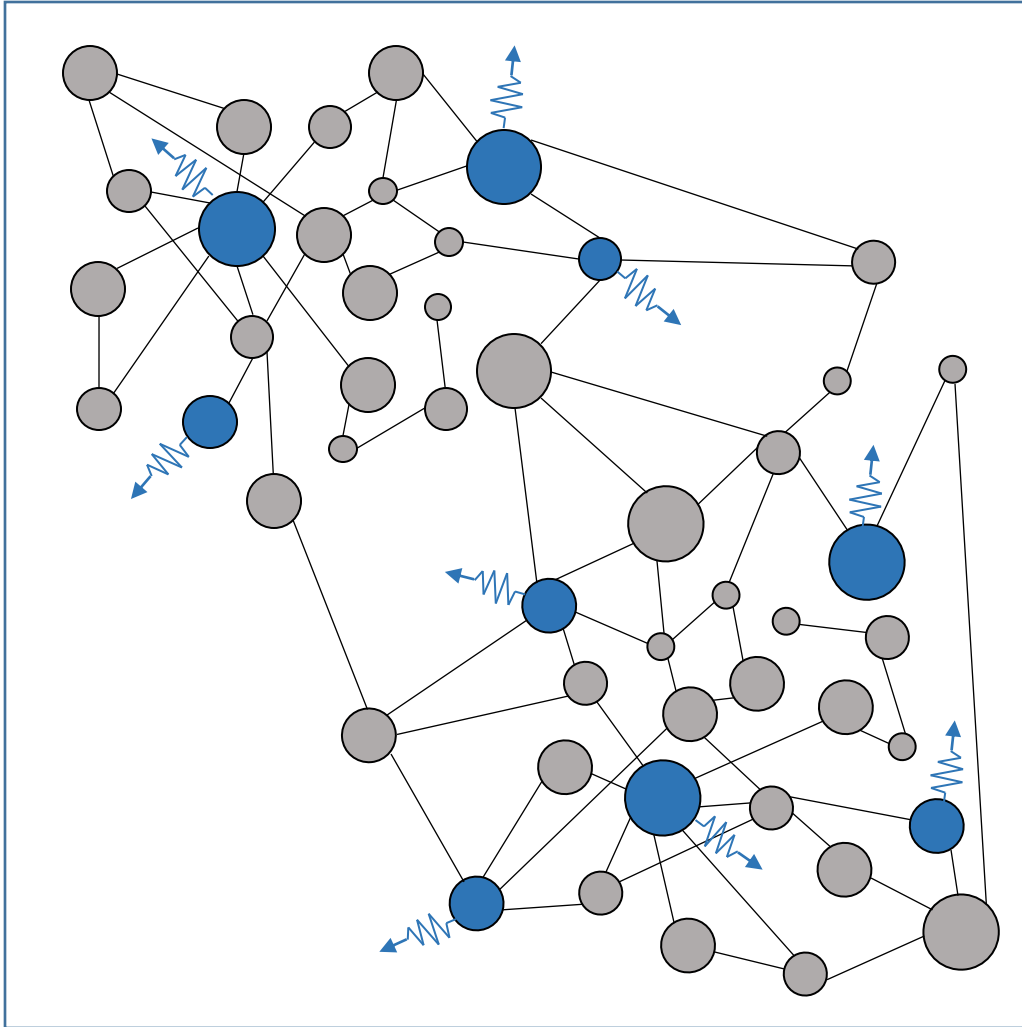
contraction



dilation



Observability of network systems



$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} = C\mathbf{x} \end{cases}$$

inference graph $\mathcal{G}(A)$
set of state variables $\mathcal{X}$
set of **sensor nodes** $\mathcal{S}$

pair (C, A) is observable

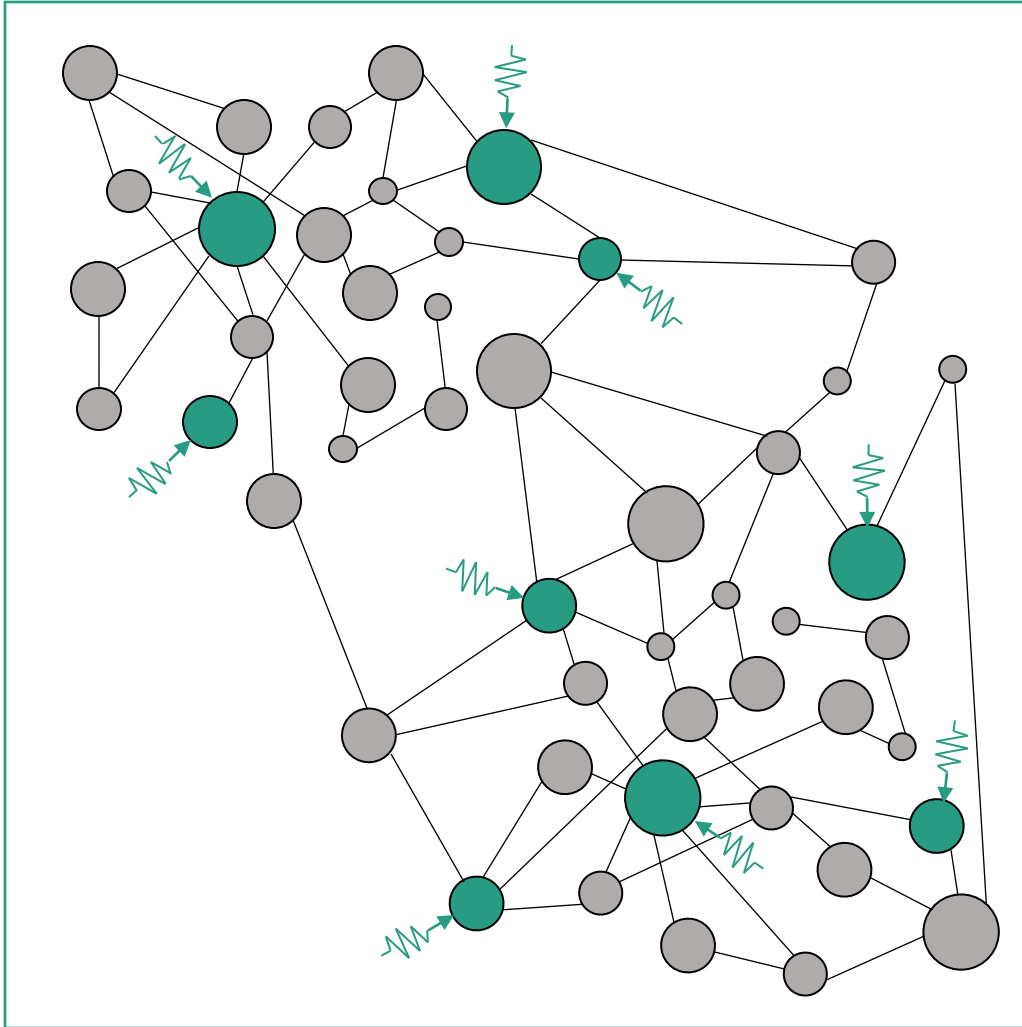
$$\mathcal{O} = \begin{bmatrix} C \\ CA \\ CA^2 \\ \vdots \\ CA^{n-1} \end{bmatrix}$$
$$\text{rank}(\mathcal{O}) = n$$

1) every node $x_i \in \mathcal{X}$ has
a path to some sensor
node $y_j \in \mathcal{S}$

2) graph has no contractions

CT Lin. Structural controllability.
IEEE Transactions on Automatic Control (1974).

Controllability of network systems



$$\begin{cases} \dot{\mathbf{x}} = A\mathbf{x} + B\mathbf{u} \\ \mathbf{y} = C\mathbf{x} \end{cases}$$

inference graph $\mathcal{G}(A)$
set of state variables $\mathcal{X}$
set of driver nodes $\mathcal{D}$

pair (A, B) is controllable iff (A^T, C^T) is observable

$$C = [B \ AB \ \dots \ A^{n-1}B]$$

$$\text{rank}(C) = n$$

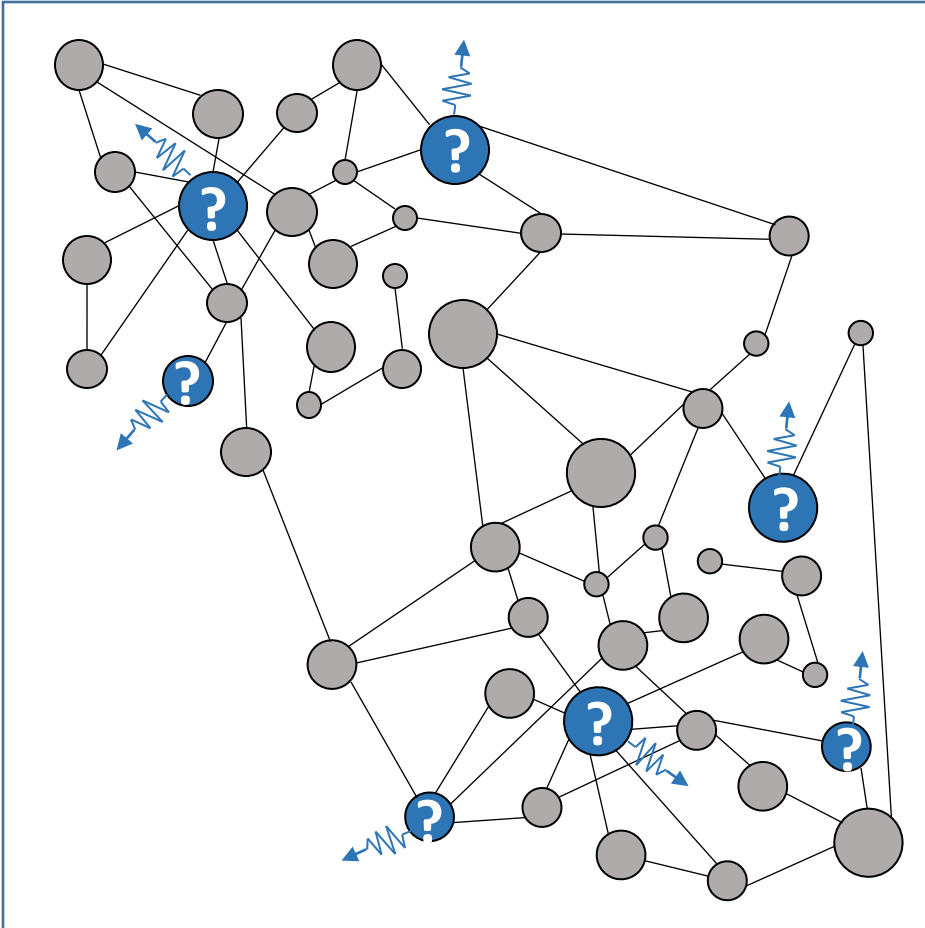
1) for every $x_i \in \mathcal{X}$, some driver node $u_j \in \mathcal{D}$ has a path to x_i

2) graph has no dilations

CT Lin. Structural controllability.
IEEE Transactions on Automatic Control (1974).

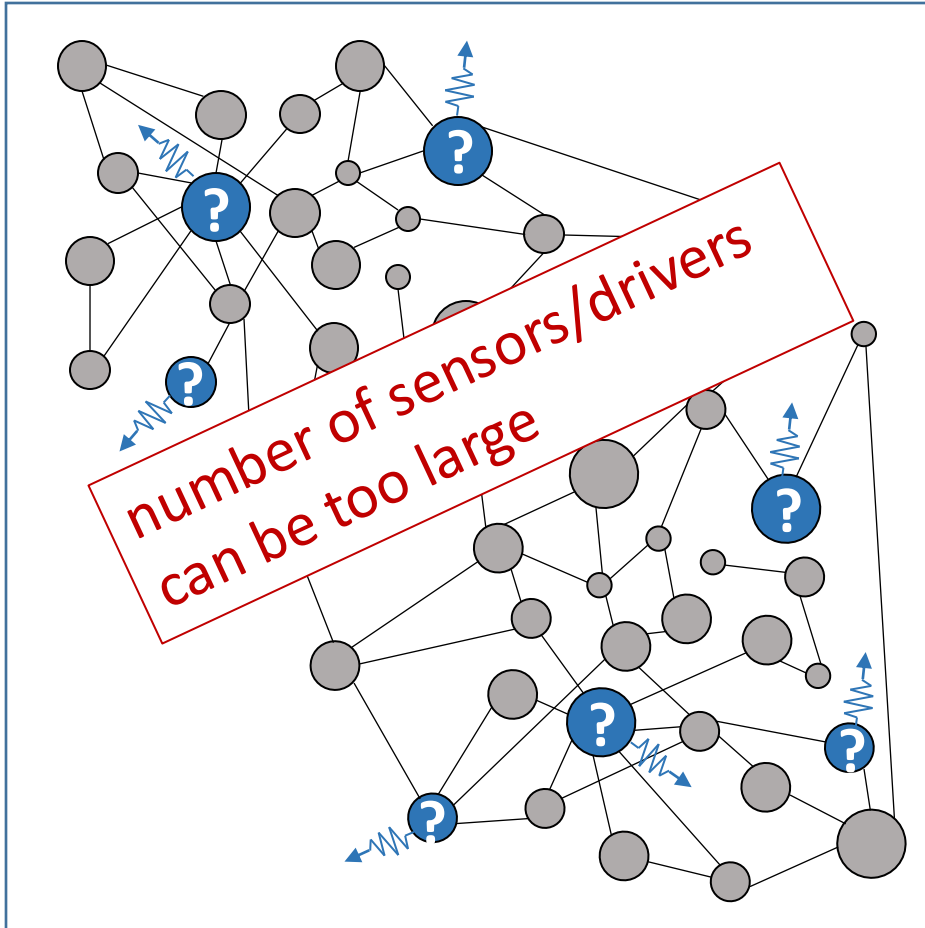
Challenges in large-scale networks

1) Minimum component placement



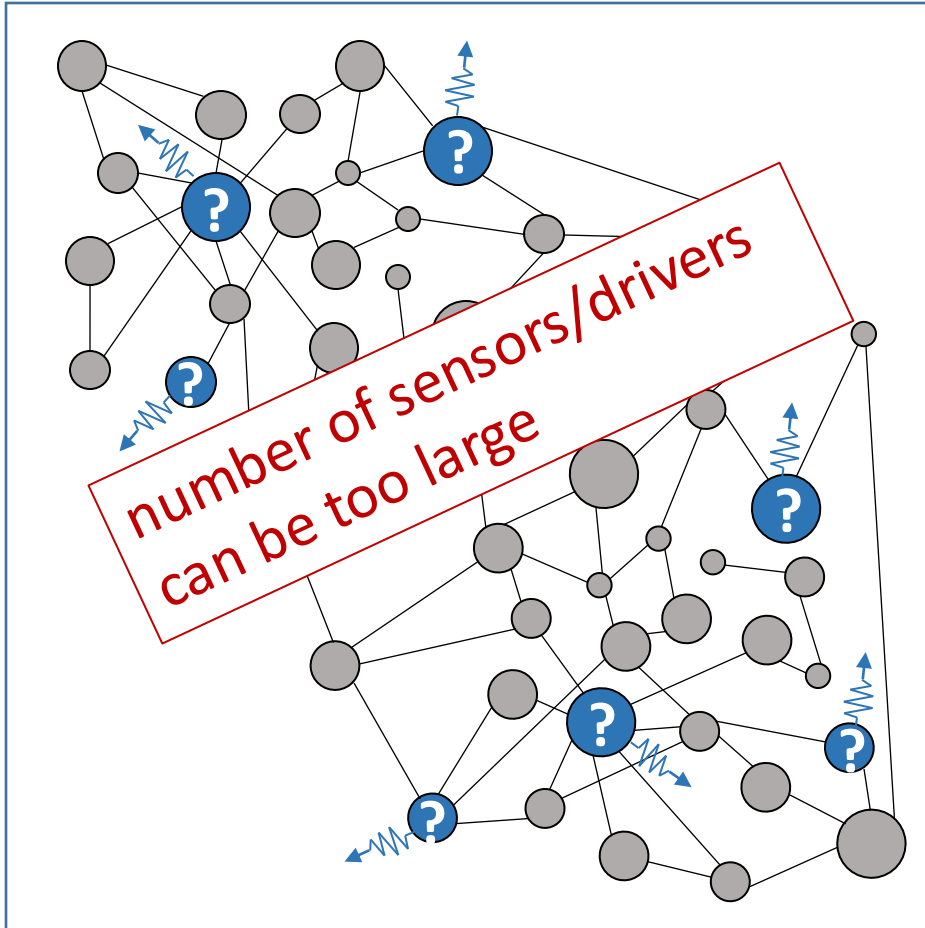
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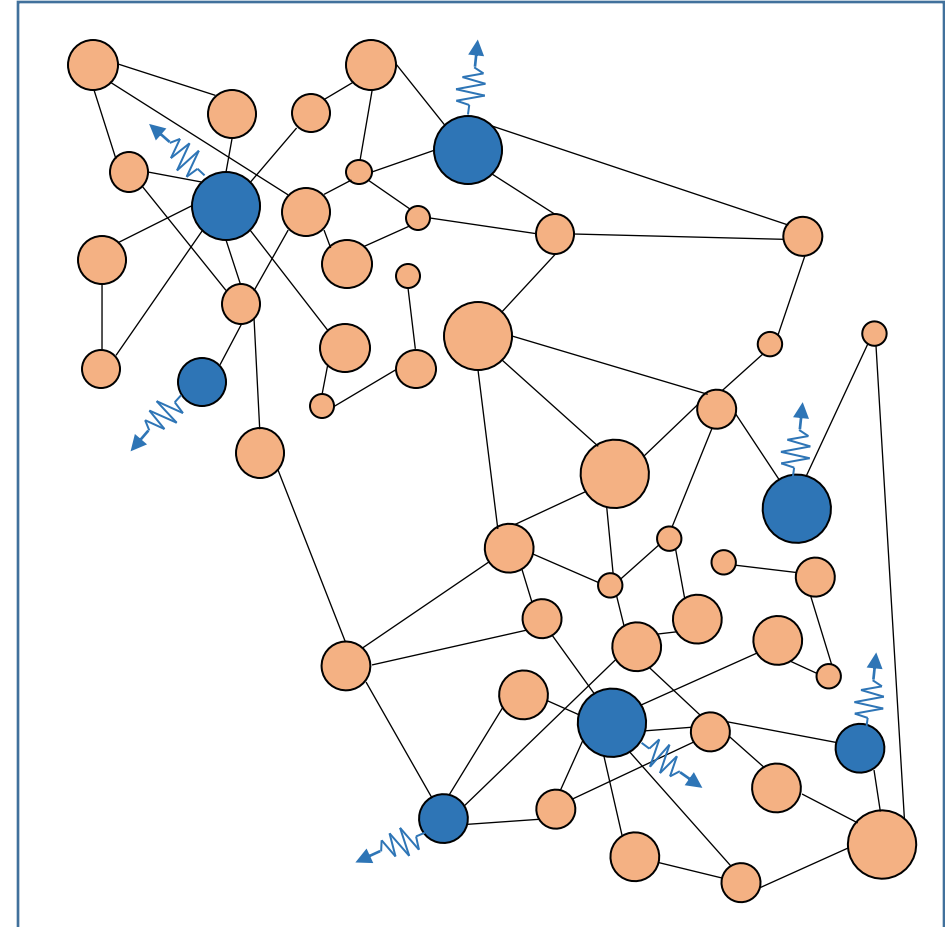


Challenges in large-scale networks

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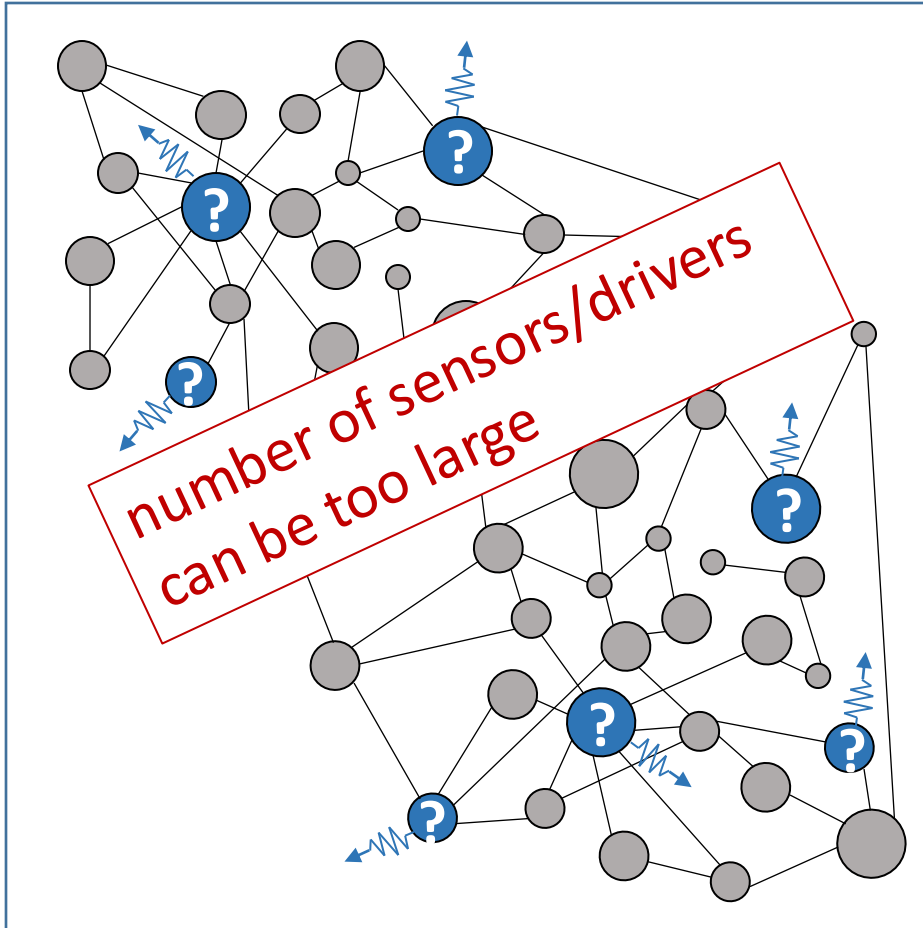


2) Observer or controller design

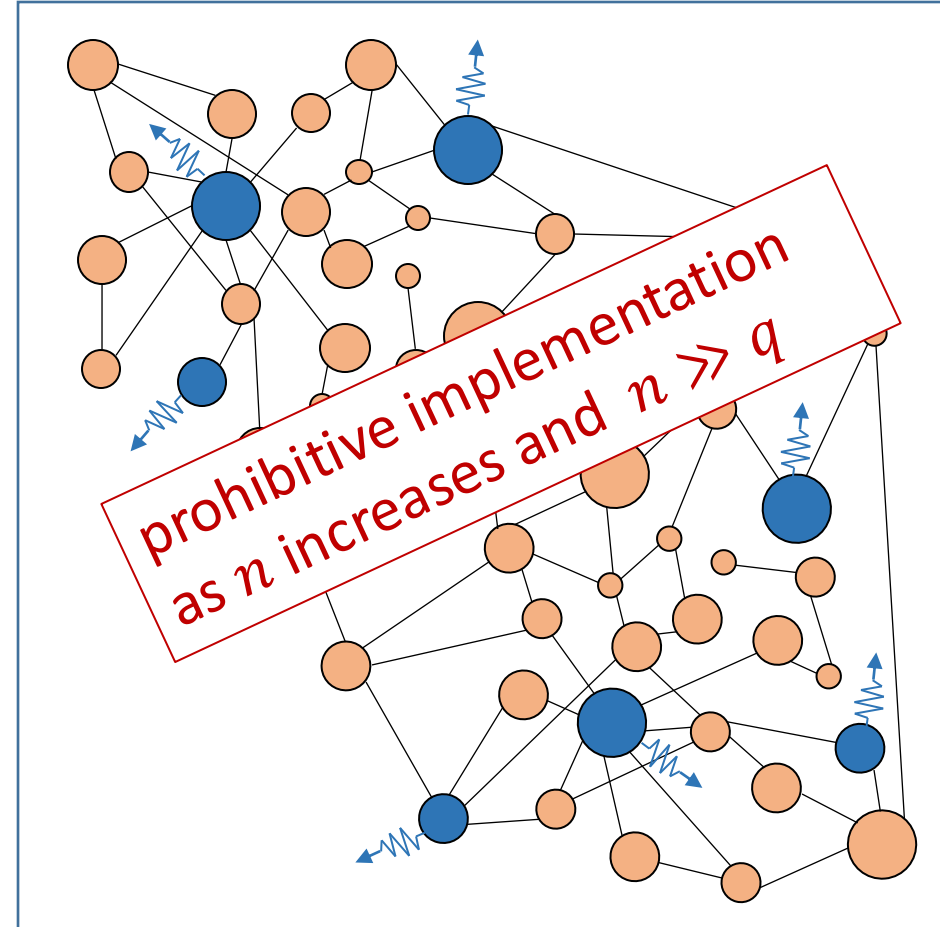


Challenges in large-scale systems

1) Minimum component placement



2) Observer or controller design





monitoring



control

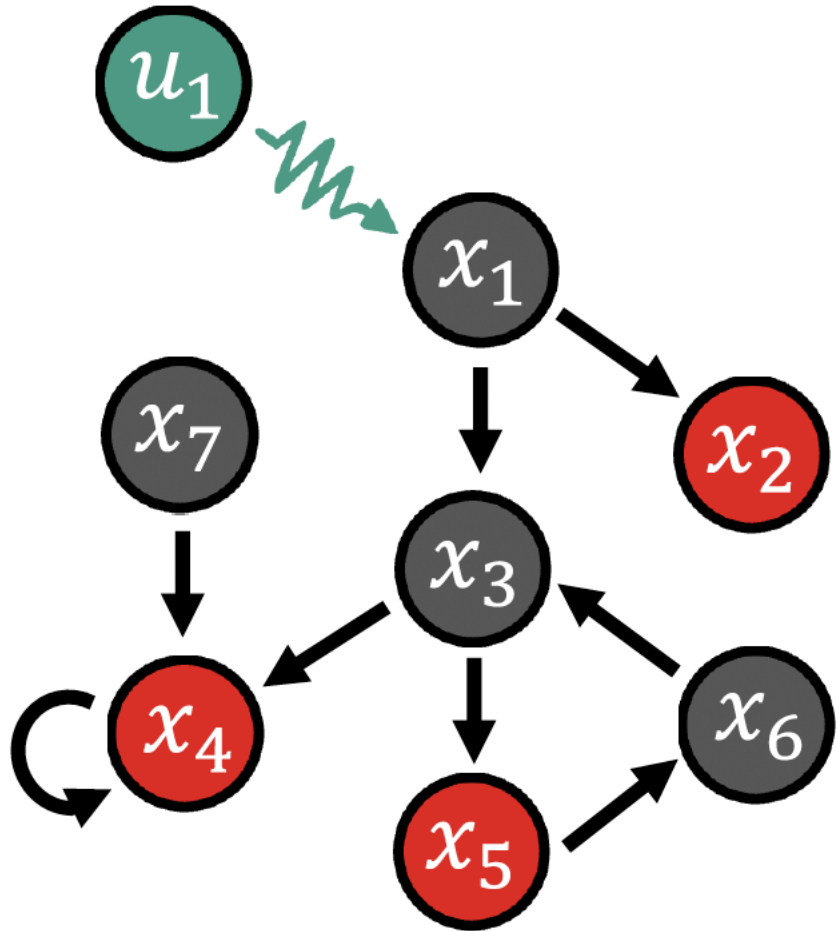


intervention

desired to **control/estimate**
only a **target** subset of state variables



Target (or output) controllability

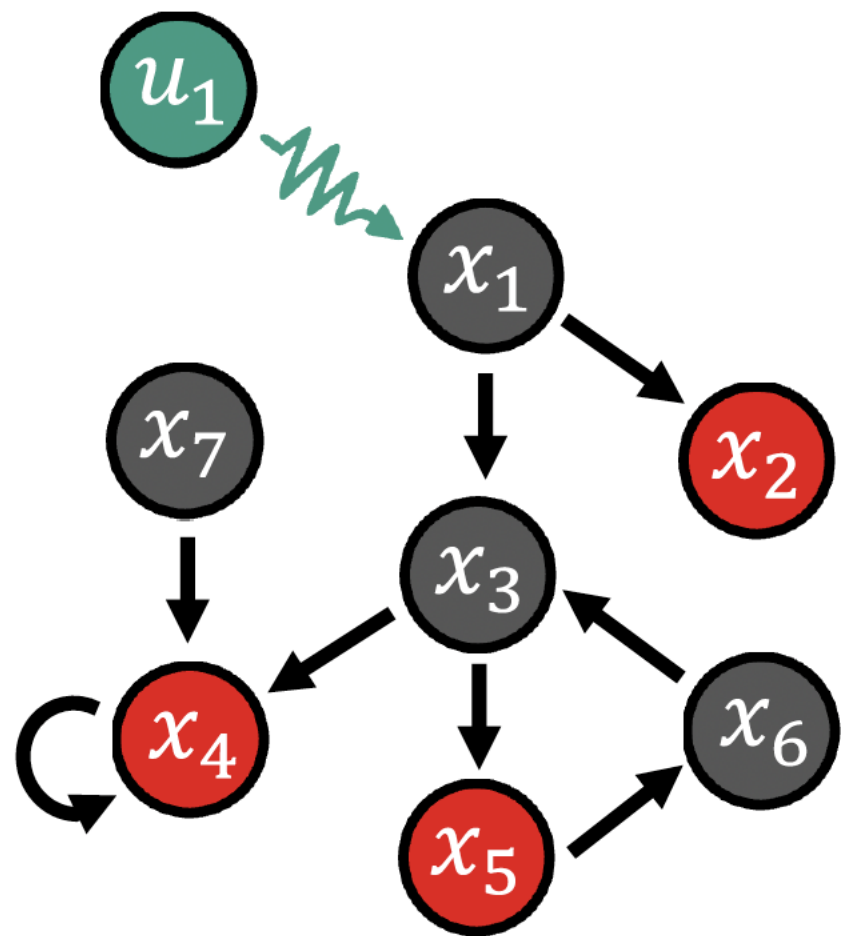


$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \\ z = Fx \end{cases}$$

inference graph $G(A)$
set of state variables $\mathcal{X}$
set of driver nodes $\mathcal{D}$
set of target nodes $\mathcal{T}$



Target (or output) controllability



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \\ z = Fx \end{cases}$$

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system $(A, B; F)$ is target controllable

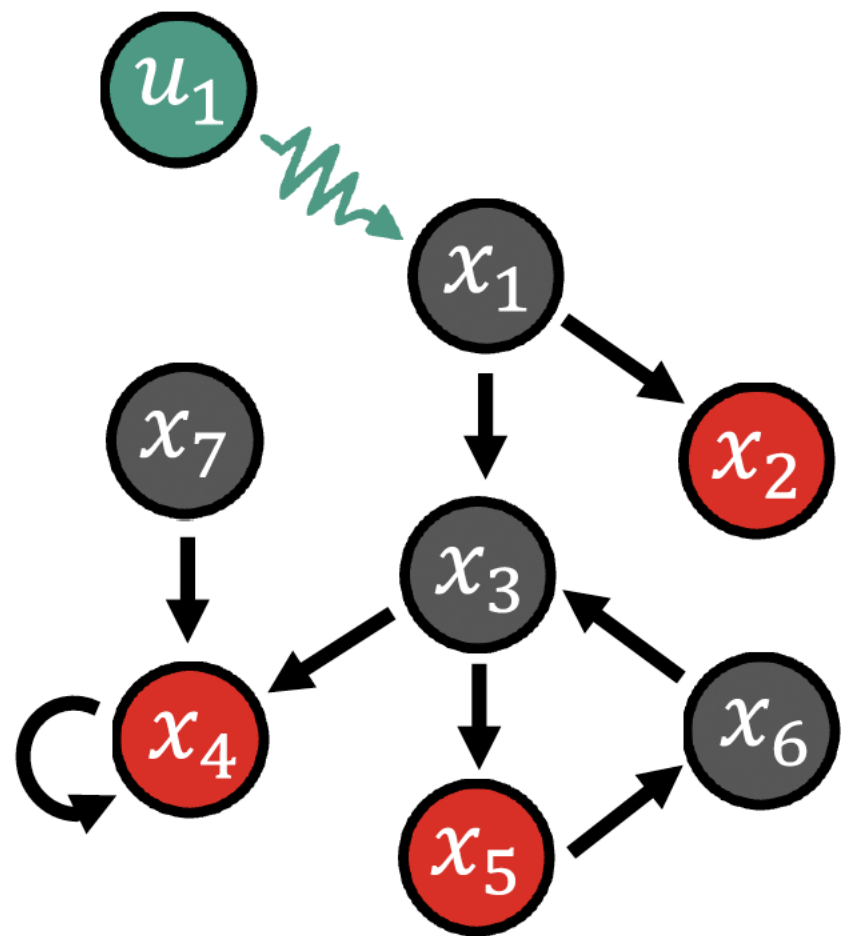
condition

$$\text{rank}(FC) = \text{rank}(F)$$

JE Bertram, PE Sarachik.
On optimal computer control.
IFAC Proceedings Volumes (1960).



Target (or output) controllability



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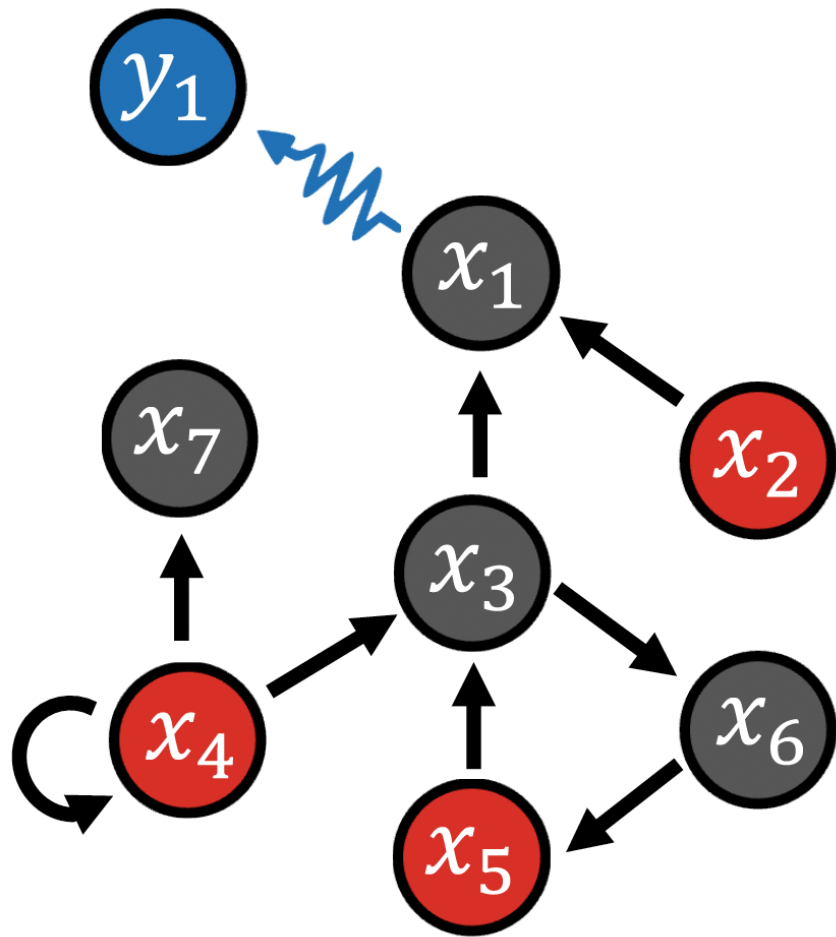
inference graph $\mathcal{G}(A)$
set of state variables $\mathcal{X}$
set of driver nodes $\mathcal{D}$
set of target nodes $\mathcal{T}$

- 1) for each target $x_i \in \mathcal{T}$,
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has a path to x_i
- 2) no subset $\mathcal{T}_\ell \subseteq \mathcal{T}$ in the
subgraph $\mathcal{G}'$ has a dilation

AN Montanari, C Duan, AE Motter.
Target controllability and target observability
of structured network systems.
IEEE Control Systems Letters (2023).



Functional observability



$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx \\ z = Fx \end{cases}$$

inference graph $\mathcal{G}(A)$
set of state variables $\mathcal{X}$
set of **sensor nodes** $\mathcal{S}$
set of **target nodes** $\mathcal{T}$

system $(C, A; F)$ is functionally observable

condition

$$\text{rank} \left(\begin{bmatrix} \mathcal{O} \\ F \end{bmatrix} \right) = \text{rank}(\mathcal{O})$$

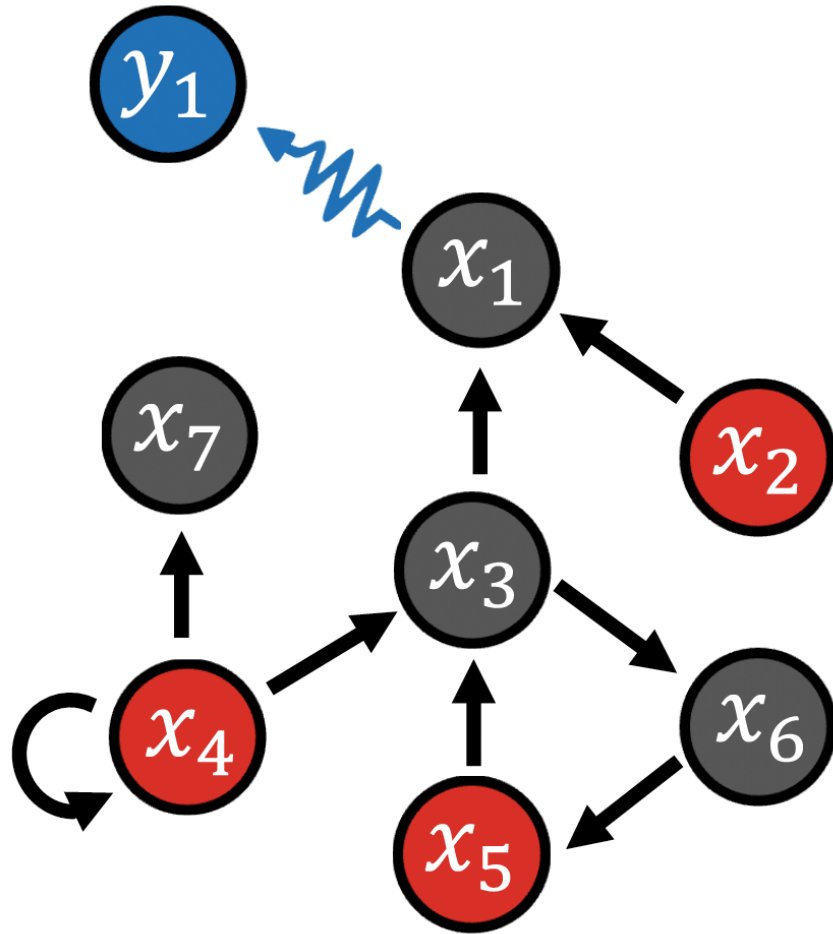
LS Jennings, TL Fernando, HM Trinh.

Existence conditions for functional observability from an eigenspace perspective.

IEEE Transactions on Automatic Control (2011).



Functional observability



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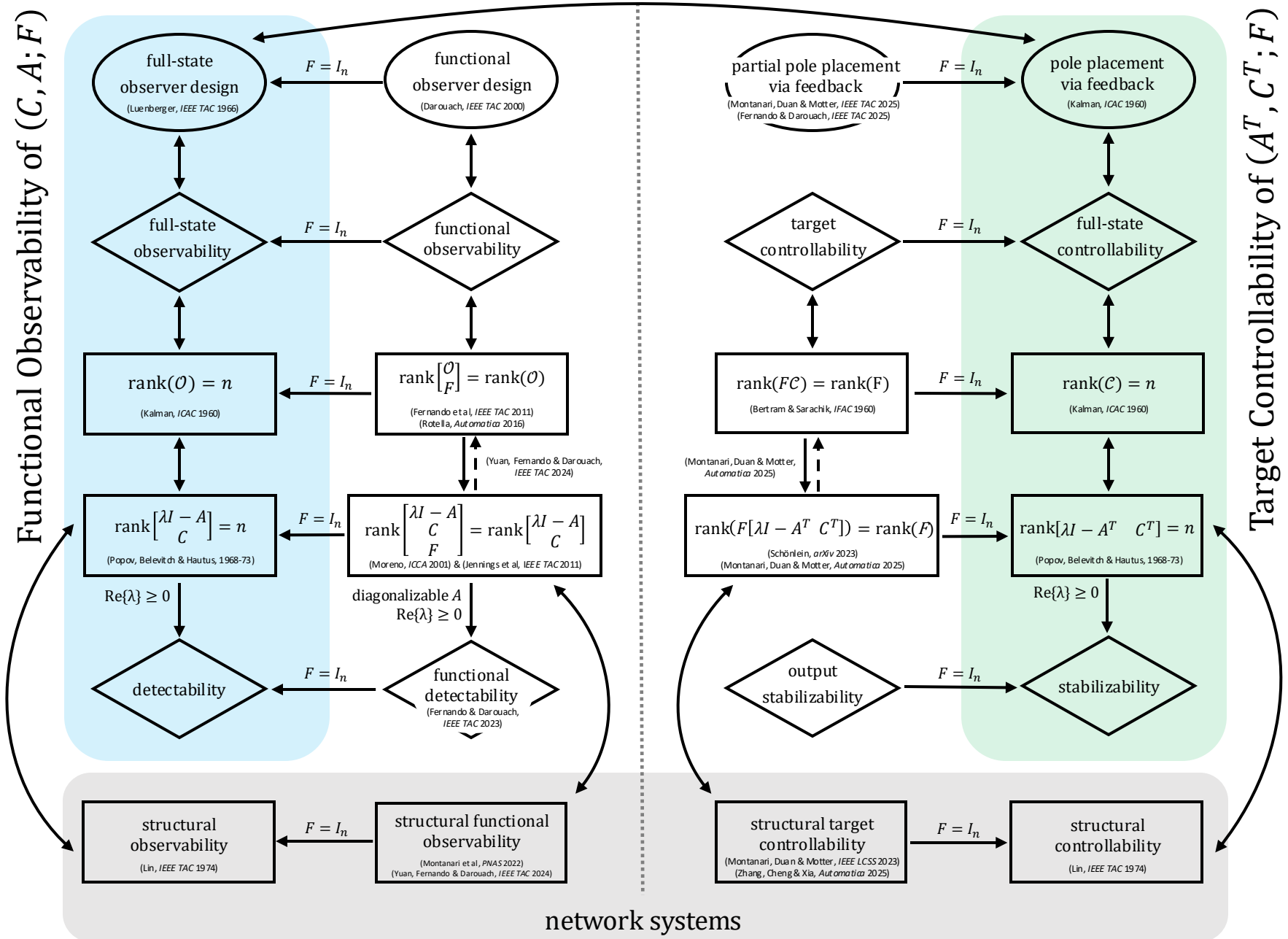
system $(C, A; F)$ is functionally observable

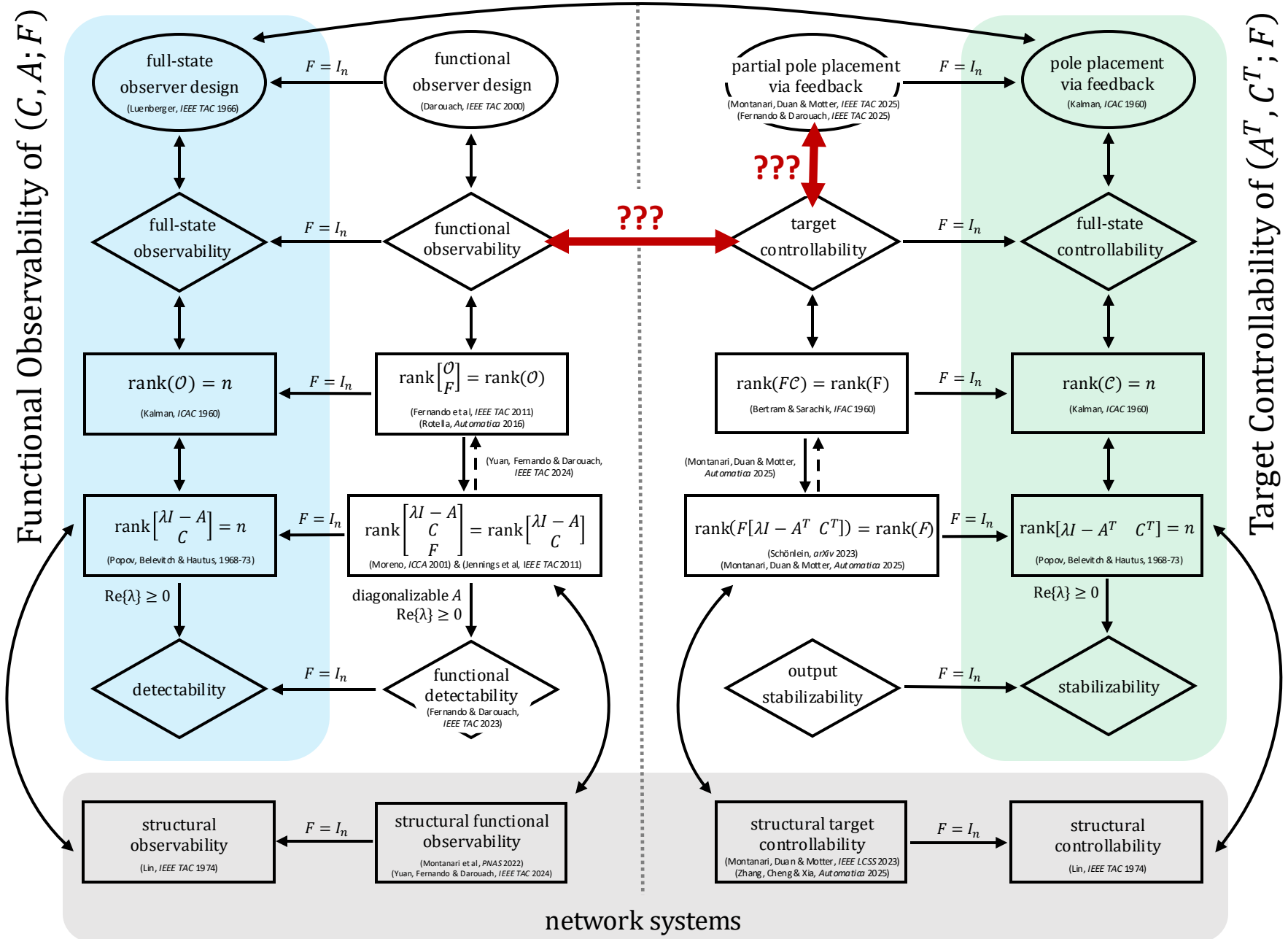
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$$\text{rank} \left(\begin{bmatrix} \mathcal{O} \\ F \end{bmatrix} \right) = \text{rank}(\mathcal{O})$$

- 1) every **target** $x_i \in \mathcal{T}$ has a path to sensor $y_j \in \mathcal{S}$
- 2) $\mathcal{T} \cap \mathcal{K} = \emptyset$, where $\mathcal{K}$ is the minimal contraction set

AN Montanari, C Duan, LA Aguirre, AE Motter.
Functional observability and target state
estimation in large-scale networks.
PNAS (2022).





Lack of duality

target controllability

$$\text{rank}(FC) = \text{rank}(F)$$

- 1) for each target $x_i \in \mathcal{T}$, some driver node $u_j \in \mathcal{D}$ has a path to x_i
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functional observability

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Generalized duality principle

Duality Principle

target

controllability

subspace of target
controllable states

$$\mathbf{z}(t_f) \in \mathcal{C}_F$$

functional

observability

subspace of functionally
observable states

$$\mathbf{z}(0) \in \mathcal{O}_F$$

AN Montanari, C Duan, AE Motter. Duality between controllability and observability for target control and estimation in networks.
IEEE Transactions on Automatic Control (2025).

Generalized duality principle

target

controllability

subspace of target
controllable states

$$\mathbf{z}(t_f) \in \mathcal{C}_F$$

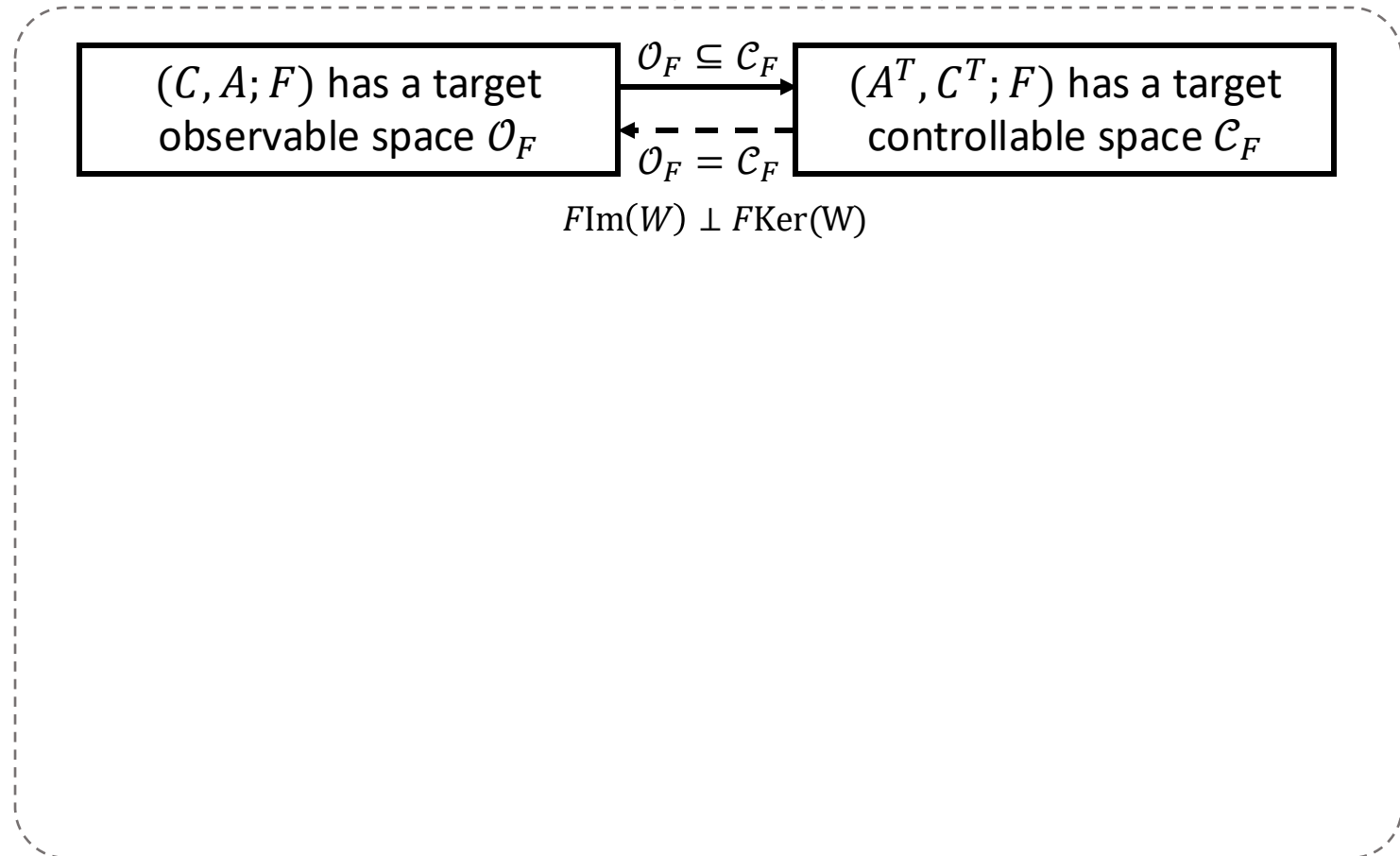
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Duality Principle



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Generalized duality principle

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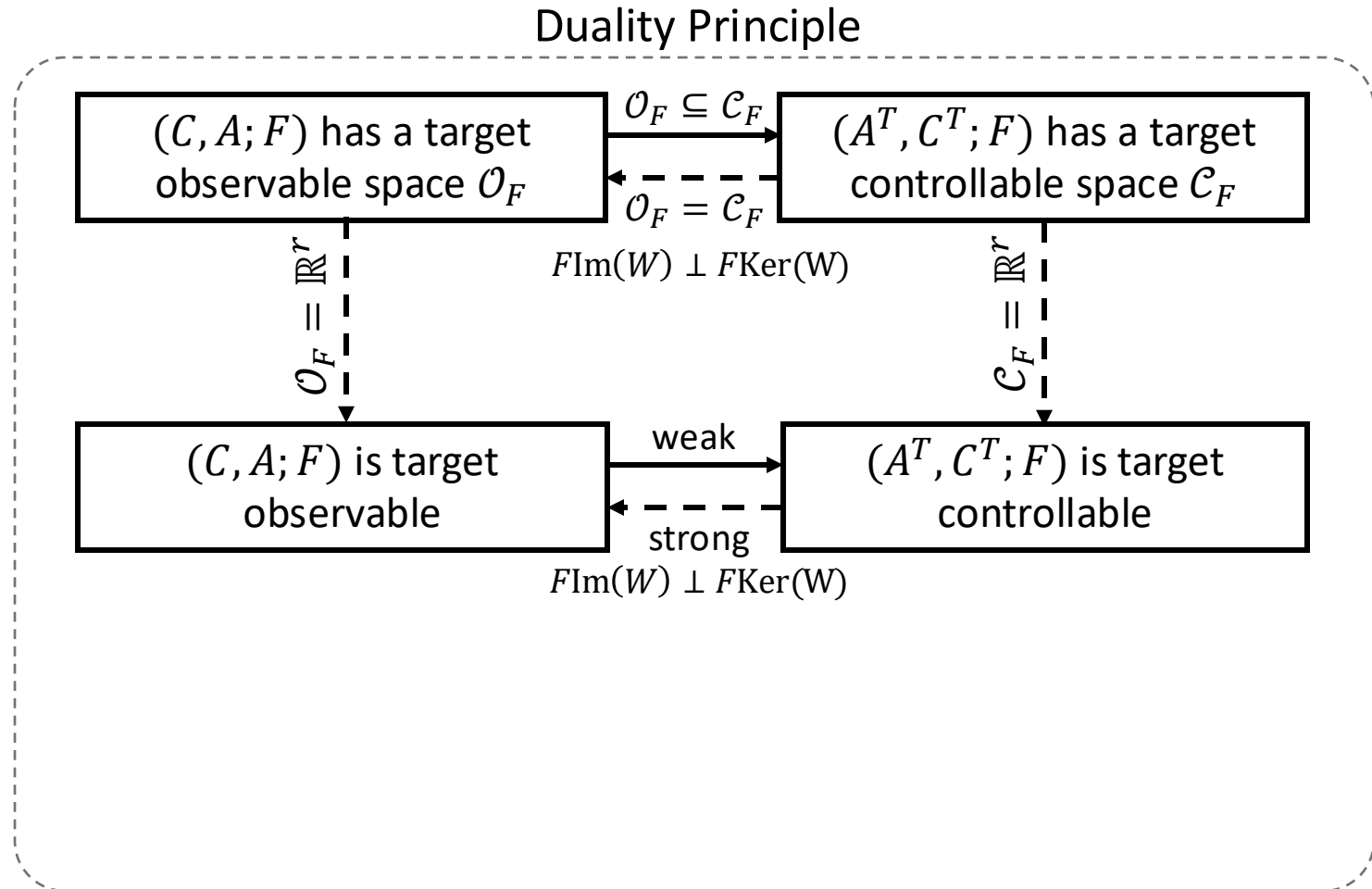
$$\mathbf{z}(t_f) \in \mathcal{C}_F$$

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AN Montanari, C Duan, AE Motter. Duality between controllability and observability for target control and estimation in networks.
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Generalized duality principle

target

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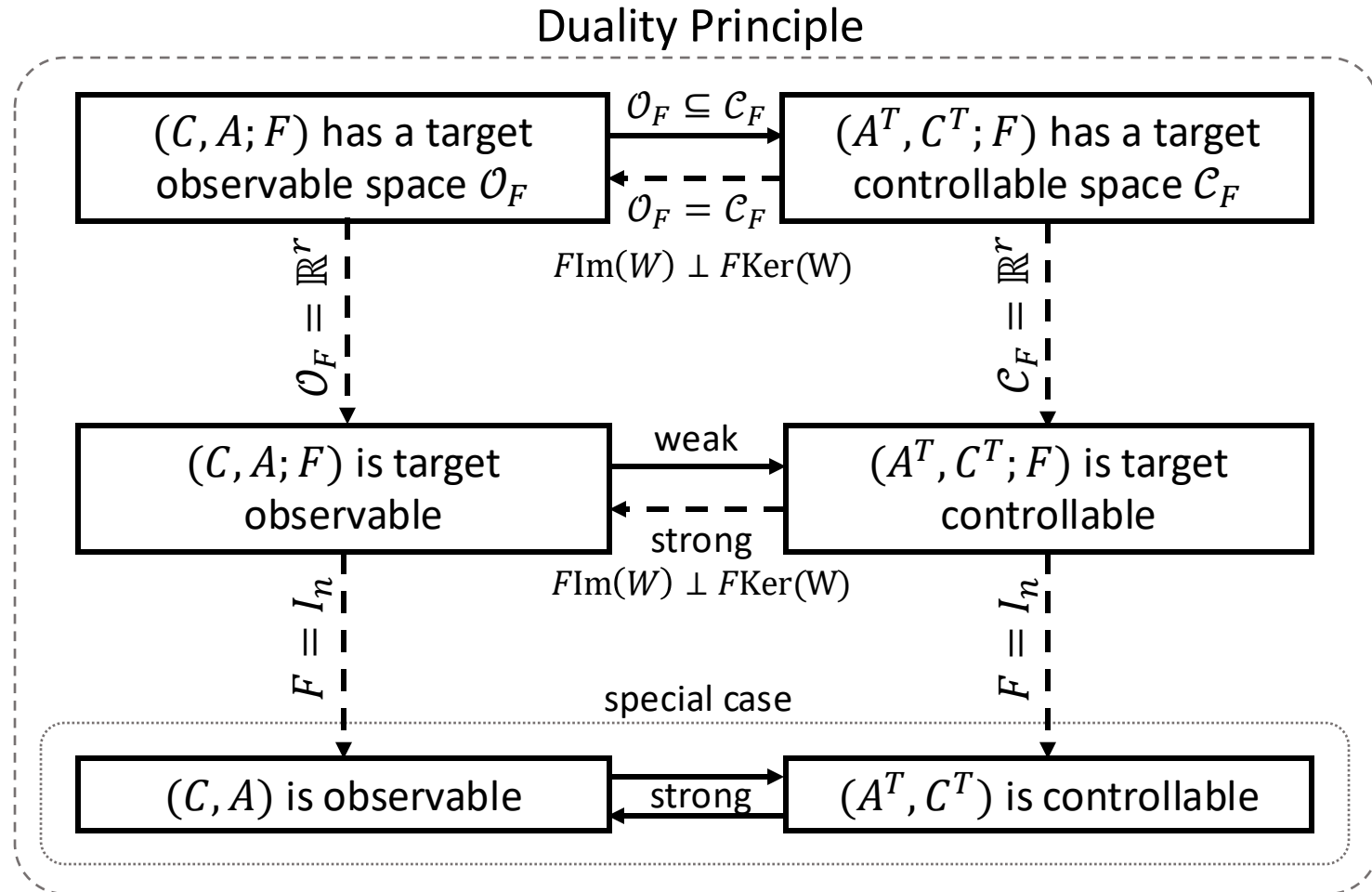
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functional

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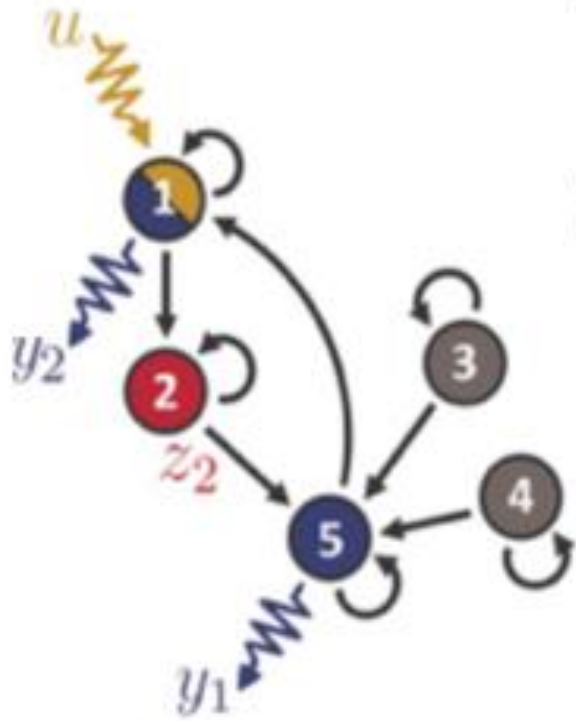
subspace of functionally
observable states

$$\mathbf{z}(0) \in \mathcal{O}_F$$



AN Montanari, C Duan, AE Motter. Duality between controllability and observability for target control and estimation in networks. *IEEE Transactions on Automatic Control* (2025).

Partial state control via feedback



$$\text{rank}[\mathcal{C}] < n$$

condition ???

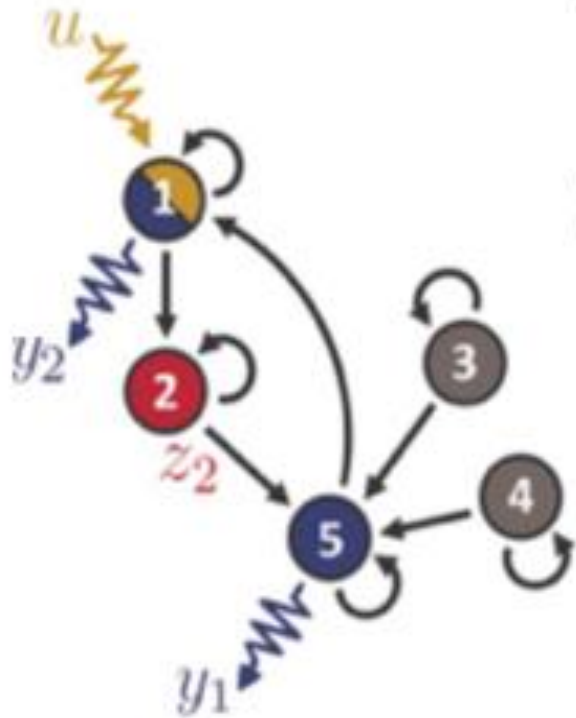
uncontrollable system
 (A, B)

functionally controllable
system $(A, B; F)$

$x(t)$ cannot be steered
via feedback $u = -Kx$

$z(t)$ can be steered
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Partial state control via feedback



$$\text{rank}[\mathcal{C}] < n$$

uncontrollable system
 (A, B)

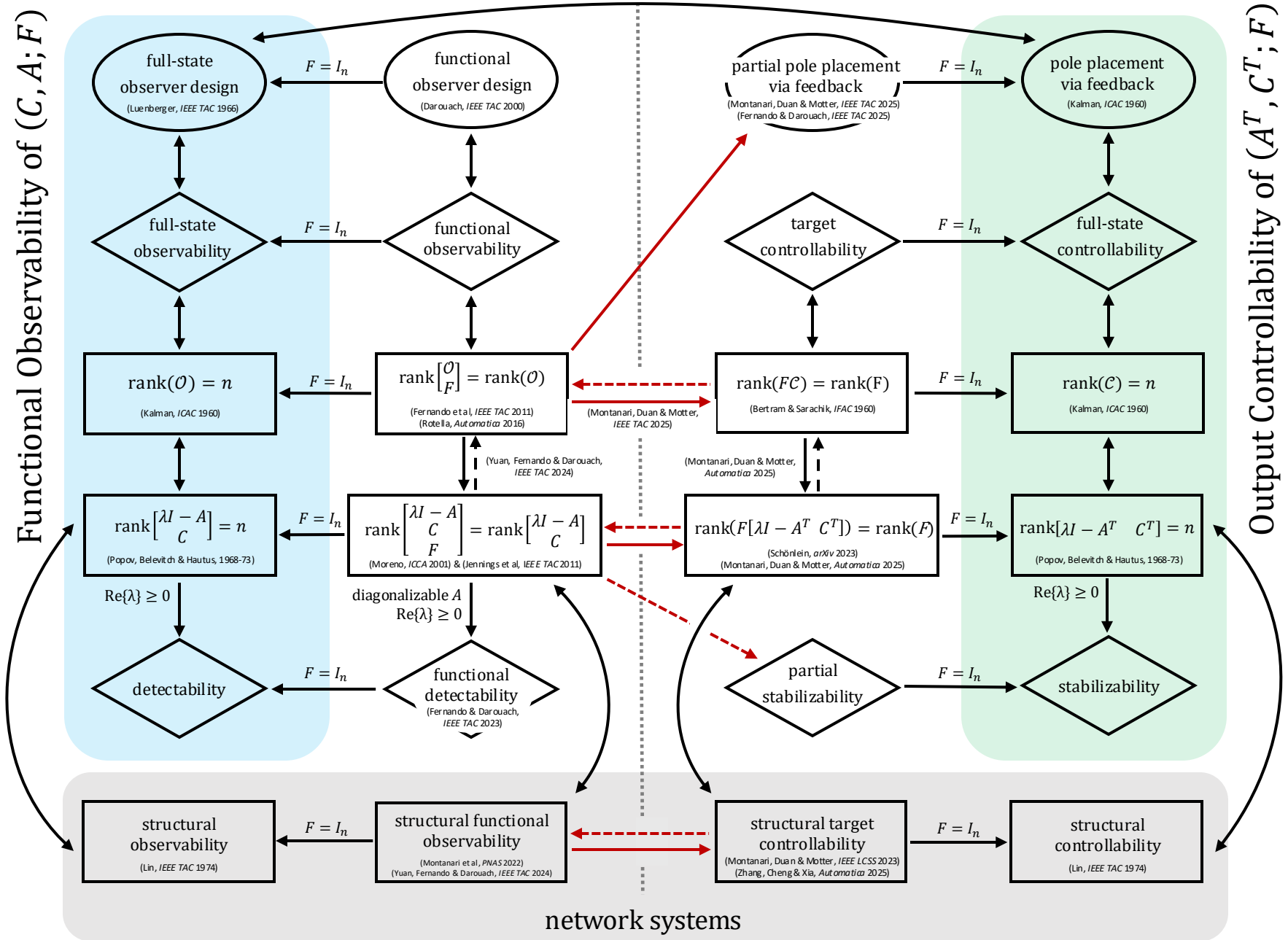
$x(t)$ cannot be steered
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$$\text{rank}[\mathcal{C} \ F] = \text{rank}[\mathcal{C}] \quad \text{functionally controllable system } (A, B; F)$$

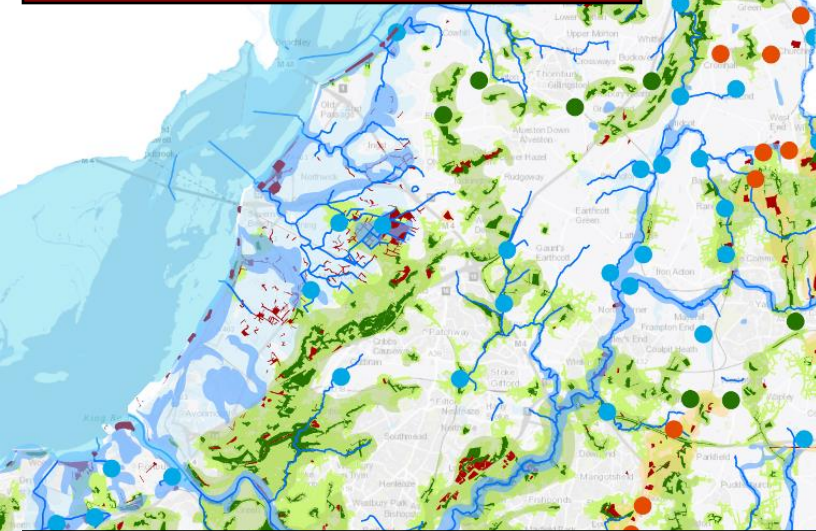
duality by
transposition

$$\text{rank} \begin{bmatrix} \mathcal{O} \\ F \end{bmatrix} = \text{rank}[\mathcal{O}] \quad \text{functionally observable dual system } (B^T, A^T; F)$$

$z(t)$ can be steered
via feedback $u = -Kx$



monitoring



Sousa, Messai, Manamanni.
Int J Crit Infrastruct Prot (2022)

MacLaren, Barzel, Masuda.
Nat Comms (2025)

Zhang, Cheng, Luo, Xia, Wang, Liu.
IEEE Trans Net Sci Eng (2025)

Kravaris. *IFAC* (2025)

data-driven



Zhang, Xu, Darouach, T
Fernando.
arXiv:2505.20750 (2025)

Zhang, Luo, Xu, Wu, Cao,
Cheng, Qin, Xia, Darouach, Li,
Tyrone Fernando.
arXiv:2512.06614 (2025).

biomedicine



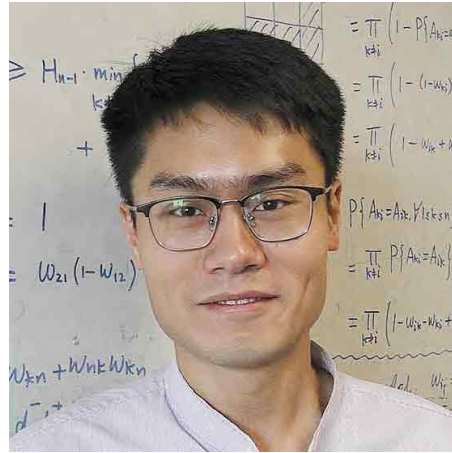
Pickard, Stansbury, Surana, Muir,
Bloch, Rajapakse.
PNAS (2025)

ANM, Freitas, Proverbio,
Gonçalves.
Phys Rev Res (2022)

Acknowledgements



Adilson Motter



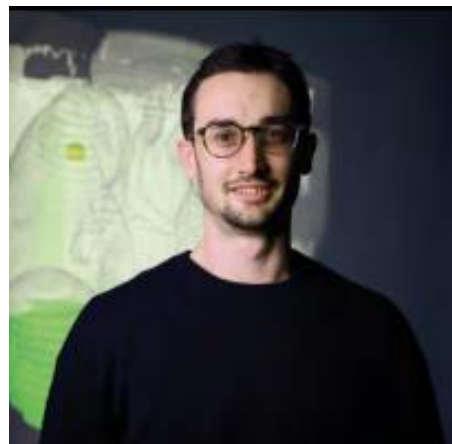
Chao Duan



Luis Aguirre



Jorge Gonçalves



Daniele Proverbio



Leandro Freitas

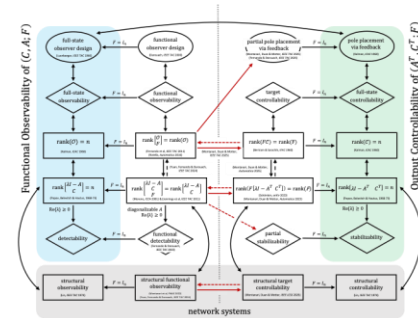




Thank you!

For slides and diagram on controllability-observability relation, see my webpage:

www.montanariarthur.com



On the duality: AN Montanari, C Duan, AE Motter. *IEEE Transactions on Automatic Control*, 70:5584-5591 (2025).

AN Montanari, C Duan, AE Motter. *Automatica*, 174:112122 (2025).

On network systems & graph theory: AN Montanari, C Duan, LA Aguirre, AE Motter. *PNAS*, 119:e2113750119 (2022).

AN Montanari, C Duan, AE Motter. *IEEE Control Systems Letters*, 7:3060-3065 (2023).

On nonlinear systems: AN Montanari, L Freitas, D Proverbio, J Gonçalves. *Physical Review Research*, 4:043195 (2022).