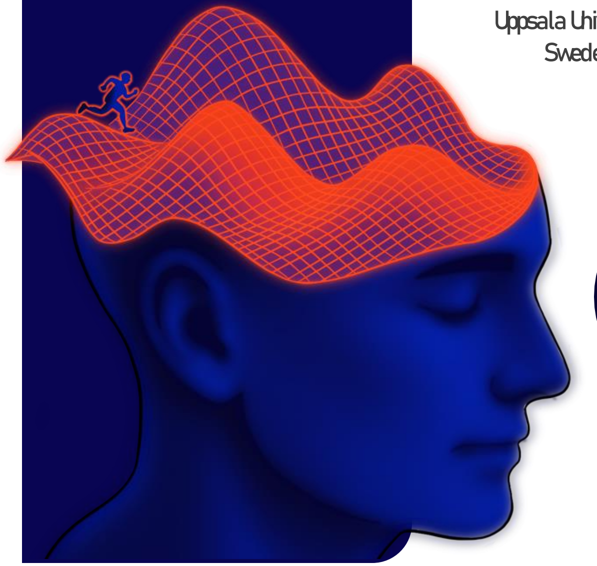
Neurocomputation & Dynamics

2025 IEEE CDC Workshop

December 9th, 2025
Rio de Janeiro, Brazil



Invited Speakers

Ahmed Allibhoy
University of California, Irvine, USA

Dmitri Chklovskii
Flatiron Institute, USA

Jorge Cortés
University of California, San Diego, USA

Leo Kozachkov
Brown University, USA

Enrique Mallada
Johns Hopkins University, USA

Antônio Ribeiro
Uppsala University, Sweden

Giovanni Russo
University of Salerno, Italy

Rodolphe Sepulchre
KU Leuven, Belgium

Ermin Wei
Northwestern University, USA

Sandro Zampieri
University of Padova, Italy

Organizers

Francesco Bullo
University of California, Santa Barbara, USA

Arthur Montanari
Northwestern University, USA

Adilson Motter
Northwestern University, USA

See program here

https://cnd.northwestern.edu/neurocomputation-workshop-2025/

Lightning Speakers

Ian Belaustegui
Princeton University, USA

Max Emerick
University of California, Santa Barbara, USA

Arthur de Oliveira
Northeastern University, USA

Daniele Proverbio
University of Trento, Italy

Arthur Rodrigues
Federal University of Minas Gerais, Brazil

Uros Sutulovic
University of Trento, Italy

Program

	Time	Speaker	Title
	08:30 – 08:40	**Welcome and opening remarks**	
Associative models for computation	08:40 - 09:00	Arthur Montanari	Introduction on dynamical models for neurocomputation
	09:00 - 09:30	Rodolphe Sepulchre	Neuromorphic control: shaping the memory of a machine
	09:30 - 10:00	Sandro Zampieri	Asymmetric Hopfield networks for dynamic memory models
	10:00 - 10:30	**Coffee break**	
Neural networks & dynamics	10:30 - 11:00	Francesco Bullo	Perspectives on biologically plausible optimization
	11:00 - 11:30	Giovanni Russo	Neural policy composition from free energy minimization
	11:30 - 12:00	Leo Kozachkov	"Forget the past, fast": A simple principle for robust, input-driven multi-area brain computation
	12:00 - 14:00	**Lunch break**	
Unconventional computing	14:00 - 14:30	Ahmed Allibhoy	Computation with coupled oscillator networks
	14:30 - 15:00	Ermin Wei	Extended-variable probabilistic computing with p-dits
	15:00 - 15:30	**Lightning talks**	
	15:30 - 16:00	**Coffee break**	
Neuro-inspired & data-driven control	16:00 - 16:30	Dmitri Chklovskii	Reconceptualizing neurons as dynamical controllers
	16:30 - 17:00	Enrique Mallada	Nonparametric analysis and control of dynamical systems: Stability, safety and policy improvement
	17:00 - 17:30	Jorge Cortés	Dimensionality control in hierarchical brain networks
	17:30 - 18:00	**Open discussion and closing remarks**	

Lightning Talks

Each lightning talk will consist of 4 min presentations plus 1 minute for questions. The abstracts and list of authors are included at the end of the document.

Speaker	Title
Arthur de Oliveira	Remarks on the Polyak-Lojasiewicz inequality and the convergence of gradient systems
Uros Sutulovic	gPC-based robustness analysis of neural systems through probabilistic recurrence metrics
Max Emerick	Classification of limit solutions of a mean-field oscillator Ising model
Arthur Rodrigues	Recurrent and multi-modular computational model for conscious cognition: An integrative approach across major theories of human consciousness
Daniele Proverbio	Efficient and faithful reconstruction of dynamical attractors using homogeneous differentiators
Ian Xul Belaustegui	Tunable thresholds and frequency encoding in a spiking NOD controller

An introduction on dynamical models for neurocomputation

IEEE Conference on Decision and Control 2025

Arthur Montanari

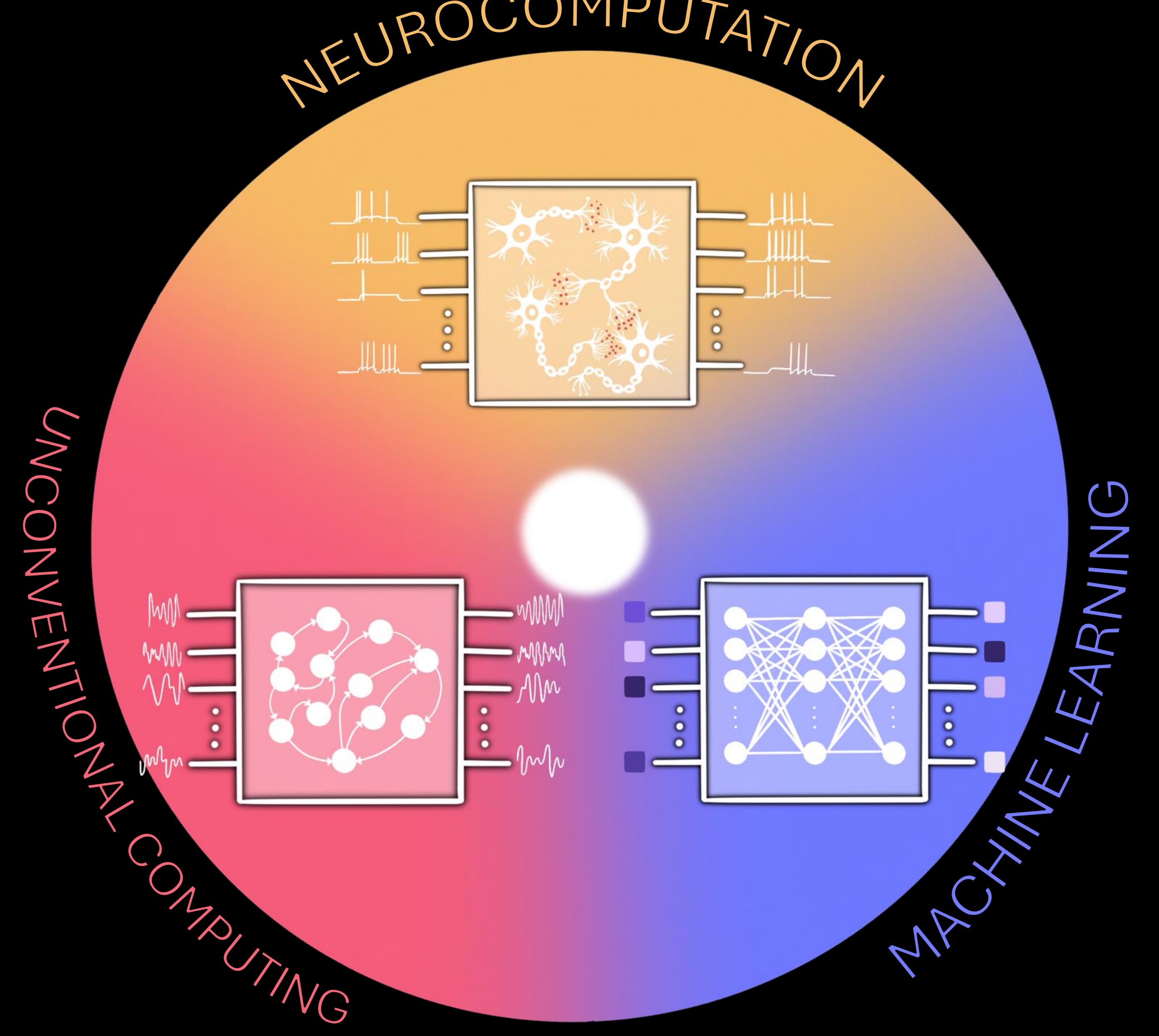
Center for Network Dynamics
Department of Physics and Astronomy
Northwestern University

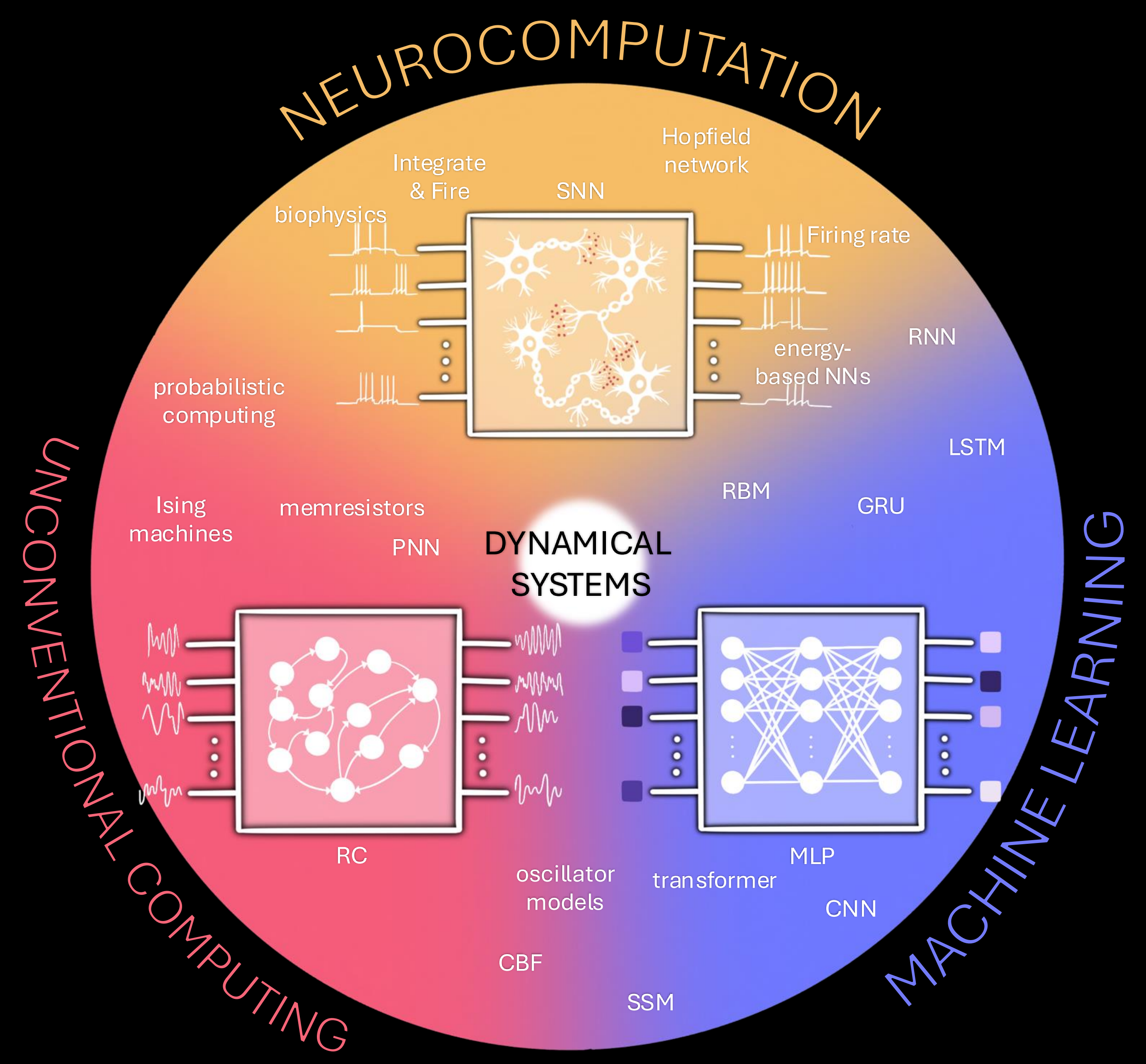
December 9, 2025

Where to start?

$$\begin{cases} \dot{x} = Ax + Bu \\ y = Cx + Du \end{cases} \quad \rightarrow \quad \begin{cases} \dot{x} = f(x, u) \\ y = h(x, u) \end{cases}$$

> stability
> controllability & observability
> feedback control & state estimation
> optimal control & filtering


NEUROCOMPUTATION
UNCONVENTIONAL COMPUTING
MACHINE LEARNING


NEUROCOMPUTATION
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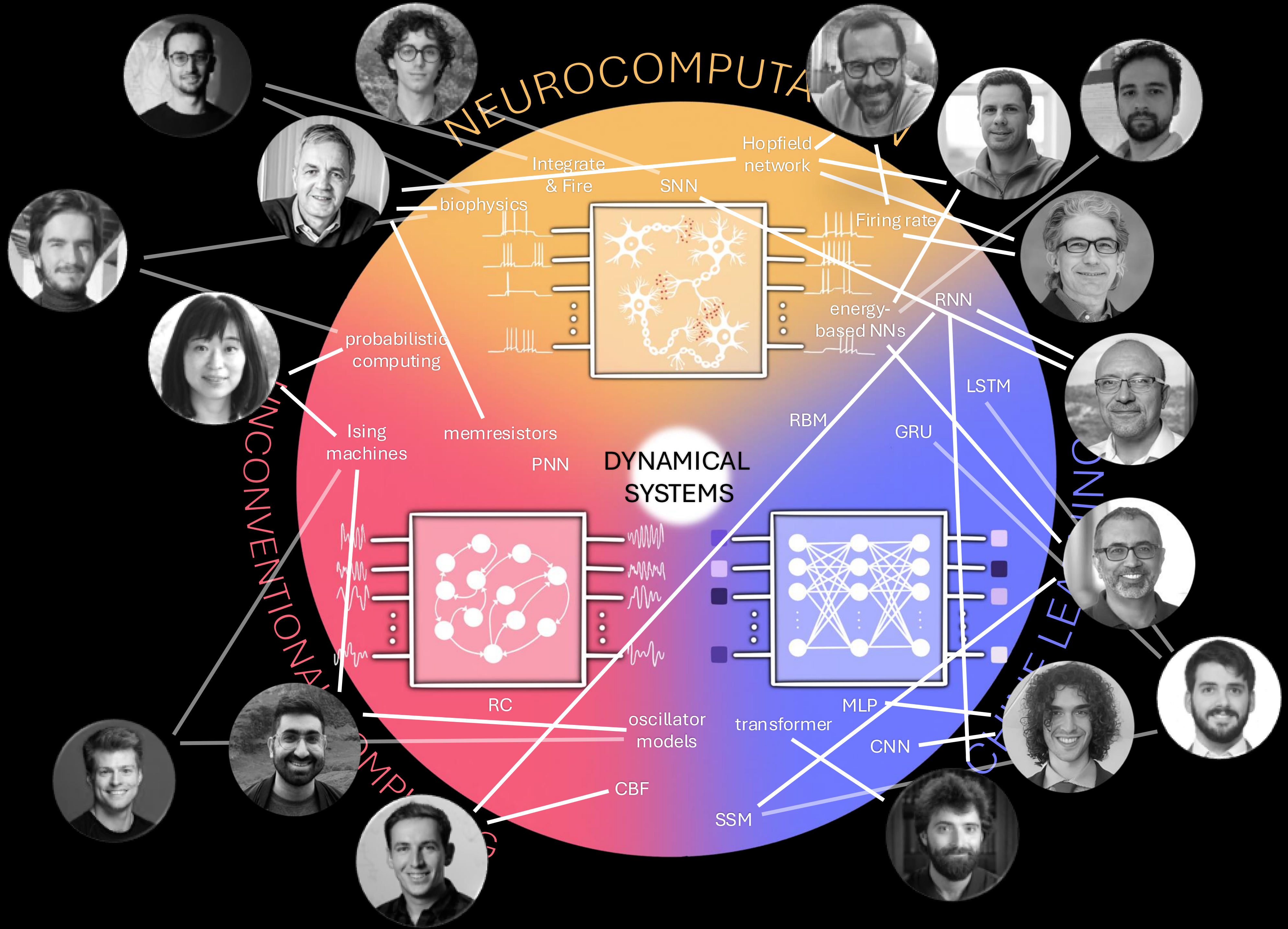
MACHINE LEARNING
DYNAMICAL SYSTEMS
Integrate & Fire
Hopfield network
SNN
biophysics
Firing rate
RNN
energy-based NNs
probabilistic computing
LSTM
RBM
GRU
Ising machines
memresistors
PNN
RC
MLP
oscillator models
transformer
CNN
CBF
SSM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Free Energy Principle

REVIEWS

The free-energy principle: a unified brain theory?

Karl Friston

environment

internal model

$p(h|s)$

what are the (hidden) causes h that explain the sensory signals s?

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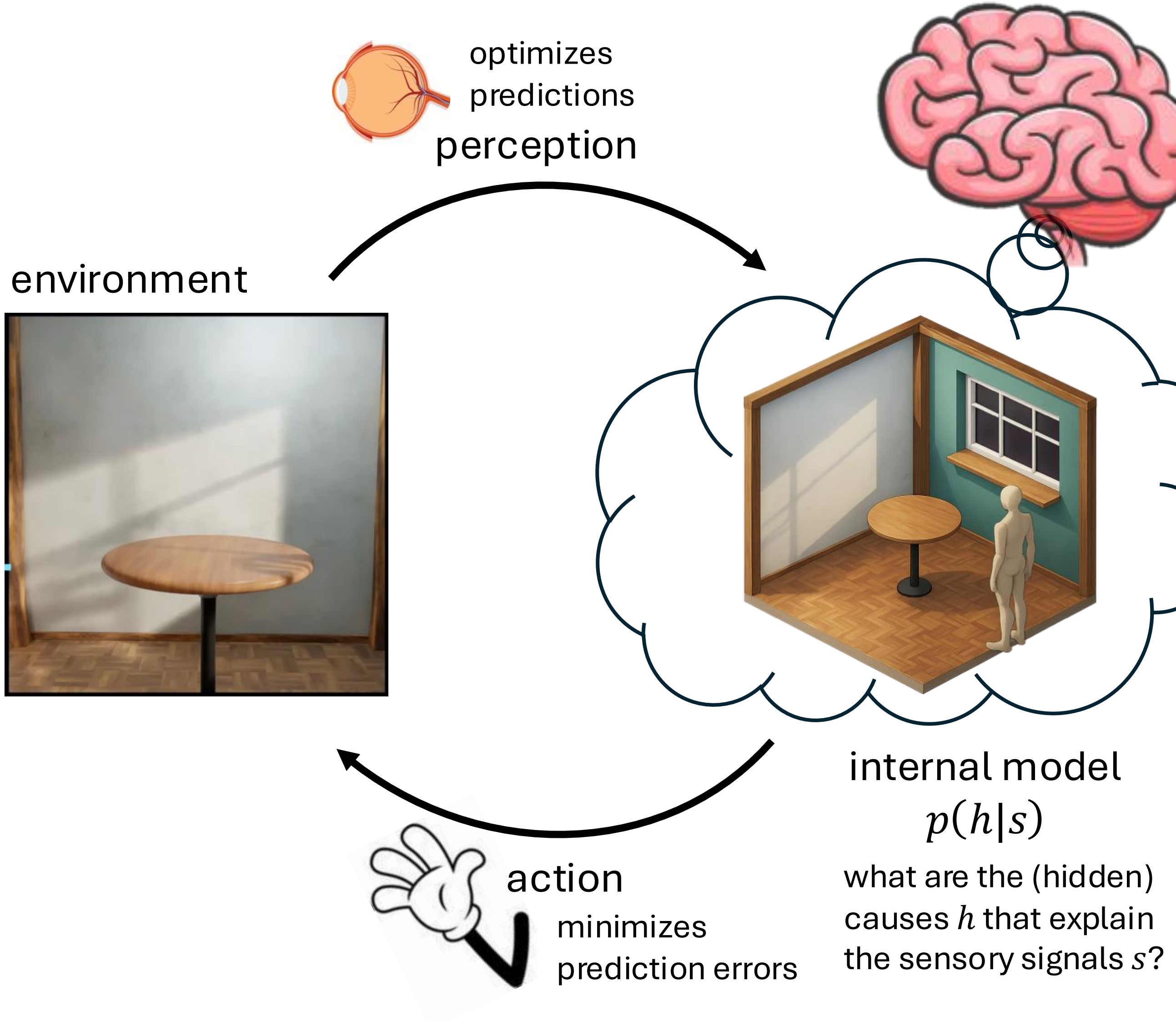

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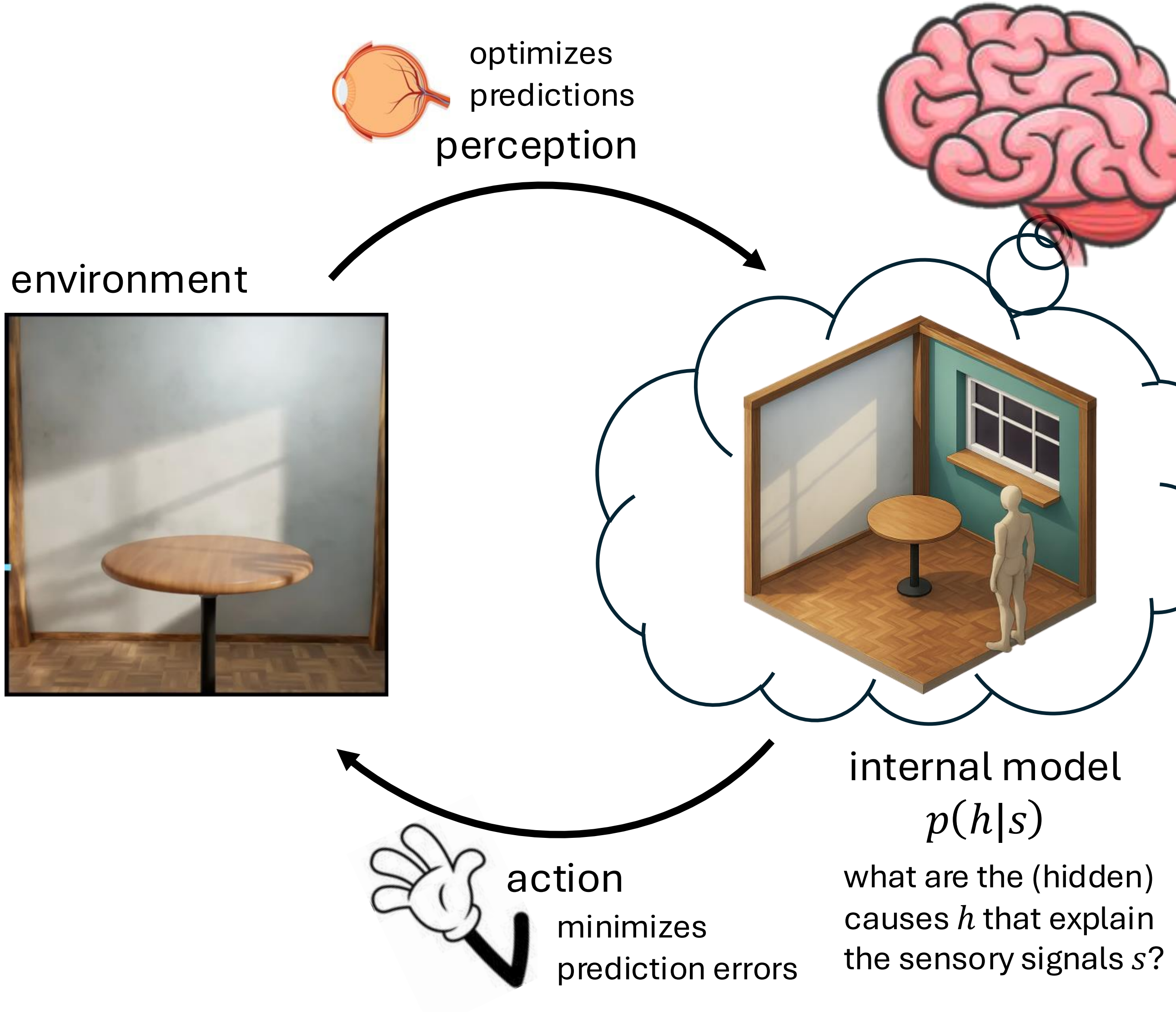
$$\underbrace{F}_{\text{free energy}} = \underbrace{\mathbb{E}[E(s,h)]}_{\substack{\text{energy}\\ \text{(probability)}}} - \underbrace{H[p(h|s)]}_{\substack{\text{entropy}\\ \text{(uncertainty)}}}$$



Free Energy Principle

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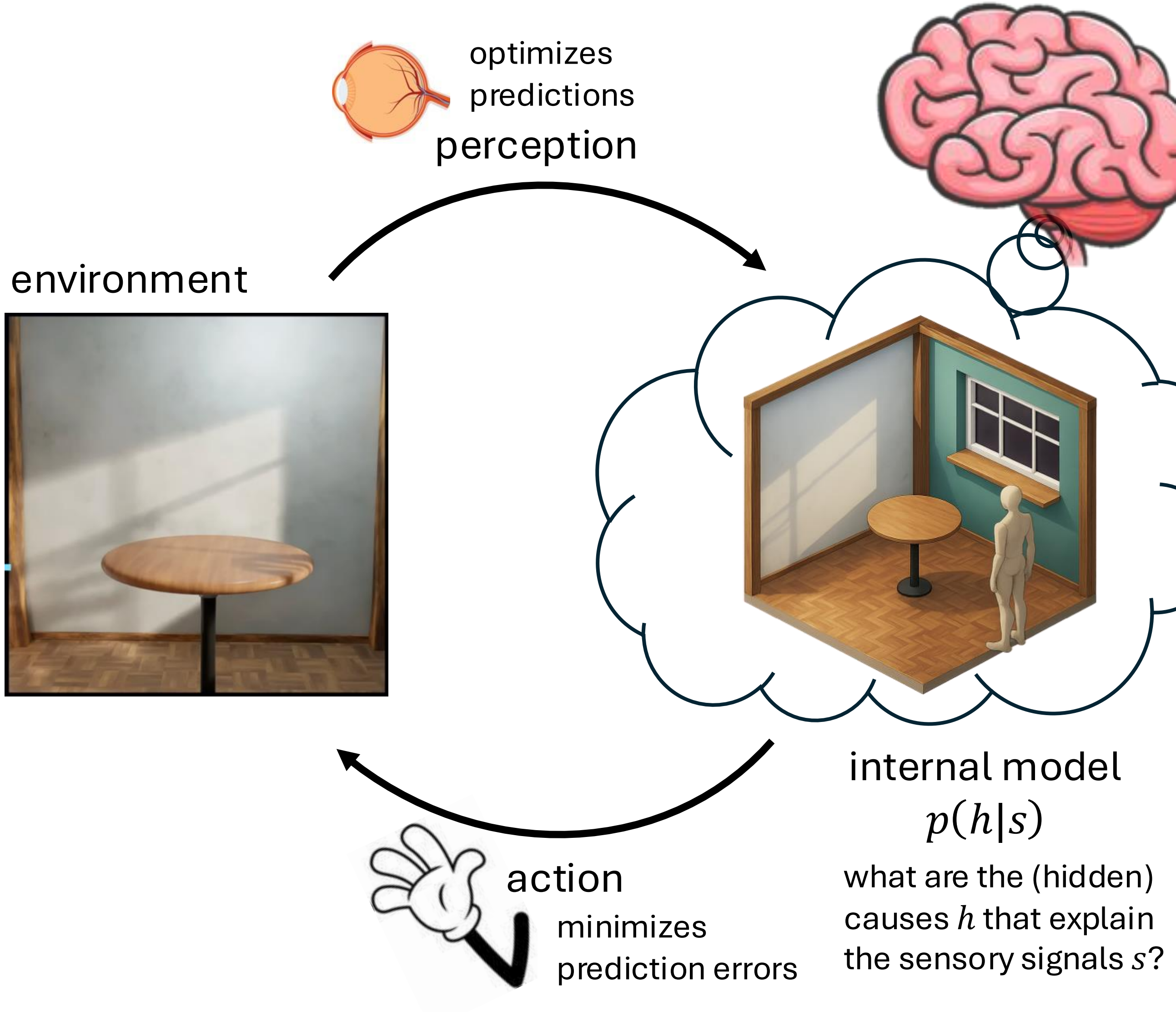
Build an internal model that minimizes (variational) free energy.

$$\underbrace{F}_{\text{variational free energy}} = \underbrace{\mathbb{E}[E(s,h)]}_{\text{energy (probability)}} - \underbrace{H[q(h)]}_{\text{entropy (uncertainty)}}$$

variational approximation

$q(h) \rightarrow p(h|s)$



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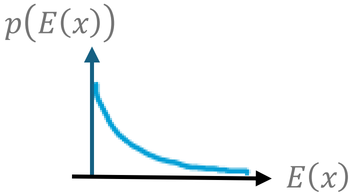
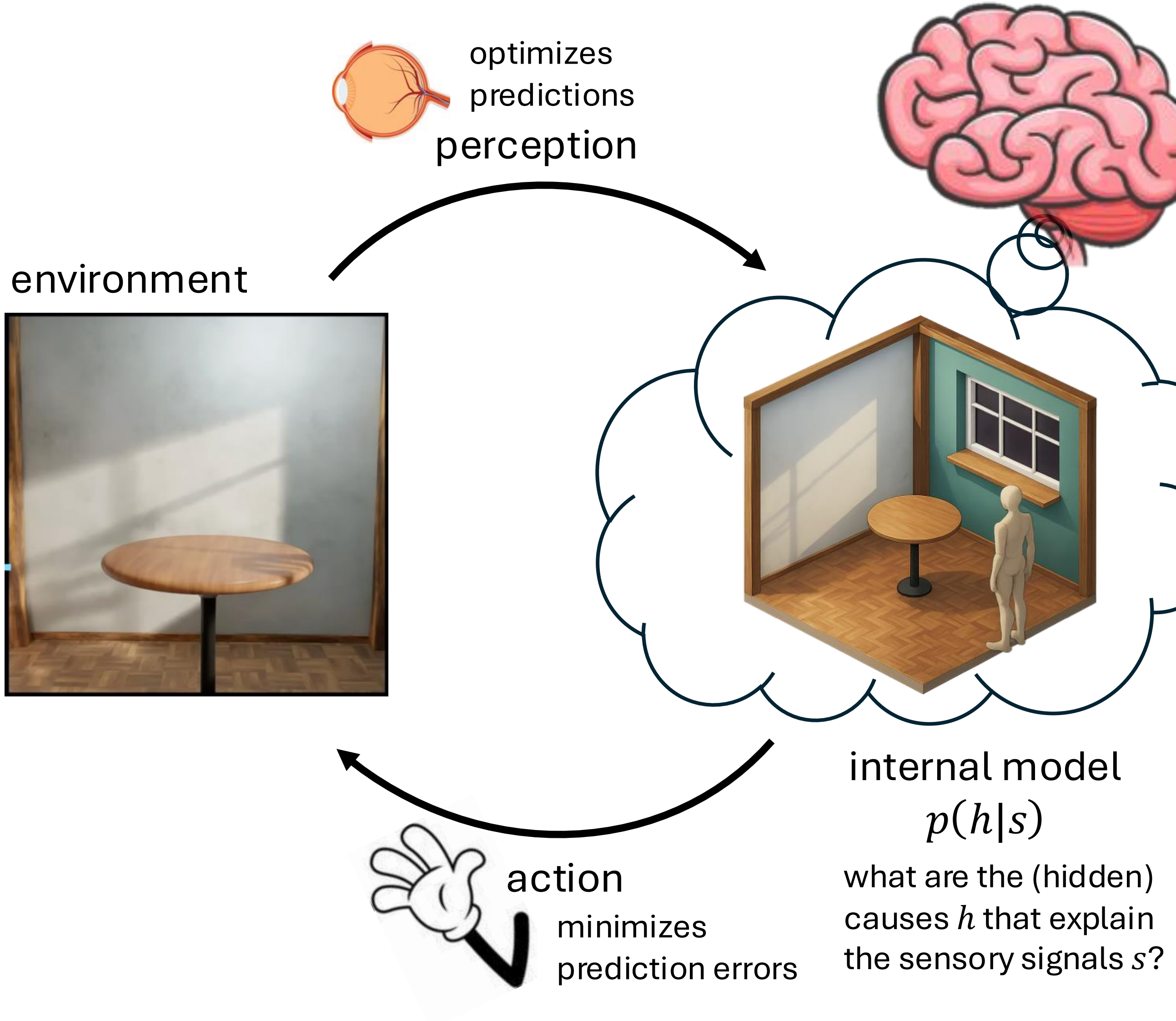


Boltzmann distribution: $p\big(E(x)\big) \propto e^{-\frac{E(x)}{kT}} \approx e^{-E(x)}$

$$F = \mathbb{E}[-\ln p(s,h)] + \sum_{h} q(h)\ln q(h)$$



Free Energy Principle

KL Downing. Gradient Expectations: Structure, Origins, and Synthesis of Predictive Neural Networks. *MIT Press*, 2023.

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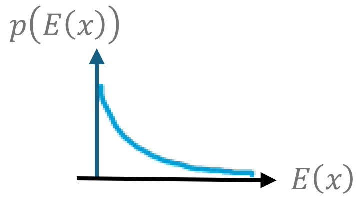


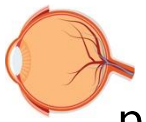
Boltzmann distribution: $p(E(x)) \propto e^{-\frac{E(x)}{kT}} \approx e^{-E(x)}$

$$F = \mathbb{E}[-\ln p(s,h)] + \sum_h q(h) \ln q(h)$$

$\vdots$ (Bayes theorem)

$$\underbrace{F}_{\text{variational free energy}} = \underbrace{-\ln p(s)}_{\text{surprisal}} + \underbrace{D_{KL}(q(h) \parallel p(h|s))}_{\text{Kullback-Leibler divergence}}$$

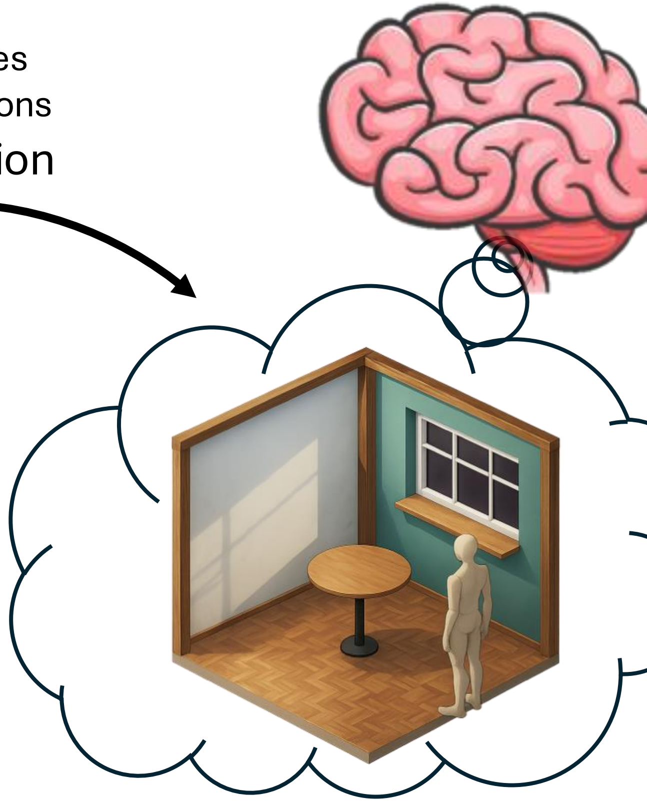
definition: $D_{\mathrm{KL}}(P \parallel Q) = \sum_{x \in \mathcal{X}} P(x) \log \frac{P(x)}{Q(x)}$.



optimizes predictions

perception

environment



internal model

$p(h|s)$

what are the (hidden) causes h that explain the sensory signals s?

action

minimizes prediction errors

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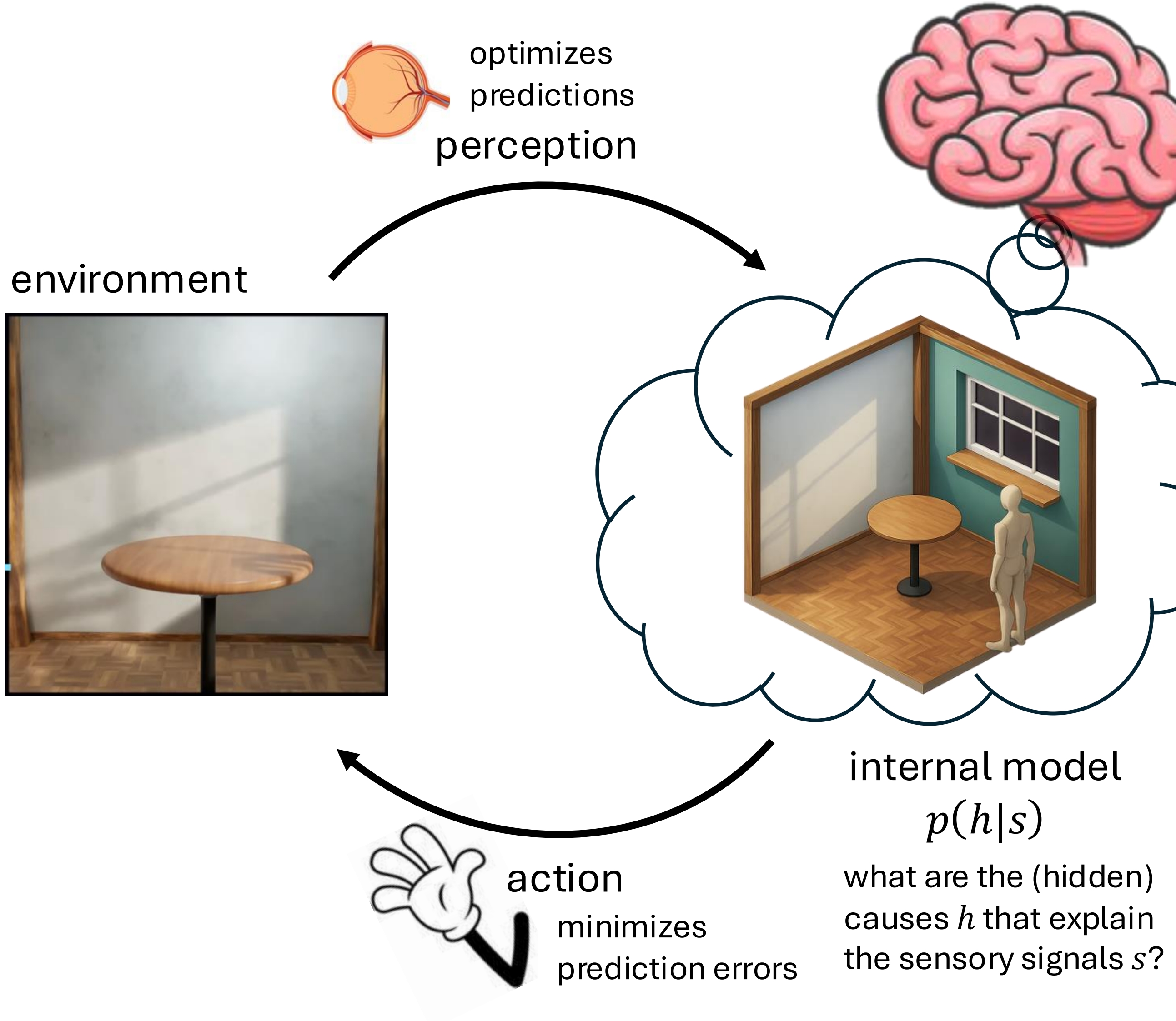
$$\underbrace{F}_{\text{variational free energy}} = \underbrace{-\ln p(s)}_{\text{surprise}} + \underbrace{D_{KL}\big(q(h) \parallel p(h|s)\big)}_{\text{divergence}}$$

Links probability, uncertainty, and energy



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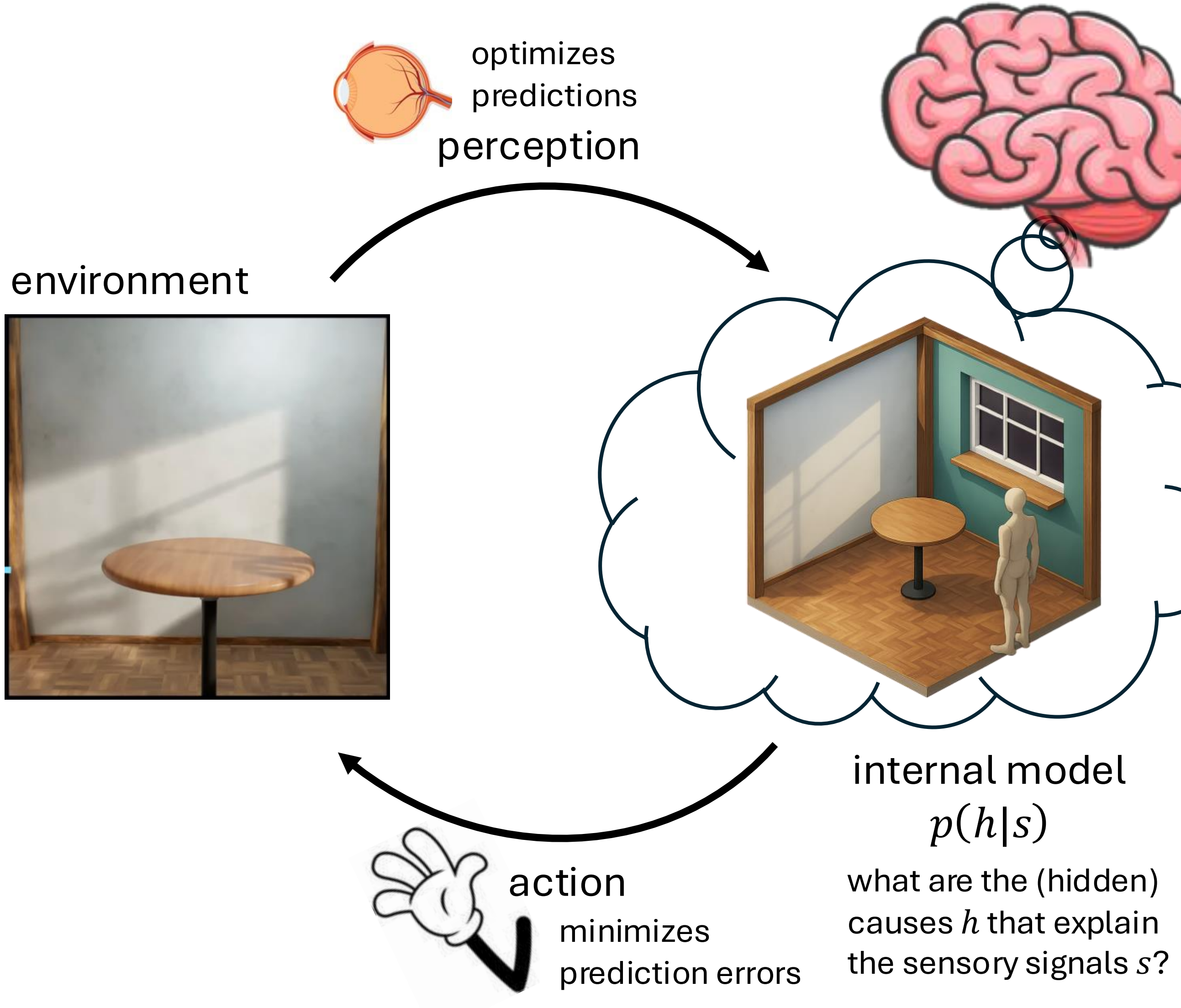
Links probability, uncertainty, and energy

Bayesian inference problem:

$$\min_{q(h)} F \leftrightarrow \min_{q(h)} D_{KL}\big(q(h) \parallel p(h|s)\big)$$

so that $q(h) \rightarrow p(h|s)$.

Build an internal model that minimizes free energy.



Free Energy Principle

F Rossi, E Garrabé, G Russo.
Free-Gate: Planning, control and policy composition via free energy gating
arXiv (2025)

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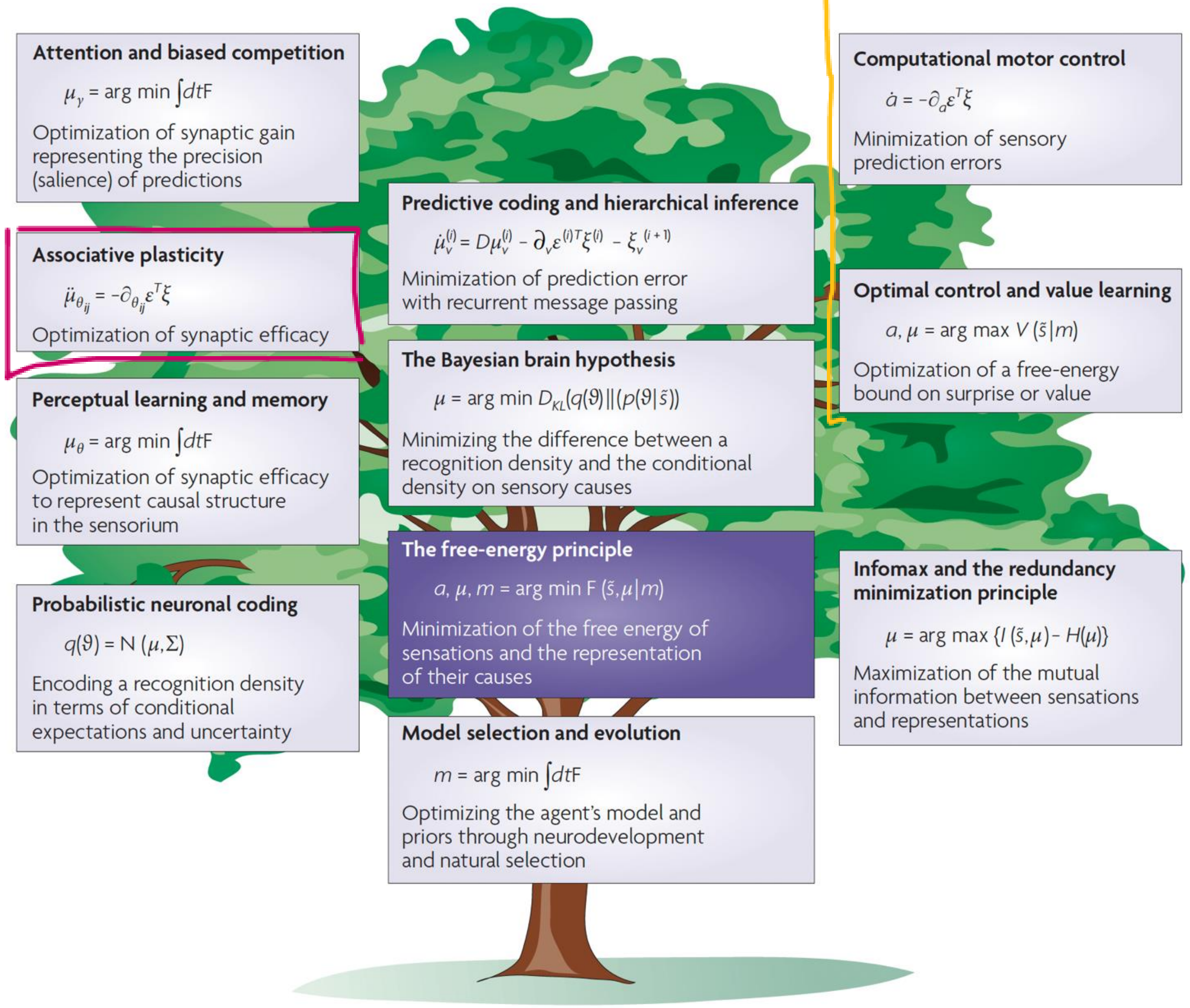
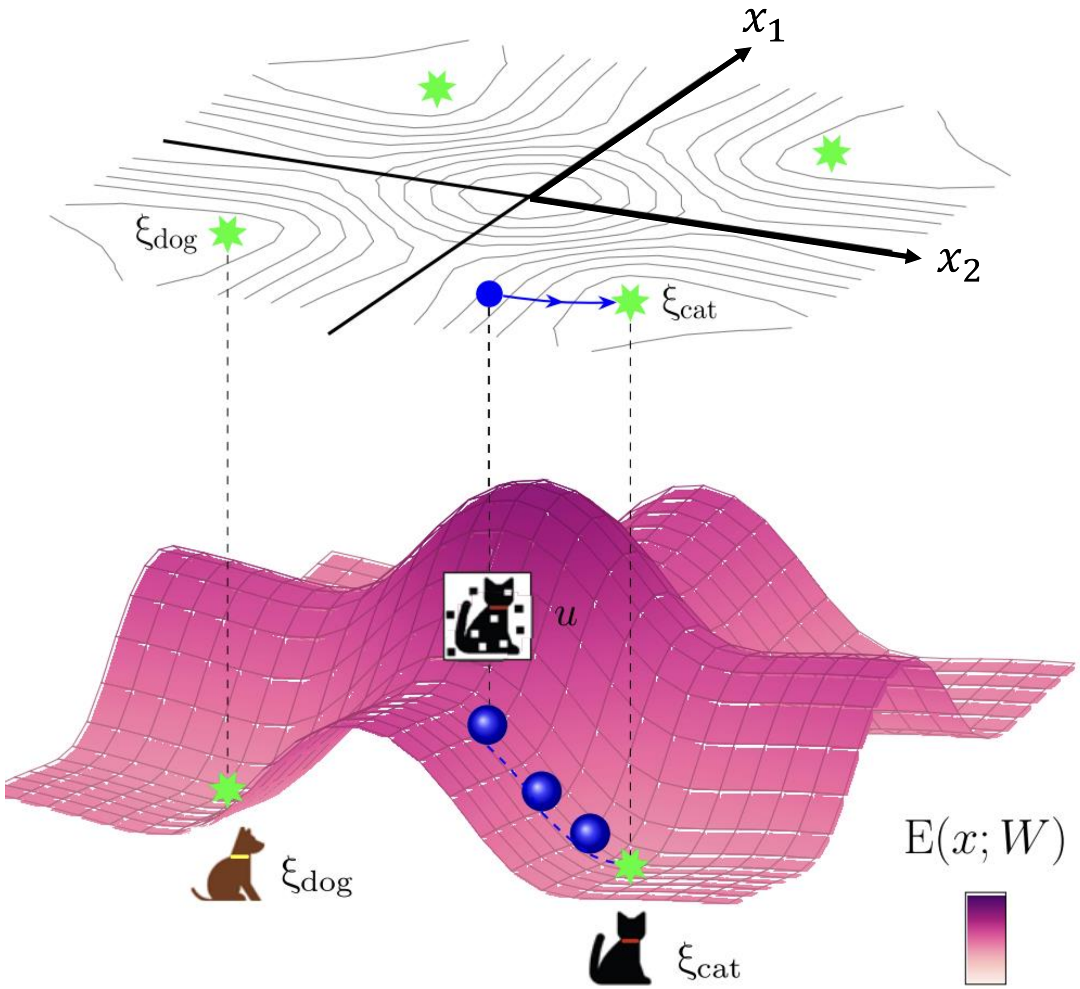
Build an internal model that minimizes free energy.

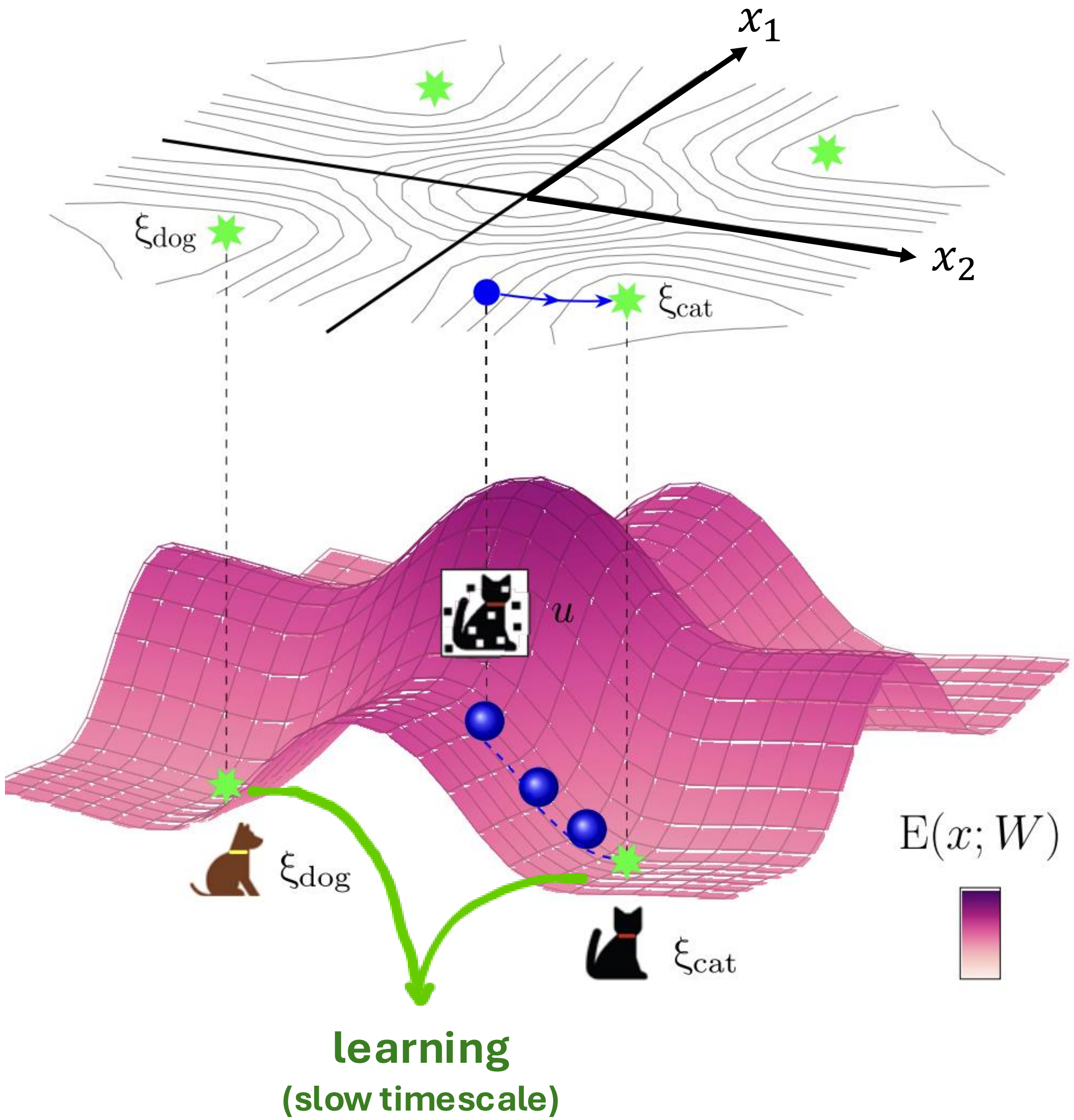


Associative memory model



S Betteti, G Baggio, F Bullo & S Zampieri.
Science Advances (2025).

Associative memory model



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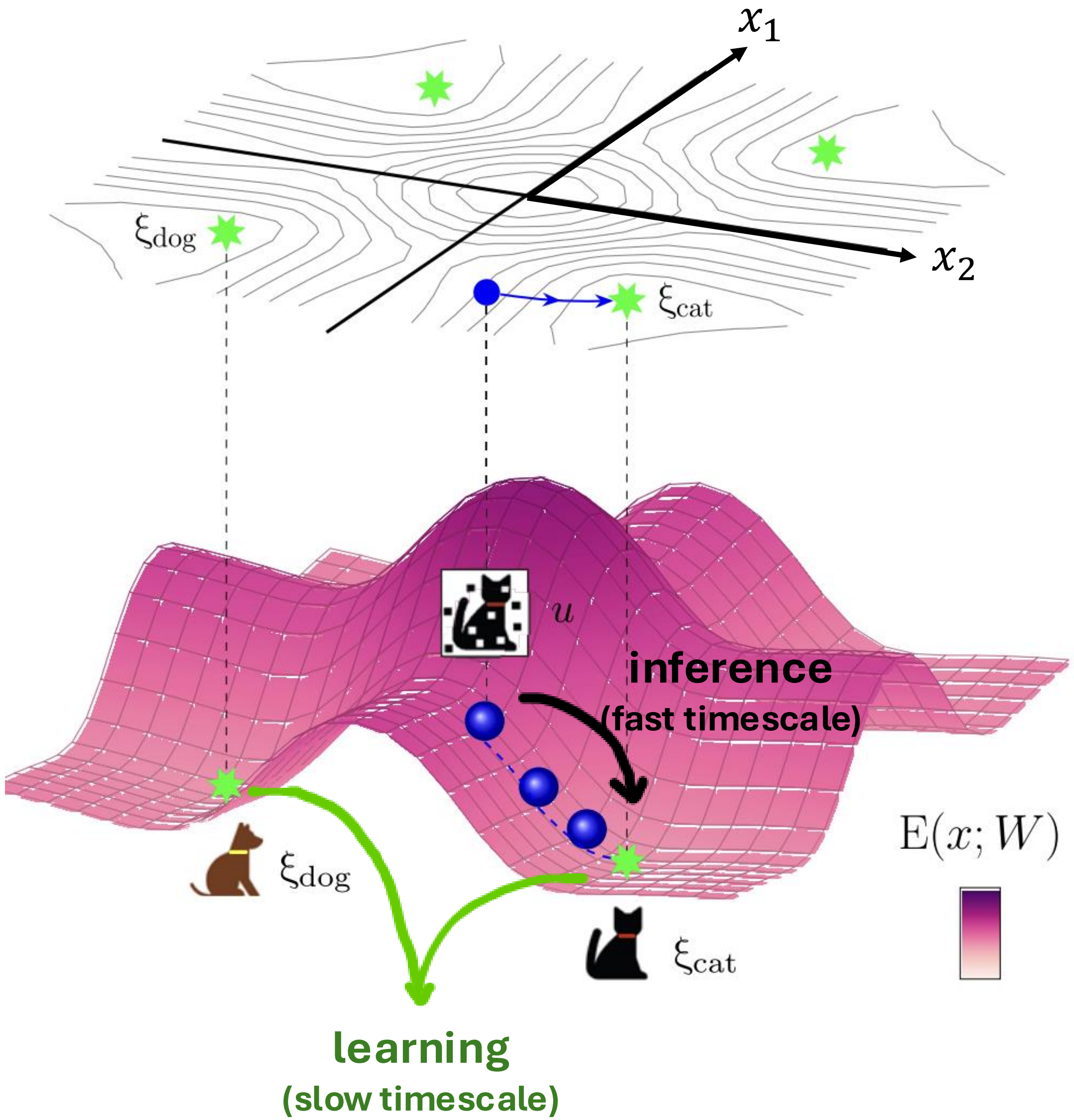

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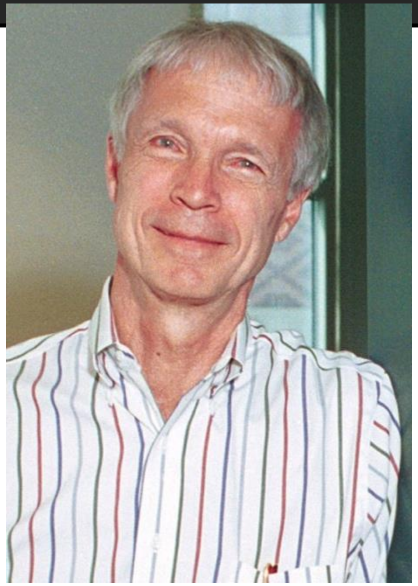
Hopfield network



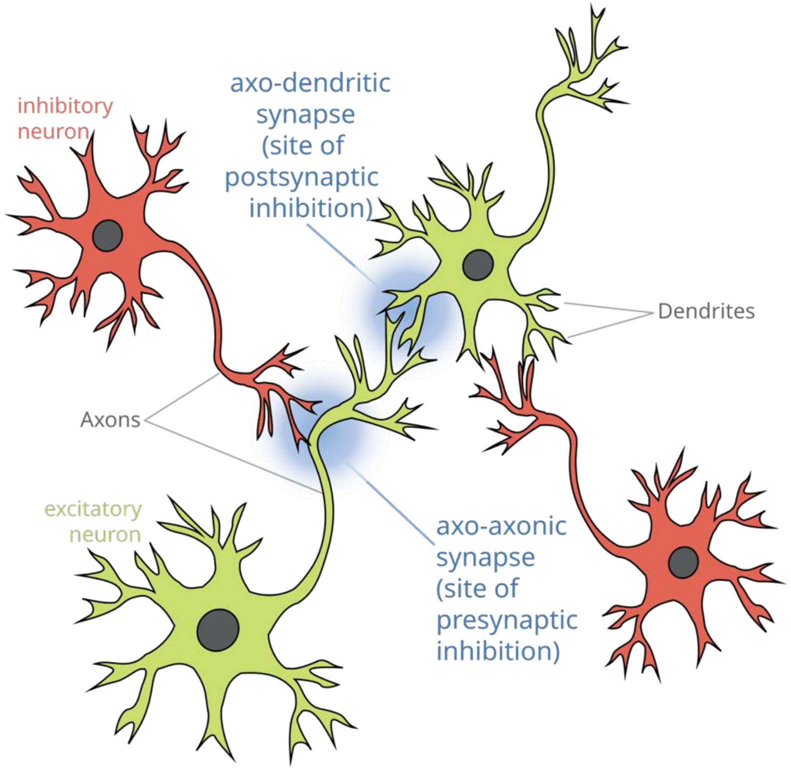
John Hopfield.
Neural networks and physical systems with emergent collective computational abilities. *PNAS* (1982).

Neuron activity:

$$x_i \in \{-1, +1\}$$

Synapse weight:

$$W_{ij} \in \mathbb{R}$$



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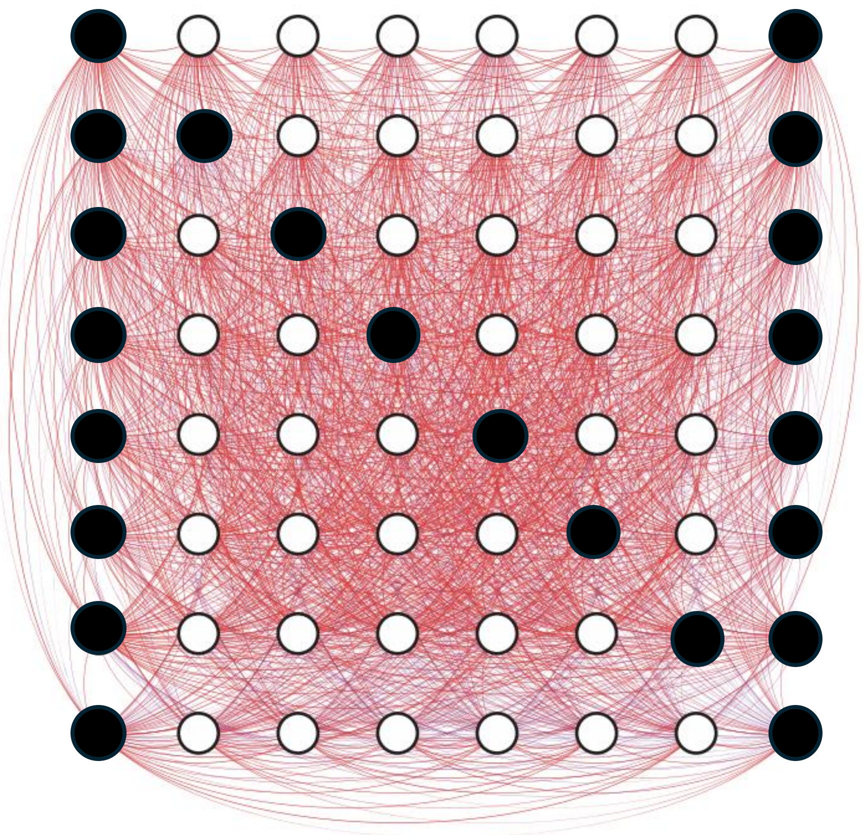

Neuron state represent patterns (e.g., images)

Neuron activity:

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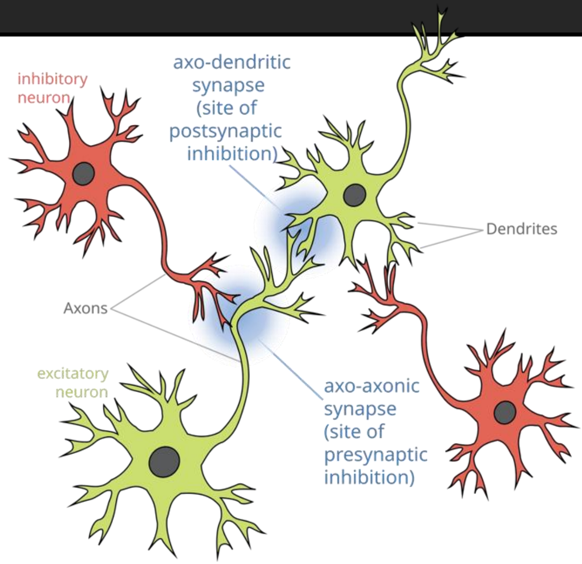

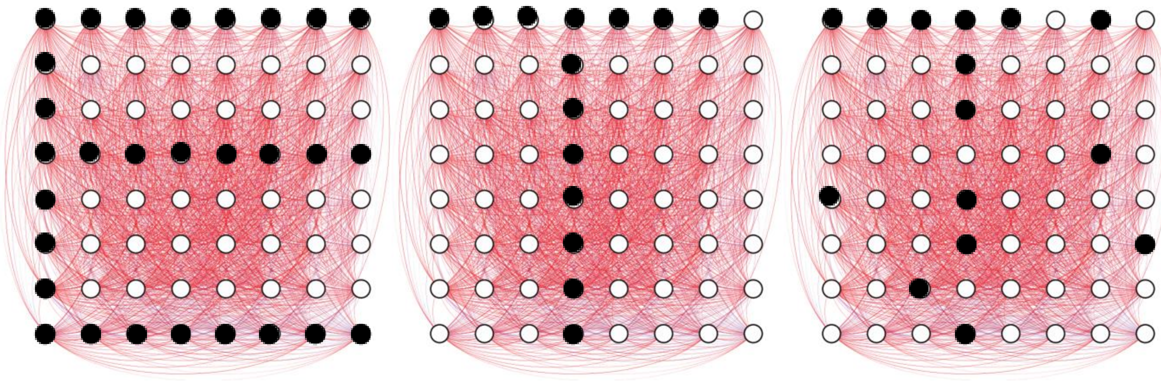

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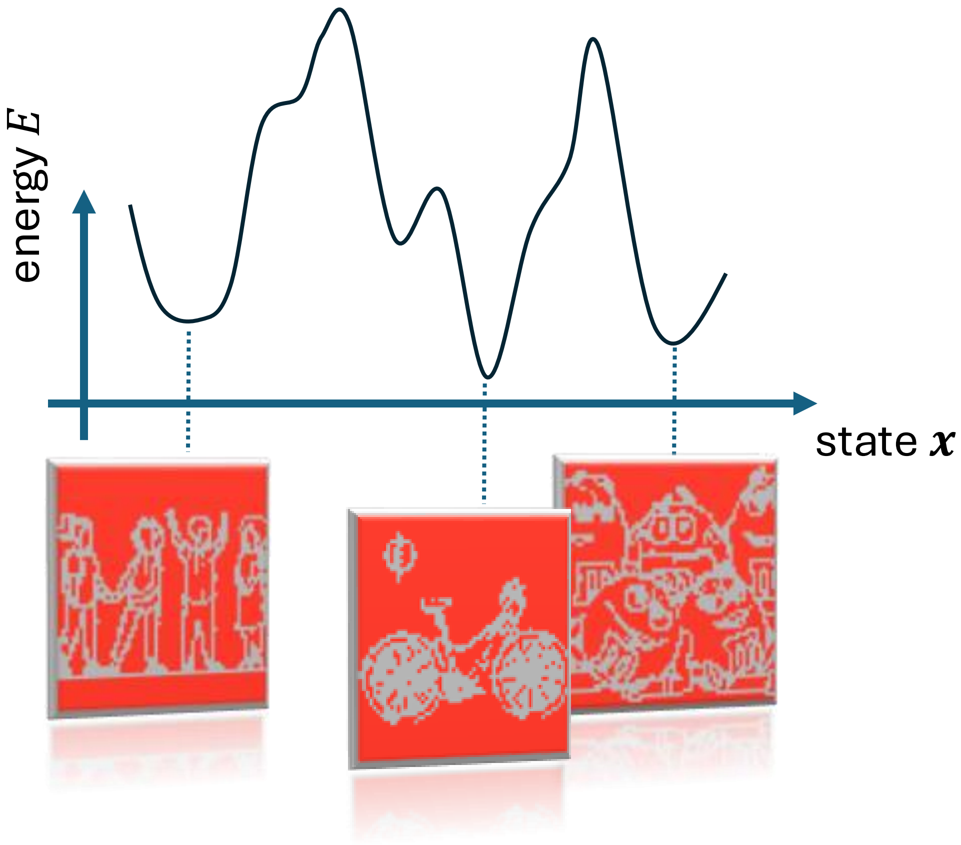

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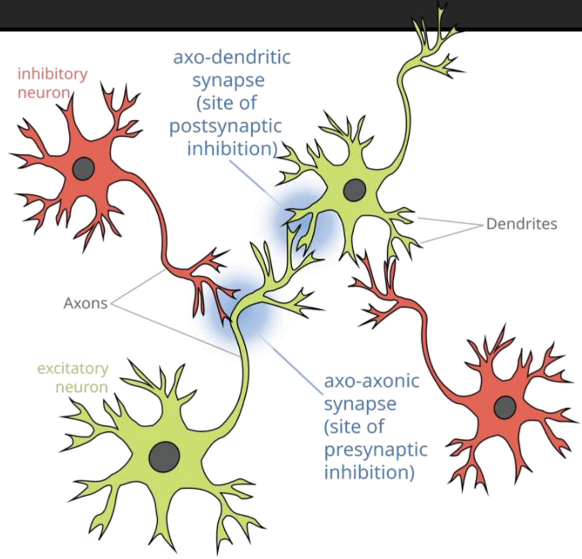
Energy function:

$$E(\boldsymbol{x}) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i$$



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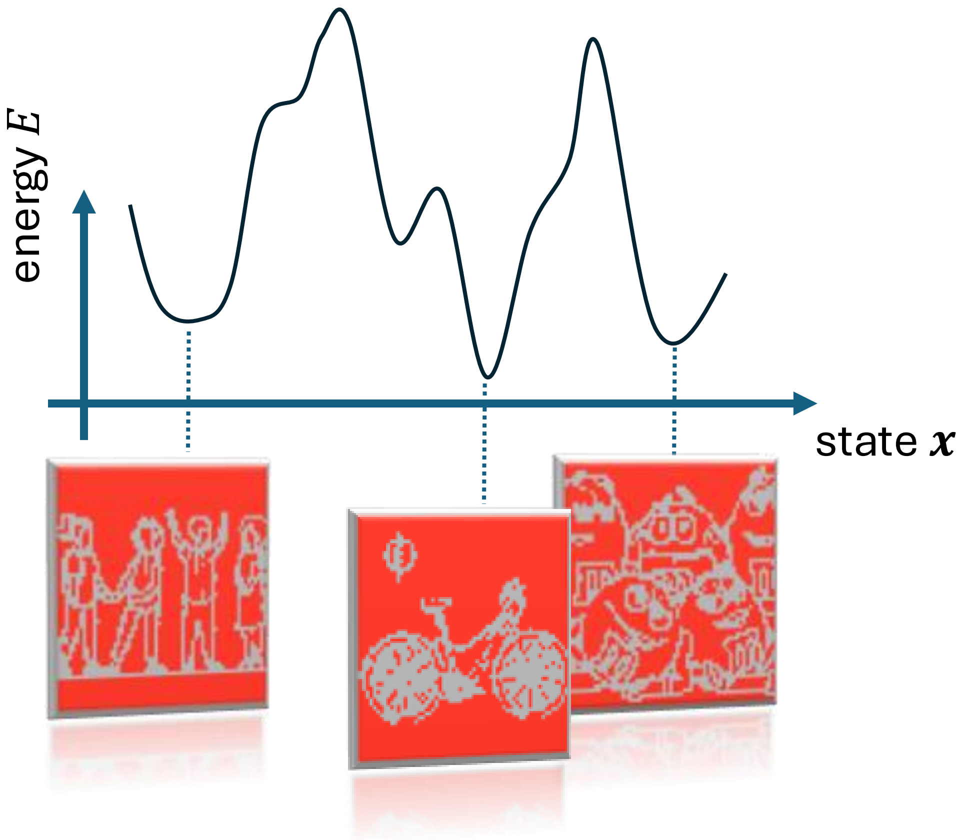

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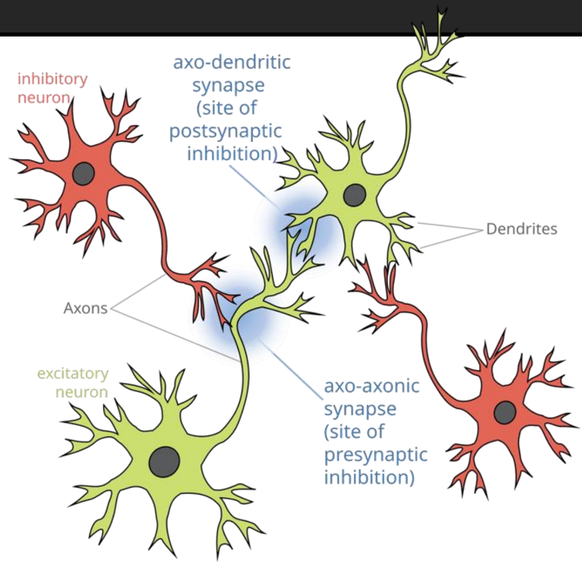
Goal:

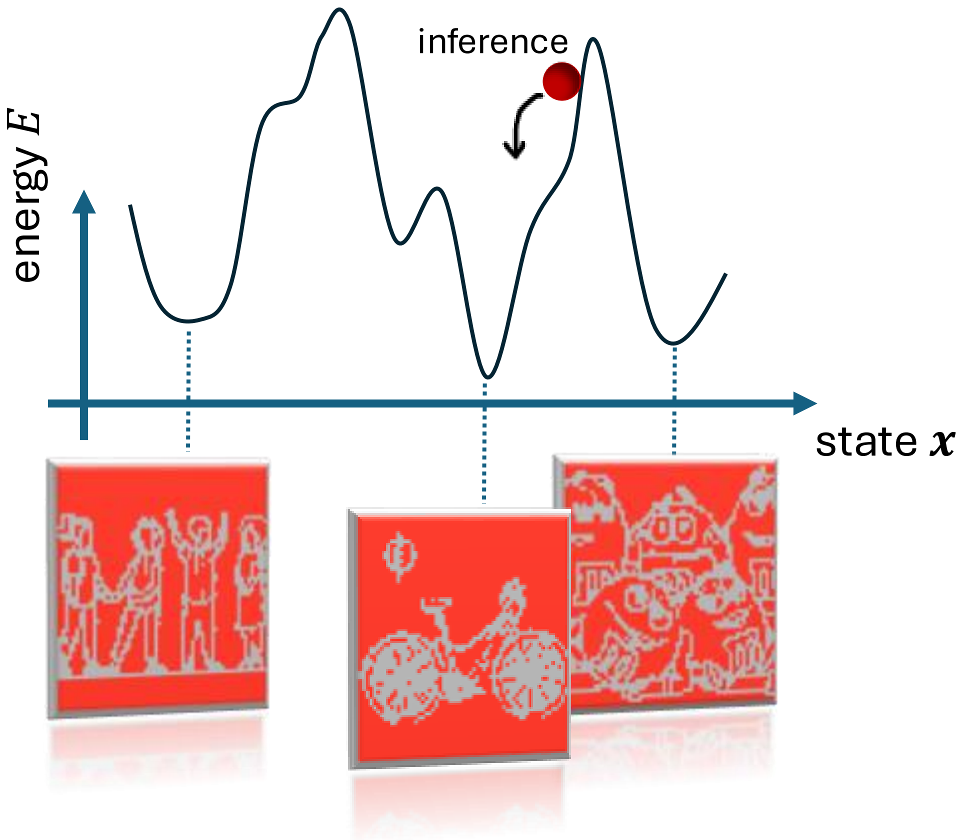
$$\min E(\boldsymbol{x})$$



energy is minimized if

- *synapse is excitatory (positive) and neurons are aligned, or*
- *synapse is inhibitory (negative) and neurons are anti-aligned*

Hopfield network: Inference



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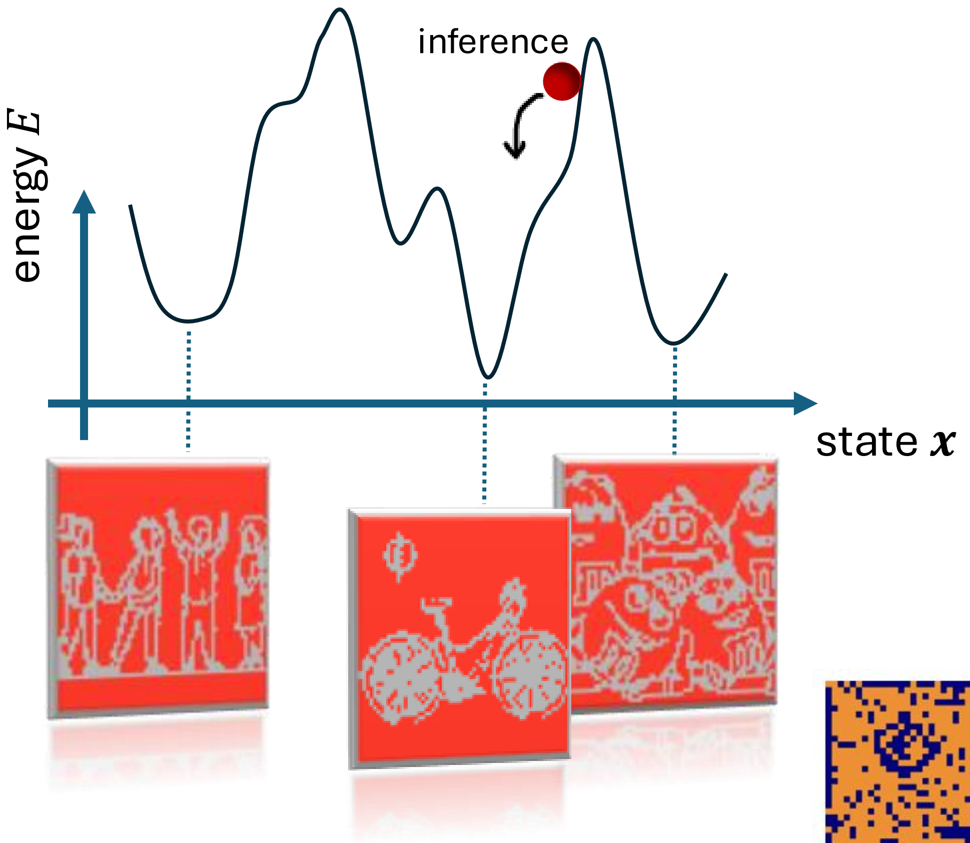
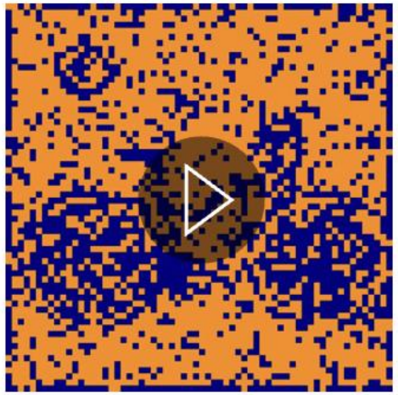
Inference: discrete-time system, asynchronous update

neuron flips its state in order to align with most of its neighbors

$$x_i = \begin{cases} +1 \text{ is preferred if } \sum_{j \neq i} W_{ij}\, x_j + b_i > 0 \\ -1 \text{ is preferred if } \sum_{j \neq i} W_{ij}\, x_j + b_i < 0 \end{cases}$$

Hopfield network: Inference





E Crouse. Hopfield networks: Neural wave machines (2022).

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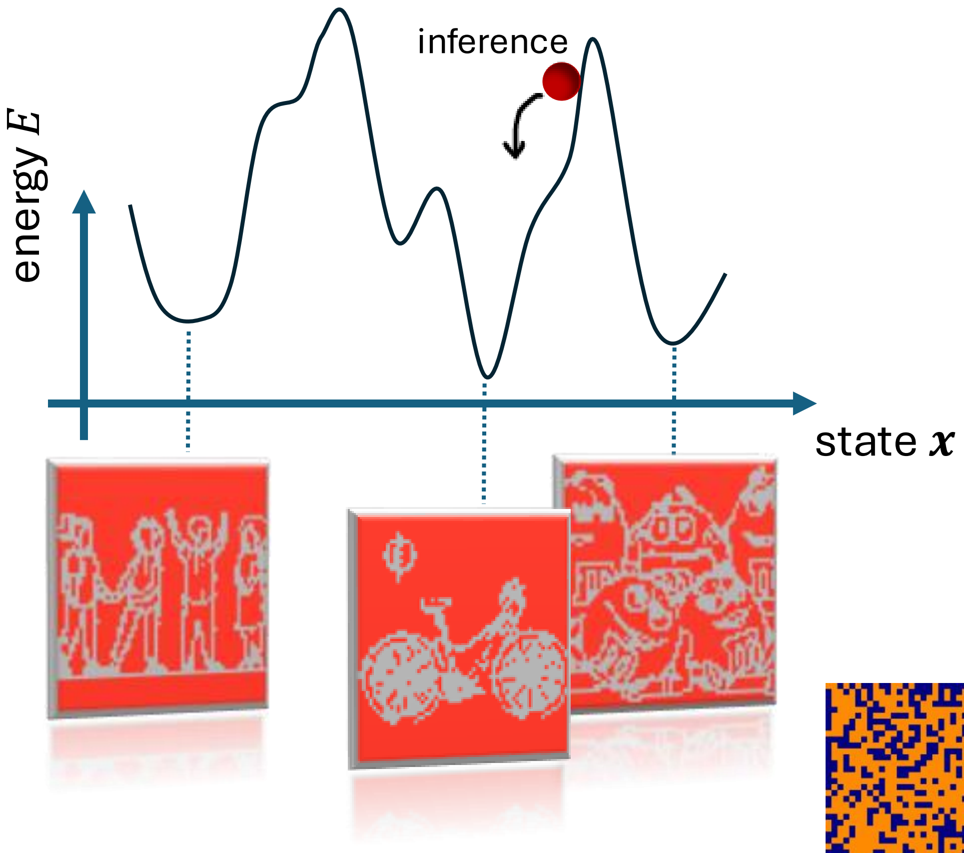

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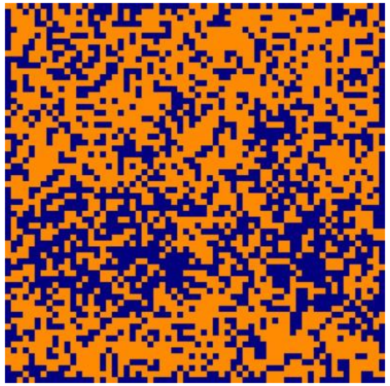

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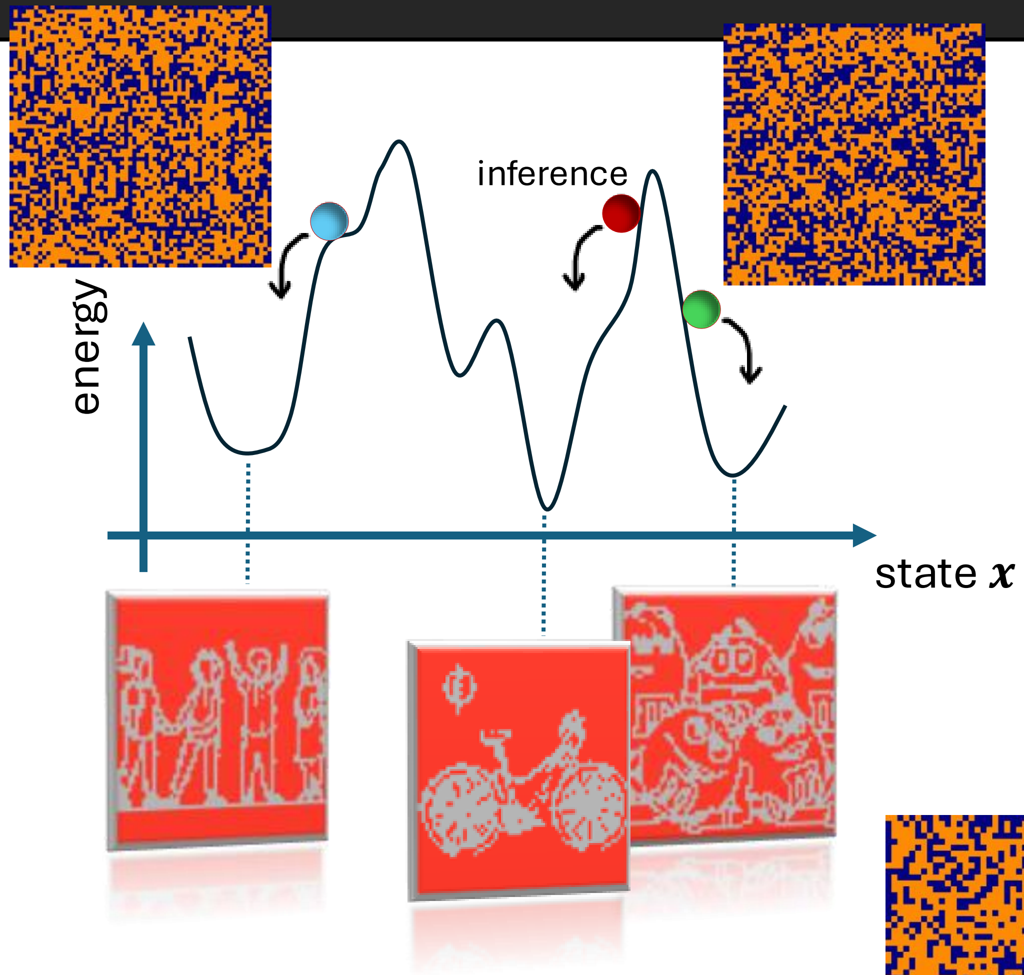

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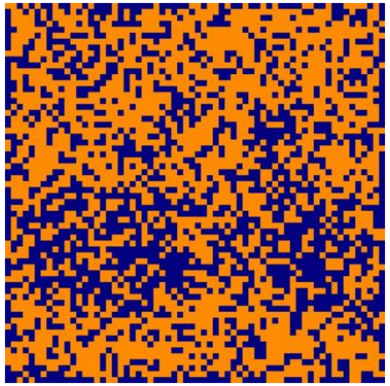

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Continuous time vs. Discrete time

Neuron activity:

$$x_i \in \mathbb{R}$$

Synapse weight:

$$W_{ij} \in \mathbb{R}$$

Energy function:

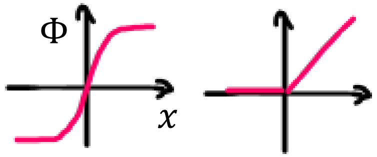
$$E(\boldsymbol{x}) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i + \sum_i \int_0^{x_i} \Phi(z)\mathrm{d}z$$

Goal:

$$\min E(\boldsymbol{x})$$

Inference: continuous-time, recurrent neural network

$$\dot{x}_i = -x_i + \sum_{j=1}^{N} W_{ij} \Phi(x_j) + b_i$$



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Neuron activity:

$$x_i \in \mathbb{R}$$

Synapse weight:

$$W_{ij} \in \mathbb{R}$$

Energy function:

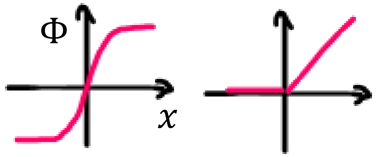
$$E(\boldsymbol{x}) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i + \sum_i \int_0^{x_i} \Phi(z) \mathrm{d}z$$

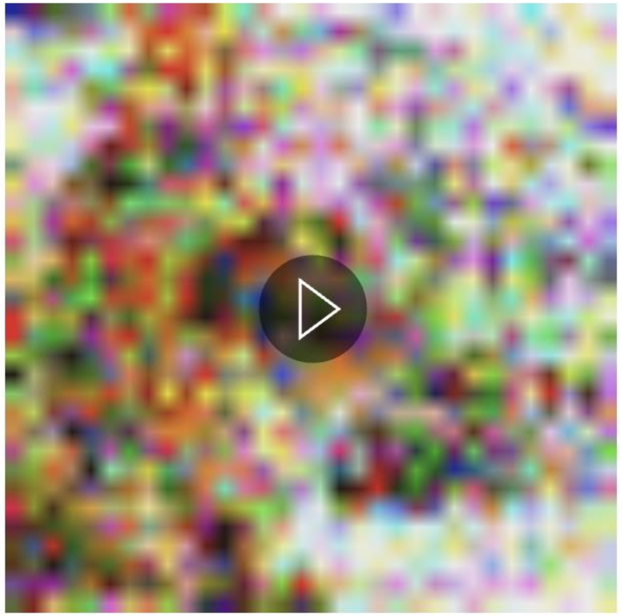
Goal:

$$\min E(\boldsymbol{x})$$

Inference: continuous-time, recurrent neural network

$$\dot{x}_i = -x_i + \sum_{j=1}^{N} W_{ij} \Phi(x_j) + b_i$$





Continuous time vs. Discrete time

Neuron activity:

$$x_i \in \mathbb{R}$$

Synapse weight:

$$W_{ij} \in \mathbb{R}$$

Energy function:

$$E(\boldsymbol{x}) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i + \sum_i \int_0^{x_i} \Phi(z) \mathrm{d}z$$

Goal:

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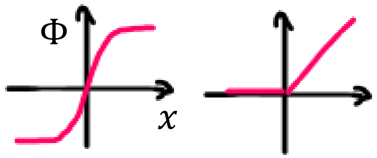
Inference: continuous-time, recurrent neural network

$$\dot{x}_i = -x_i + \sum_{j=1}^{N} W_{ij} \Phi(x_j) + b_i$$



Continuous time vs. Discrete time

Neuron activity:

$$\boldsymbol{x} \in \mathbb{R}^N$$

Synapse weight:

$$W \in \mathbb{R}^{N \times N}$$

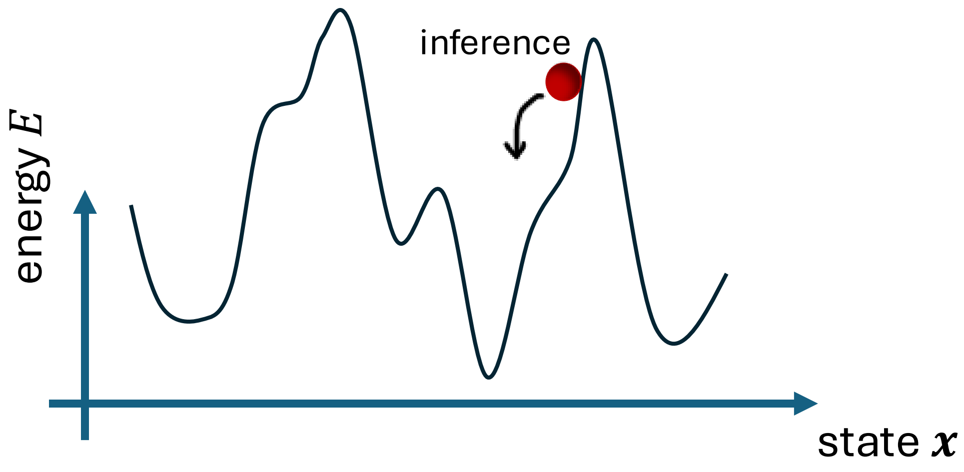
Energy function:

$$E(\boldsymbol{x}) = -\frac{1}{2}\Phi(\boldsymbol{x})^T W \Phi(\boldsymbol{x}) - \boldsymbol{b}^T \boldsymbol{x} + \sum_i \int_0^{x_i} \Phi(z)\mathrm{d}z$$

Goal:

$$\min E(\boldsymbol{x})$$

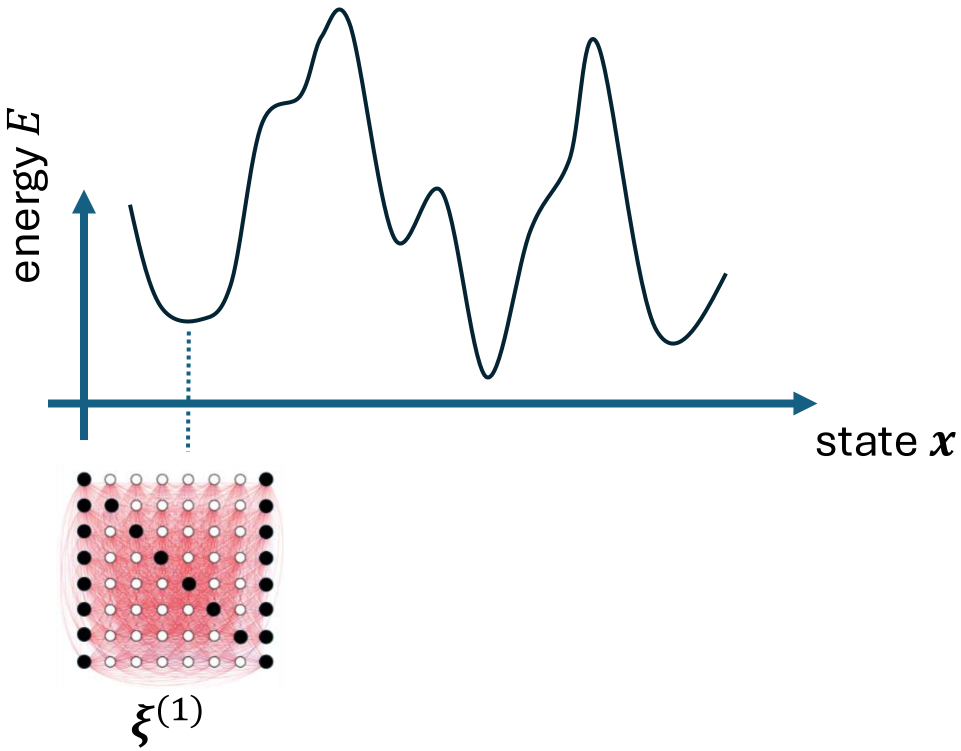
if W is symmetric, then
E is a Lyapunov function
(more in Sandro Zampieri's talk!)



Inference: continuous-time, recurrent neural network

$$\begin{aligned} \dot{\boldsymbol{x}} &= -\boldsymbol{x} + W\Phi(\boldsymbol{x}) + \boldsymbol{b} \\ &= -\nabla E(\boldsymbol{x}) \end{aligned}$$

How does it learn??



$$E(\boldsymbol{x}; W) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i$$

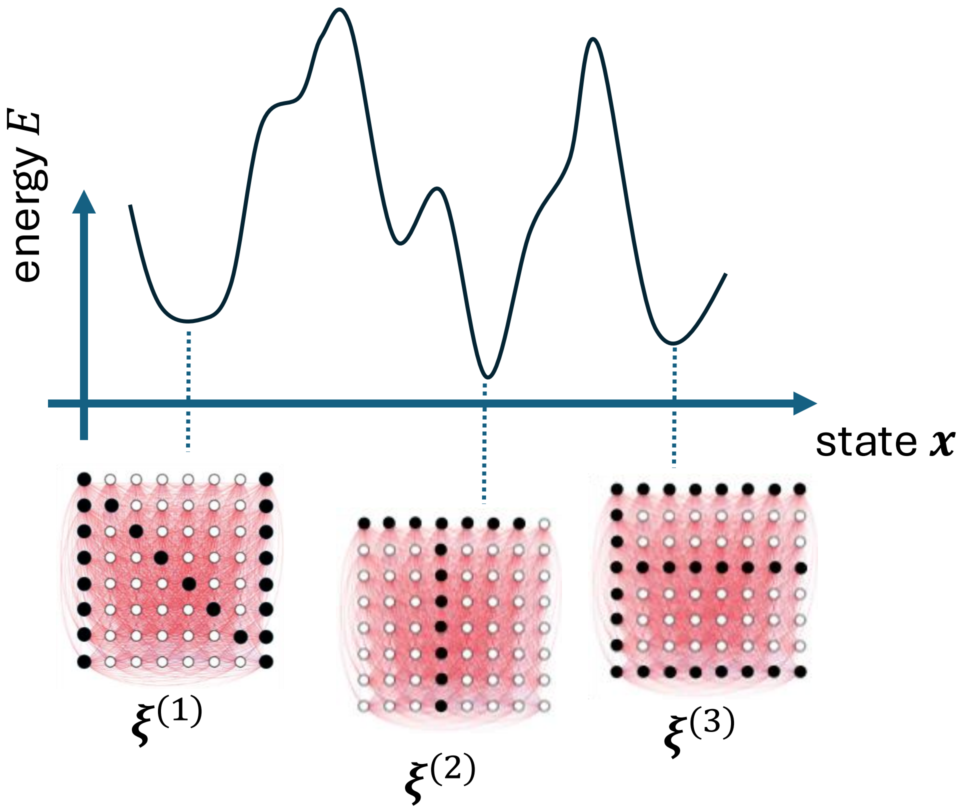
Recursive learning procedure for each pattern to find the best weight that minimizes the energy associated with a pattern $p \in \mathcal{P}$

$$\frac{\partial E^{(p)}}{\partial W_{ij}} = -x_i^{(p)} x_j^{(p)} =: \; -\xi_i^{(p)} \xi_j^{(p)}$$

Hebbian learning

Neurons that fire together, wire together

Donald Hebb



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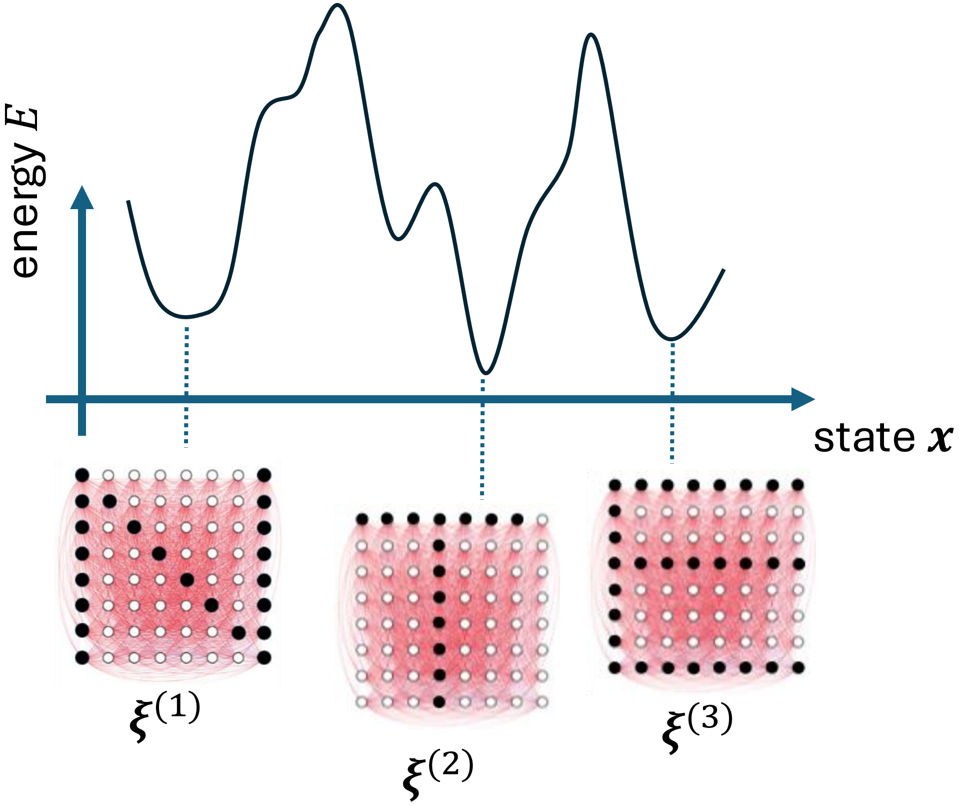
Recursive learning procedure for each pattern to find the best weight that minimizes the energy associated with a pattern $p \in \mathcal{P}$

$$\frac{\partial E^{(p)}}{\partial W_{ij}} = -x_i^{(p)} x_j^{(p)} =: \; -\xi_i^{(p)} \xi_j^{(p)}$$

if both neurons are active, then increasing W_{ij} lowers the energy of that pattern.

Hebbian learning

Donald Hebb



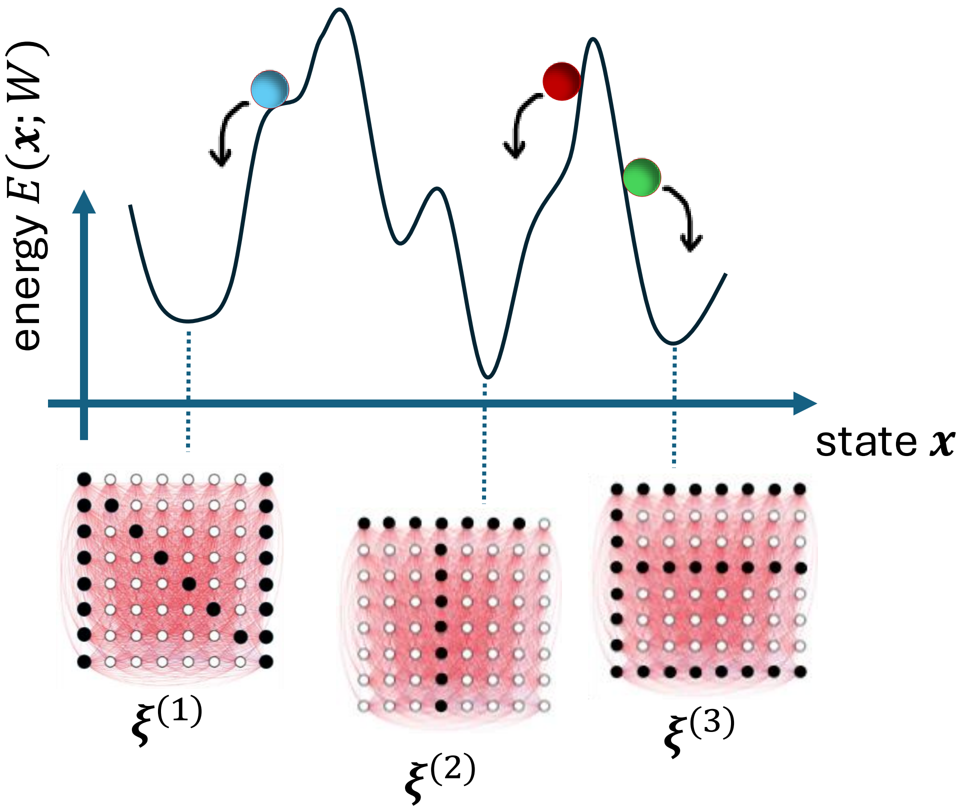
$$E(\boldsymbol{x}; W) = -\sum_{i,j} W_{ij} x_i x_j - \sum_i b_i x_i$$

Recursive learning procedure for each pattern to find the best weight that minimizes the energy associated with a pattern $p \in \mathcal{P}$

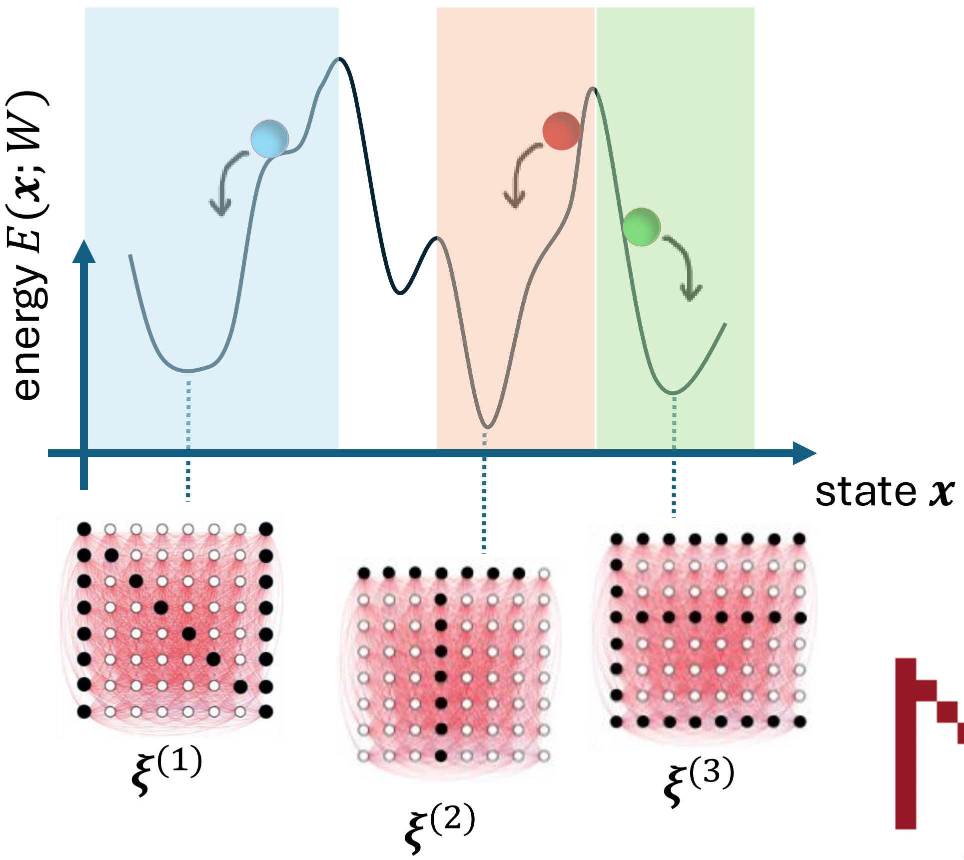
$$\frac{\partial E^{(p)}}{\partial W_{ij}} = -x_i^{(p)} x_j^{(p)} =: \; -\xi_i^{(p)} \xi_j^{(p)}$$

$$\begin{aligned} W_{ij} &= \frac{1}{N}\left(\xi_i^{(1)}\xi_j^{(1)} + \xi_i^{(2)}\xi_j^{(2)} + \ \dots\right) \\ &= \frac{1}{N}\sum_{p \in \mathcal{P}} \xi_i^{(p)} \xi_j^{(p)} \\ &= \frac{1}{N}\boldsymbol{\xi}\boldsymbol{\xi}^T \end{aligned}$$

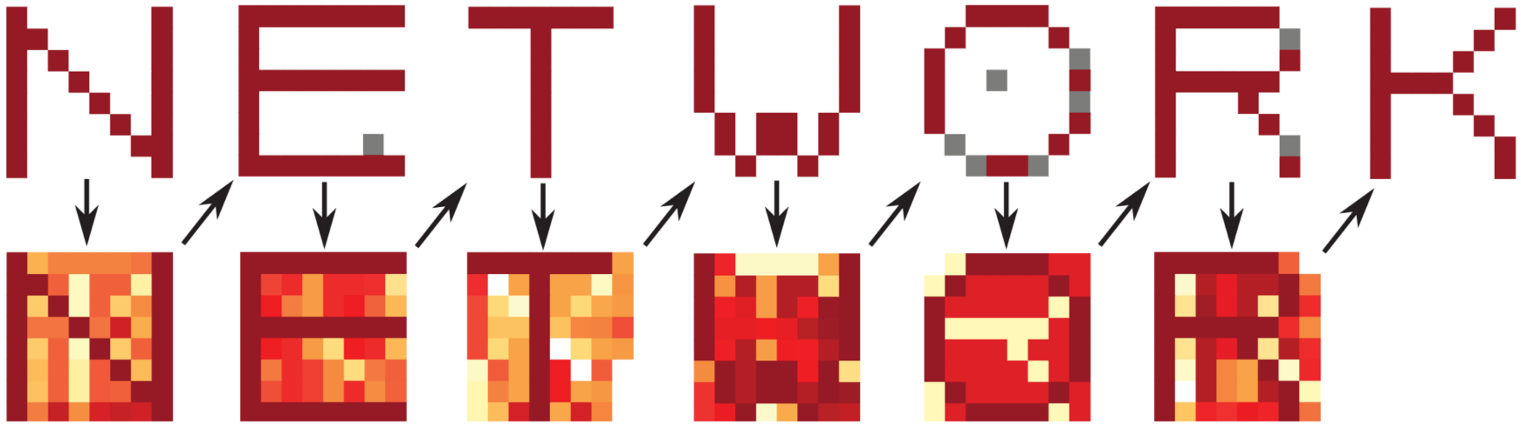
Learning, and then inference



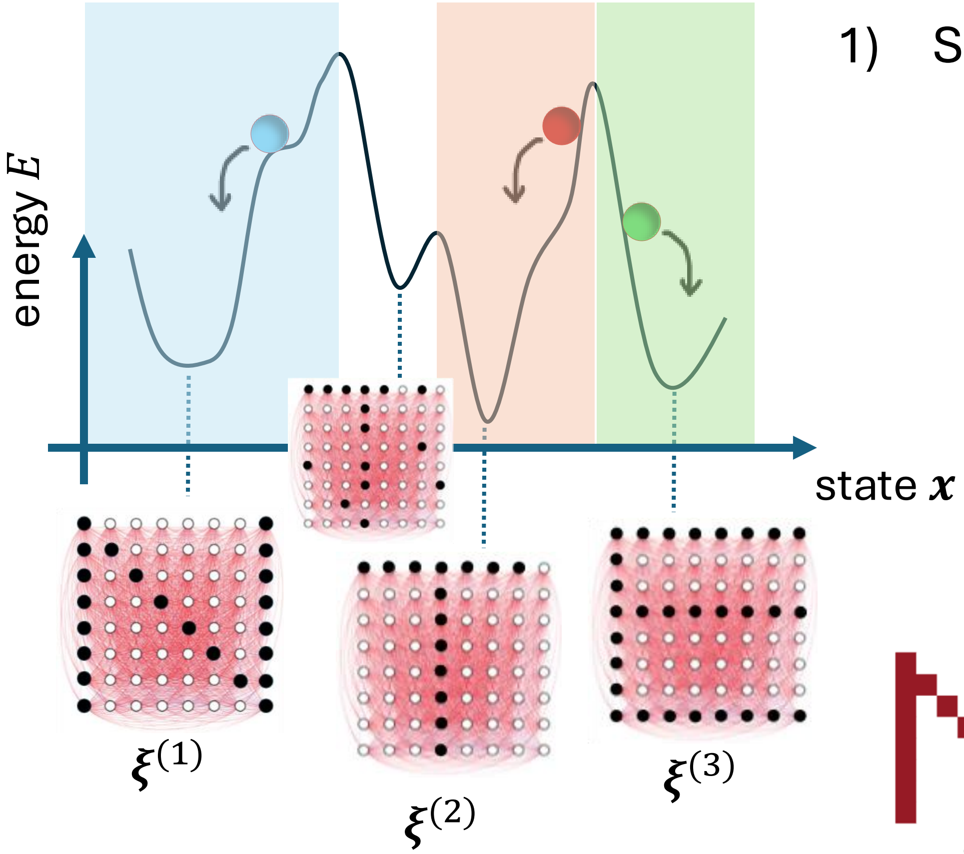
Stability, regions of attraction & control



SP Cornelius, WL Kath, AE Motter.
Nature Communications (2013).

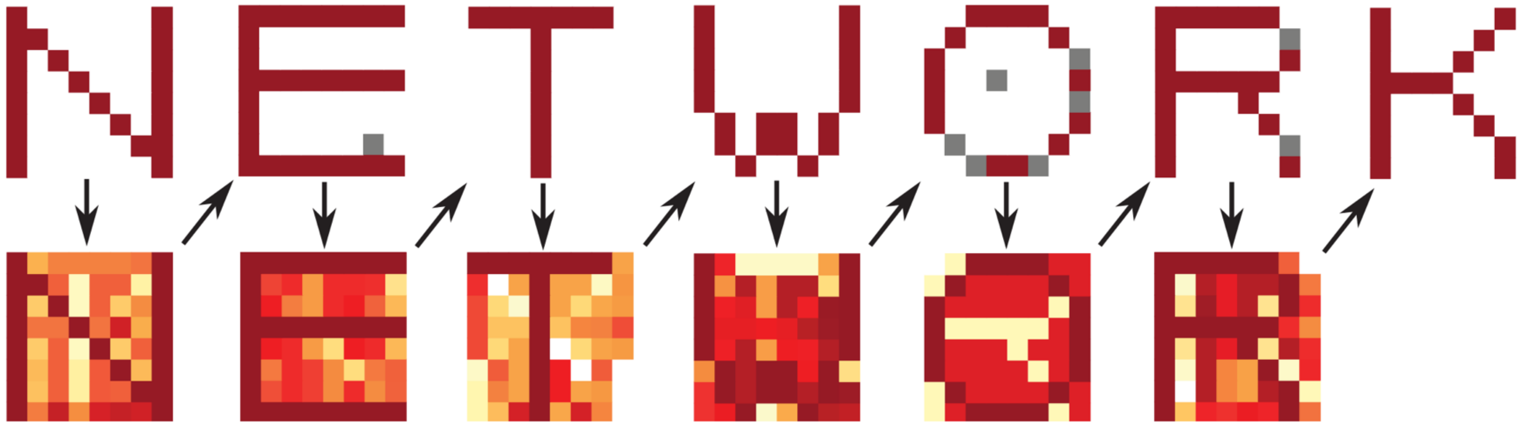


Challenges

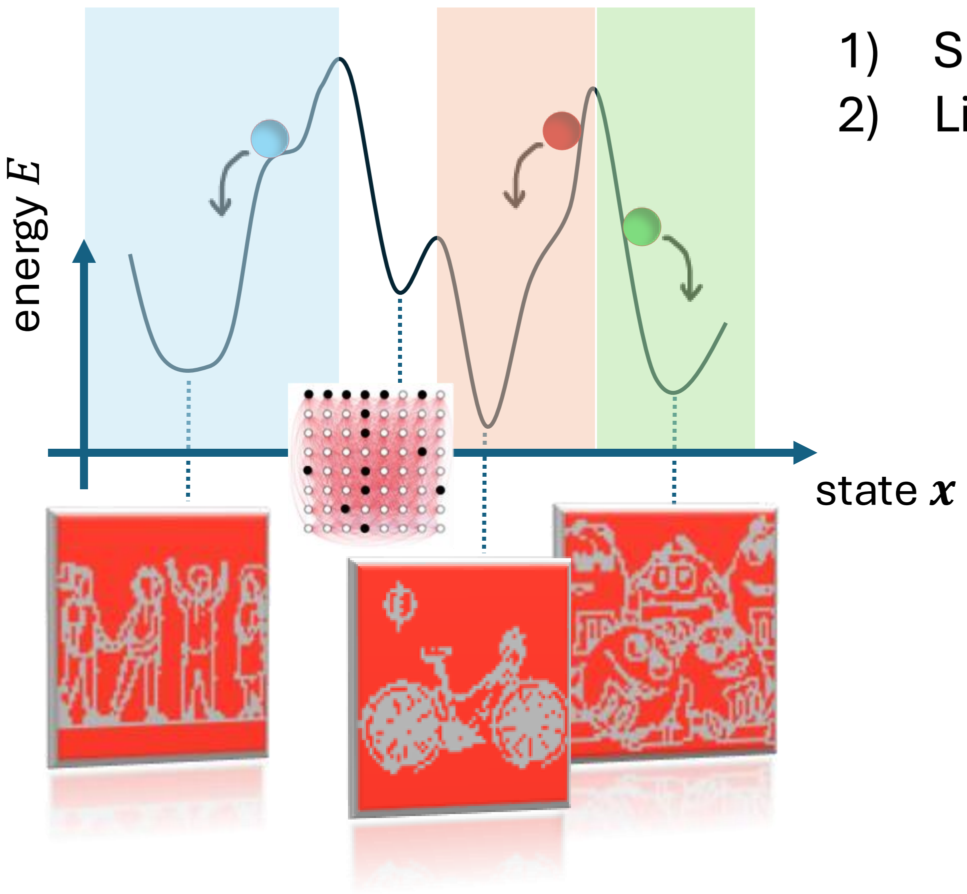


1) Spurious/parasite attractors

SP Cornelius, WL Kath, AE Motter.
Nature Communications (2013).

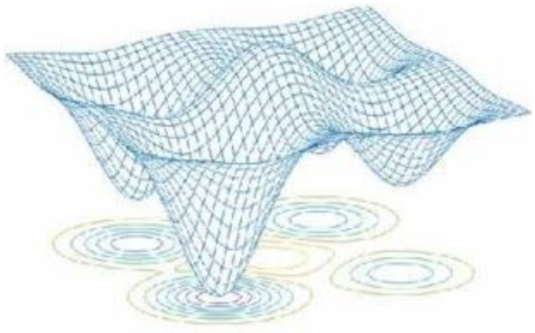


Challenges



1) Spurious/parasite attractors
2) Limited memory capacity: $C_{\max} \approx 0.14N$ for random patterns

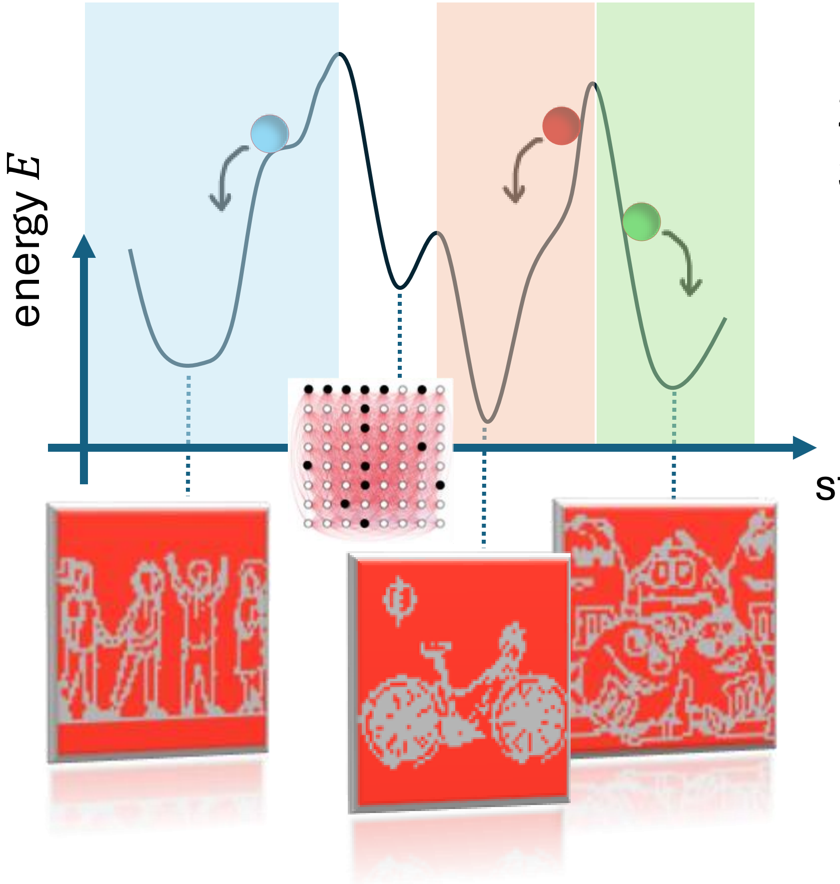
DJ Amit, H Gutfreund, H Sompolinsky. *PRL* (1985).



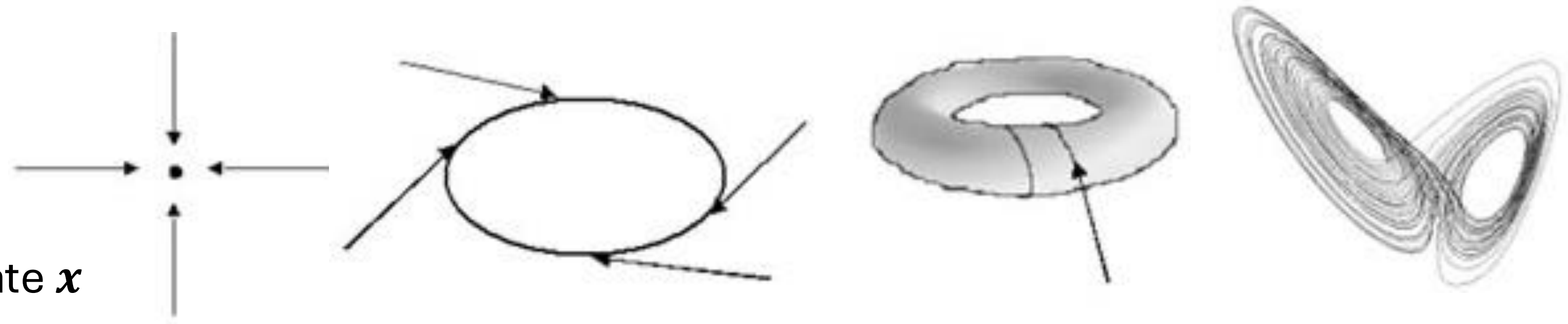
$C_{\max} \approx \dfrac{N}{f \log f}$ for sparse patterns

MV Tsodyks MV, MV Feigel'man. *Europhysics Letters* (1988).

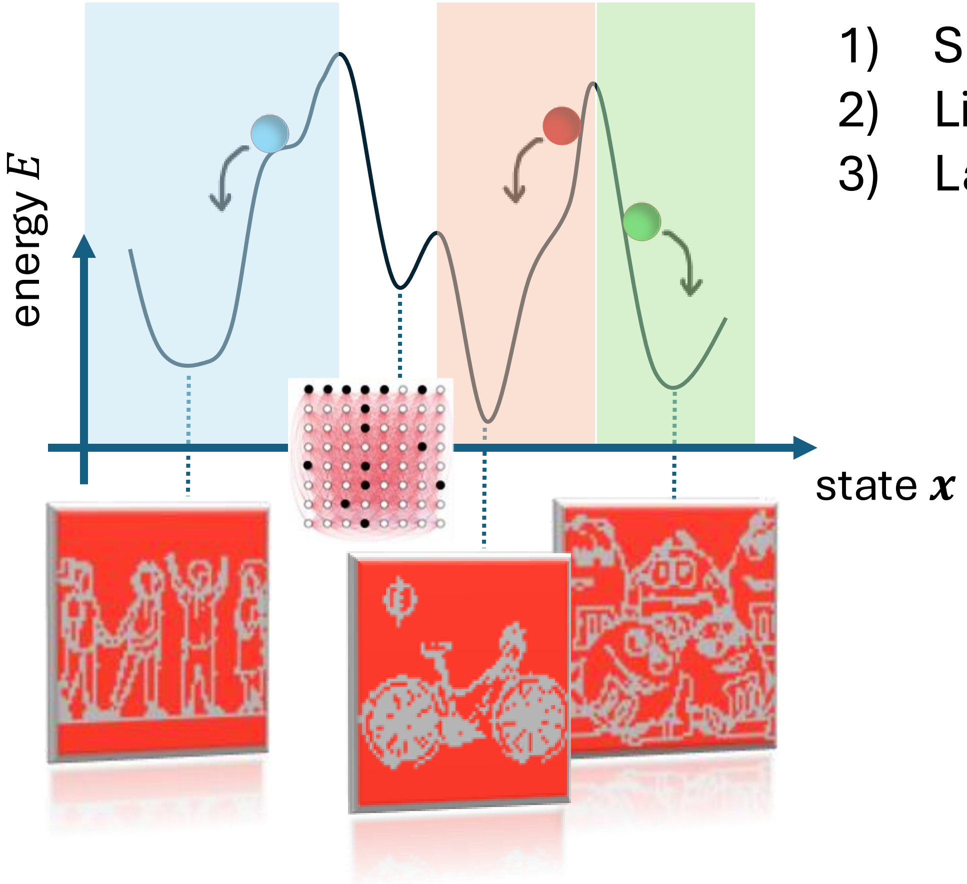
Challenges and solutions



1) Spurious/parasite attractors
2) Limited memory capacity
3) Lack of generalizability



Challenges and solutions



1) Spurious/parasite attractors
2) Limited memory capacity
3) Lack of generalizability

1) Oscillatory associative memory models

T Nishikawa, YC Lai YC, FC Hoppensteadt. *PRL* (2004).

T Guo, A Ogranovich, AR Venkatakrishnan, MR Shapiro, F Bullo, F Pasqualetti. *IEEE CDC* (2025).

2) Dense associative memory models

D Krotov & J Hopfield, *NeurIPS* 2016.

D Krotov, B Hoover, P Ram, B Pham. *arXiv:2507.0621*

3) Firing-rate models, recurrent NNs, spiking NNs

R Sepulchre. *Proc. IEEE* (2022).

TA Keller, M Welling. *ICML* (2023).

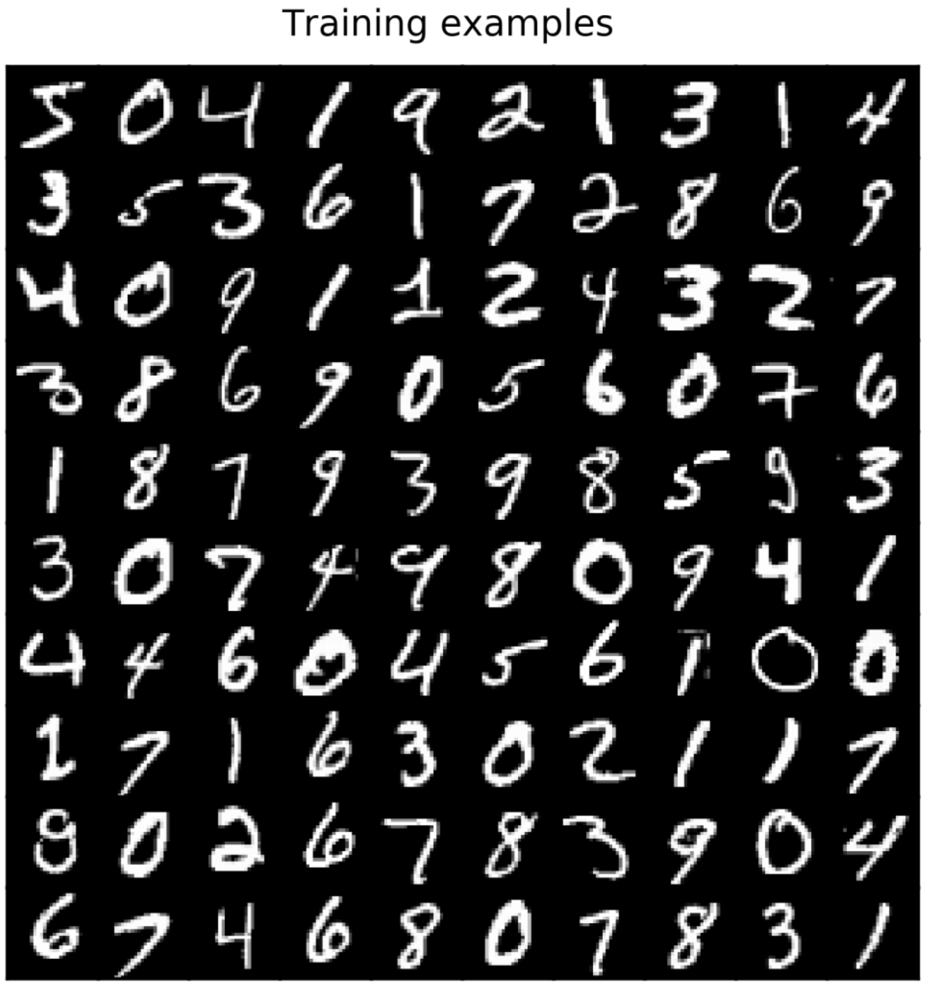
S Jafarpour, A Davydov, F Bullo. *IEEE TAC* (2023).

F Effenberger, P Carvalho, I Dubinin, W Singer. *PNAS* (2025).

L Kozachkov, M Ennis, J-J Slotine. *NeurIPS* (2022).

From Hopfield to Boltzmann machines

pattern completion *generate new patterns, never seen*

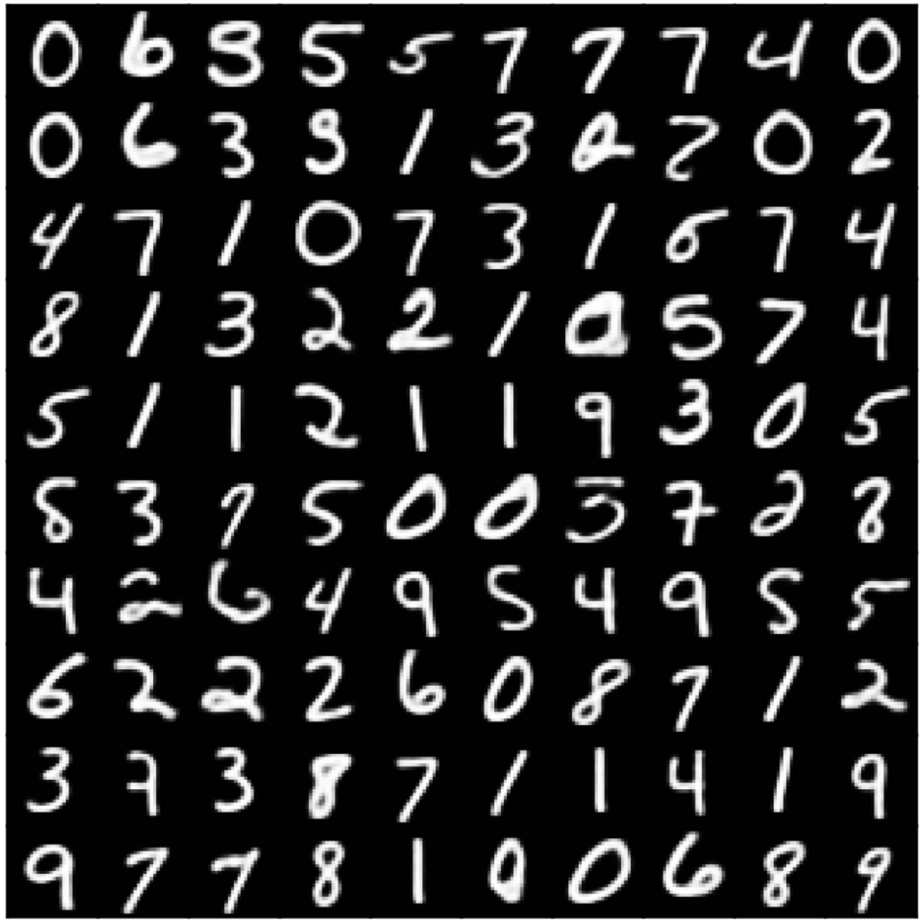


From Hopfield to Boltzmann machines

pattern completion *generate new patterns, never seen*

Training examples

Samples generated by DBM after 1337 Gibbs steps

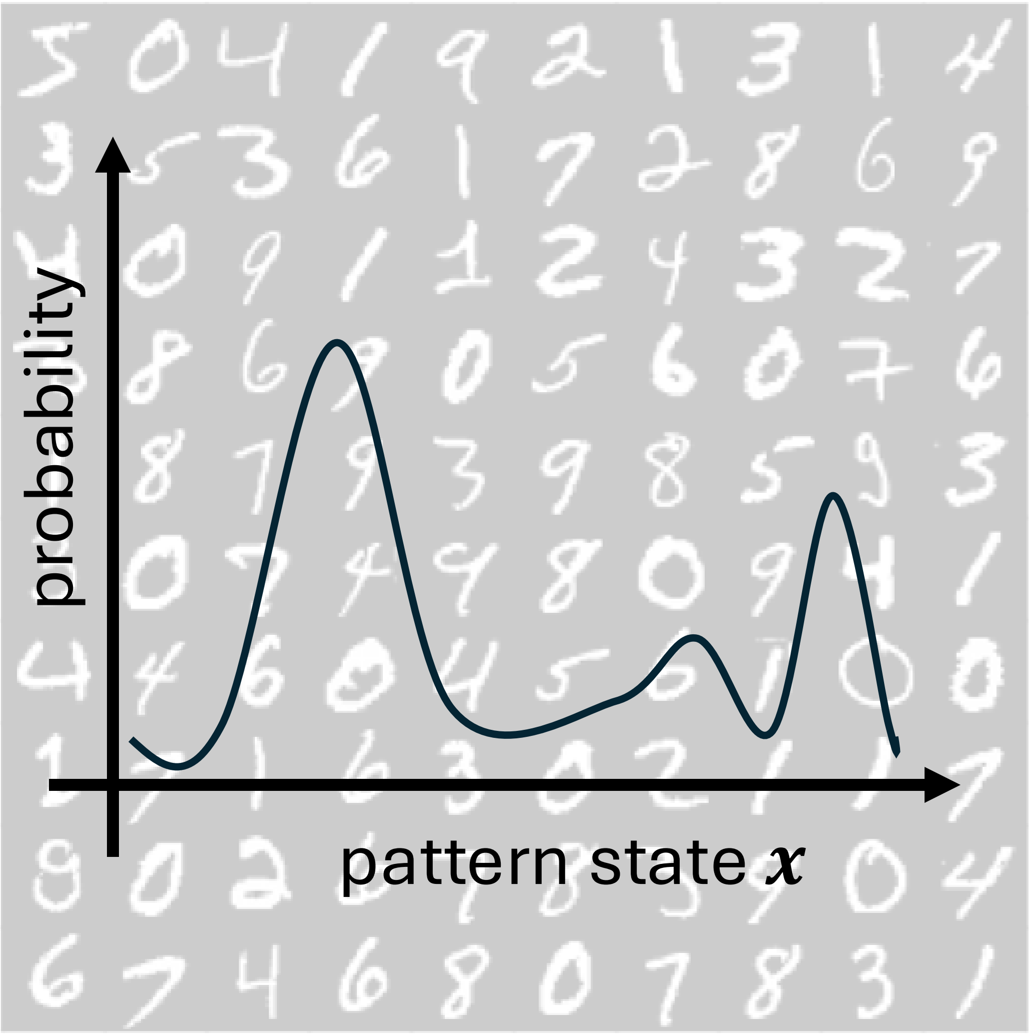


R Salakhutdinov, G Hinton. Deep Boltzmann machines. *AISTATS* (2009).

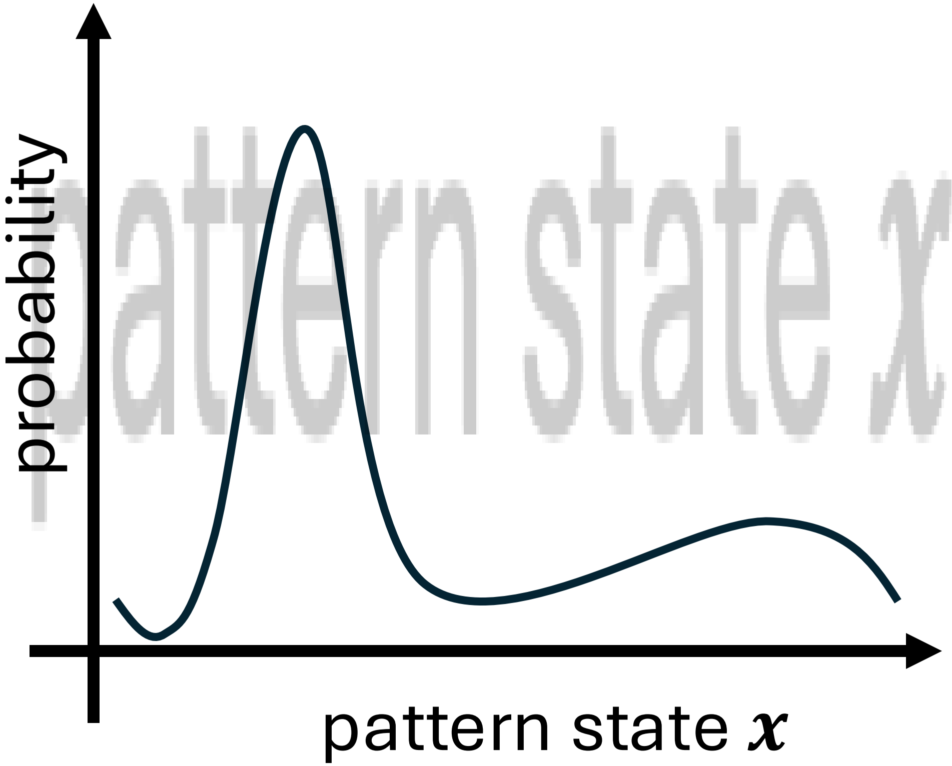
From Hopfield to Boltzmann machines

pattern completion *generate new patterns, never seen*

training data



model data



From Hopfield to Boltzmann machines

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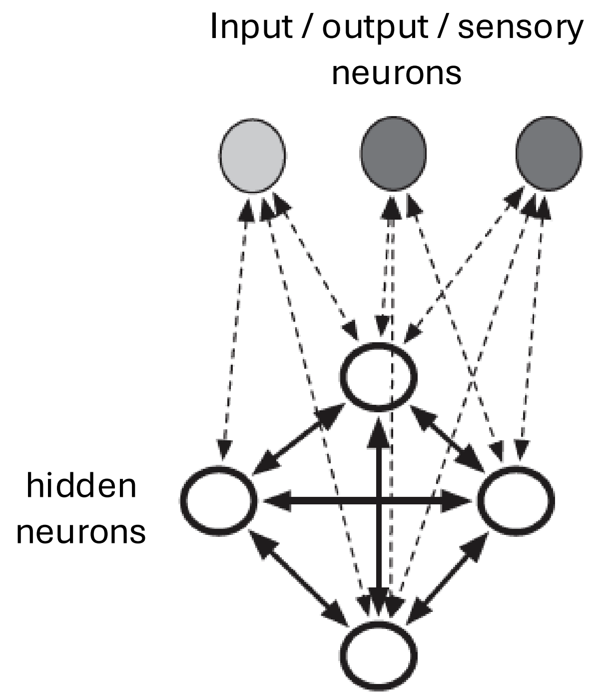
Further cognitive realism to Hopfield networks through 3 main modifications:

From Hopfield to Boltzmann machines

pattern completion *generate new patterns, never seen*

Further cognitive realism to Hopfield networks through 3 main modifications:

1) Hidden states



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Further cognitive realism to Hopfield networks through 3 main modifications:

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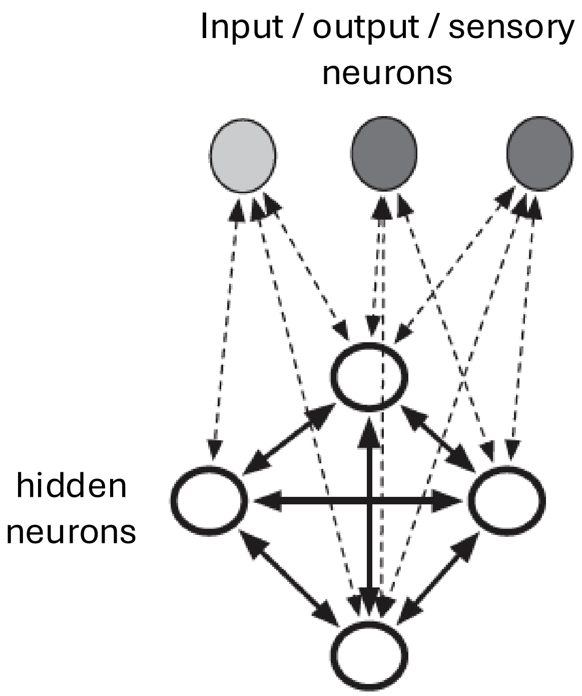
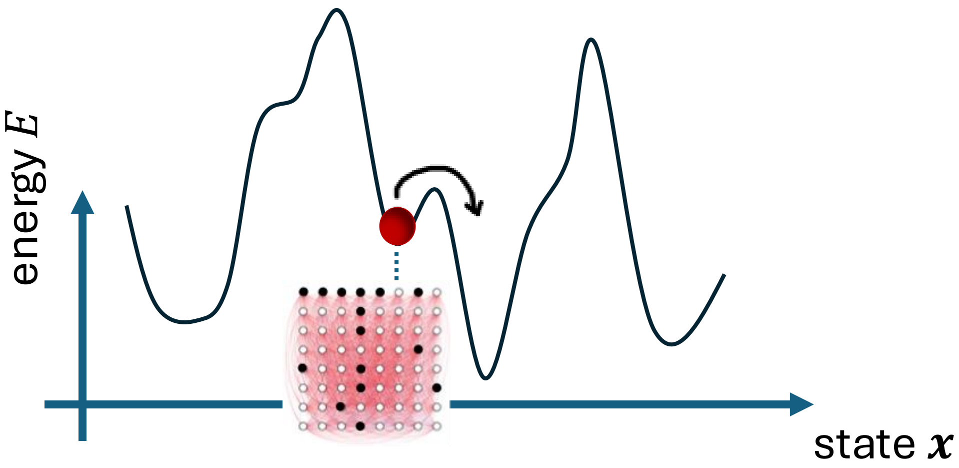


2) Stochasticity

Inference (discrete-time system)

$$x_i = \begin{cases} +1 \text{ with probability } p_{\text{on}} \\ -1 \text{ otherwise} \end{cases}$$



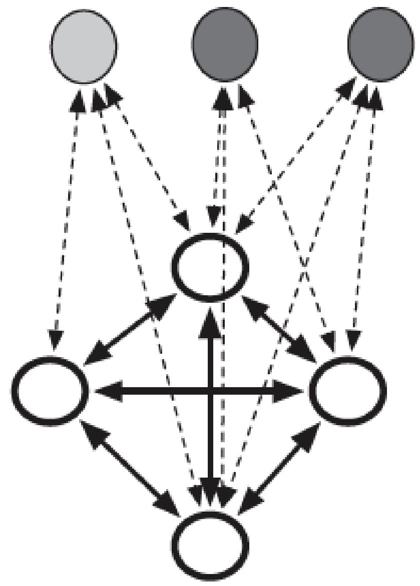
From Hopfield to Boltzmann machines

pattern completion *generate new patterns, never seen*

Further cognitive realism to Hopfield networks through 3 main modifications:

1) Hidden states

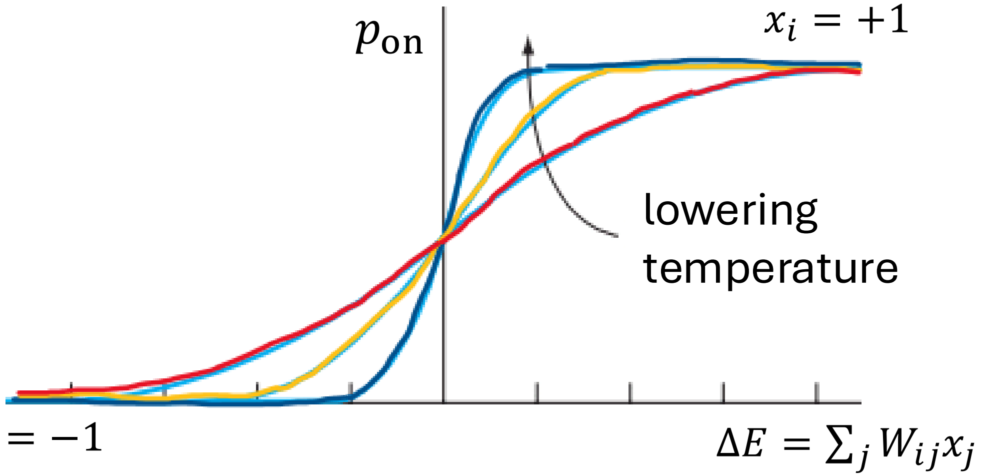
Input / output / sensory neurons



2) Stochasticity

Inference (discrete-time system)

$$x_i = \begin{cases} +1 \text{ with probability } p_{\text{on}} = \dfrac{1}{1+e^{-\frac{\Delta E}{T}}} = \dfrac{1}{1+e^{-\frac{\sum_j W_{ij}x_j}{T}}} \\ -1 \text{ otherwise} \end{cases}$$



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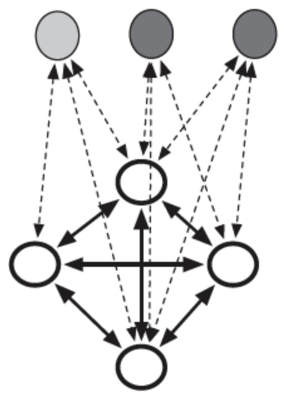
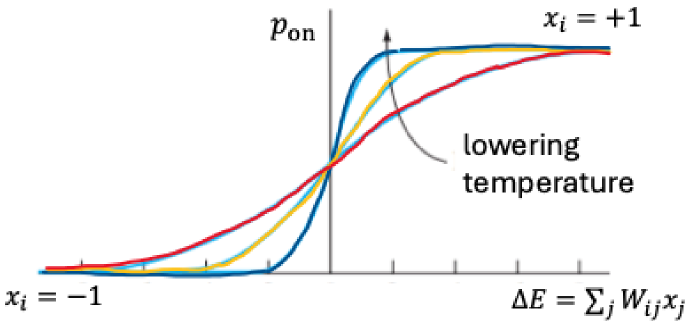


2) Stochasticity

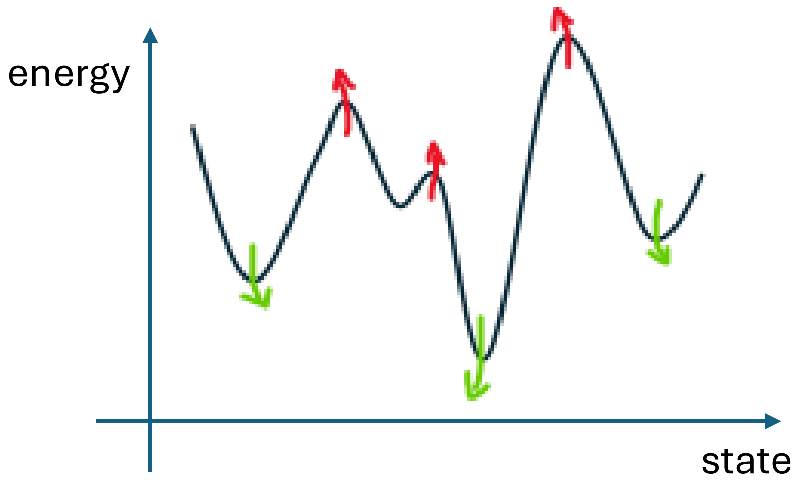


3) Contrastive Hebbian learning

$$\Delta W_{ij} = \underbrace{\left\langle \xi_i^{(p)} \xi_j^{(p)} \right\rangle_{p \,\in\, \text{dataset}}}_{\text{Hebbian term}} - \underbrace{\left\langle x_i^{(r)} x_j^{(r)} \right\rangle_{r \,\in\, \text{possible states}}}_{\text{anti-Hebbian term}}$$

Hebbian term
forces patterns to
have small energy

anti-Hebbian term
forces random states to
have high energy



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pattern completion *generate new patterns, never seen*

Further cognitive realism to Hopfield networks through 3 main modifications:

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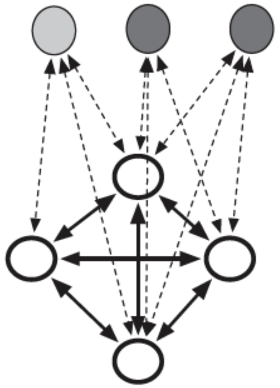
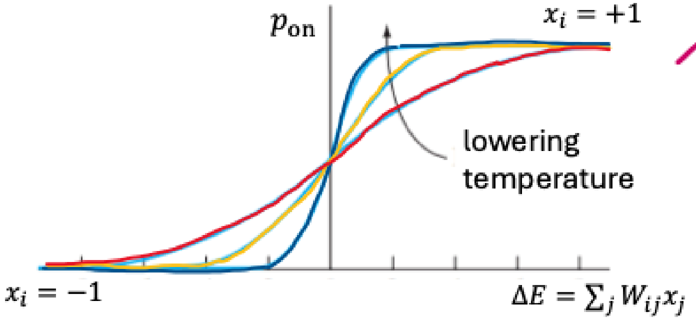


2) Stochasticity



3) Contrastive Hebbian learning

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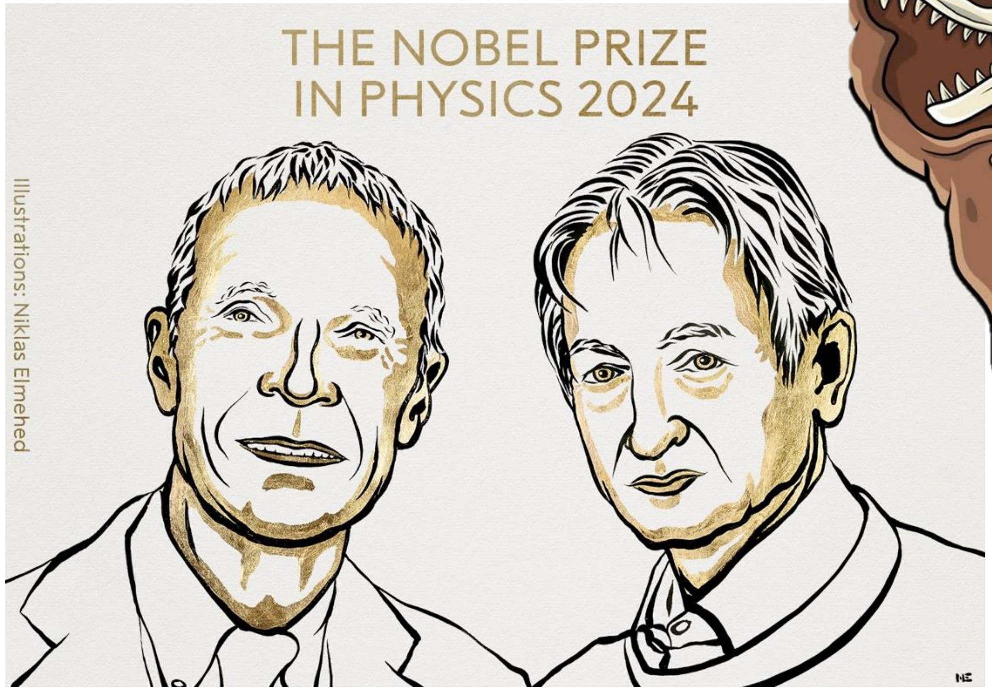
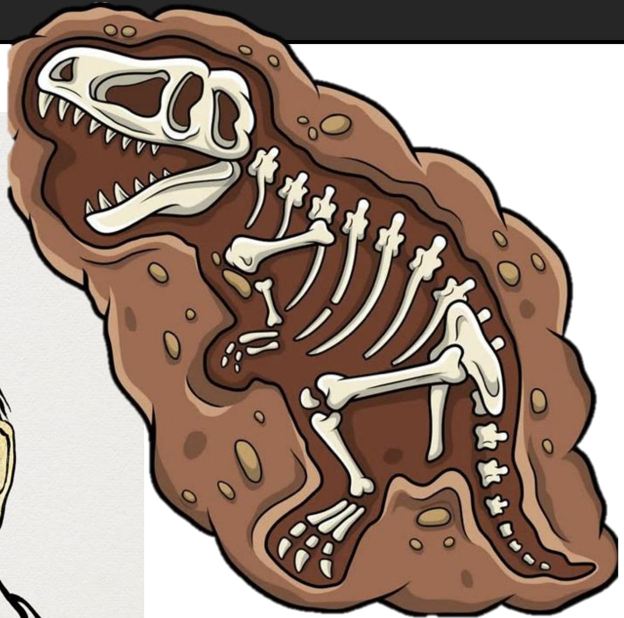
Boltzmann machine with contrastive Hebbian learning gradually learns to generate output distributions similar to the pattern data.

$$\min_W D_{KL}\big(p_{\text{data}}(s) \parallel p_{\text{model}}(s|h)\big)$$

Insights from classical work

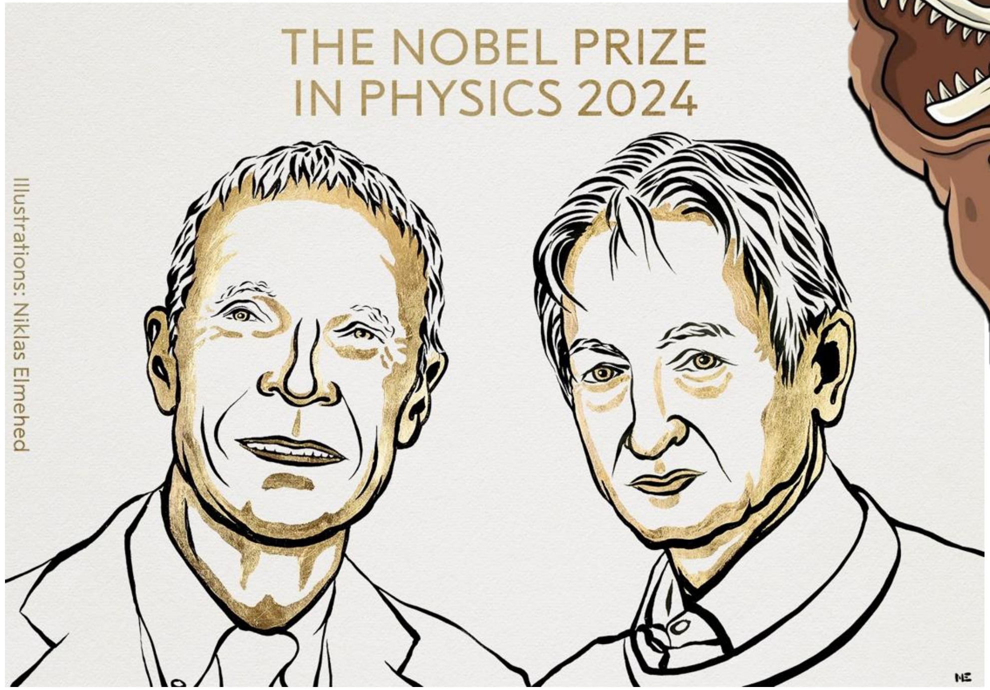




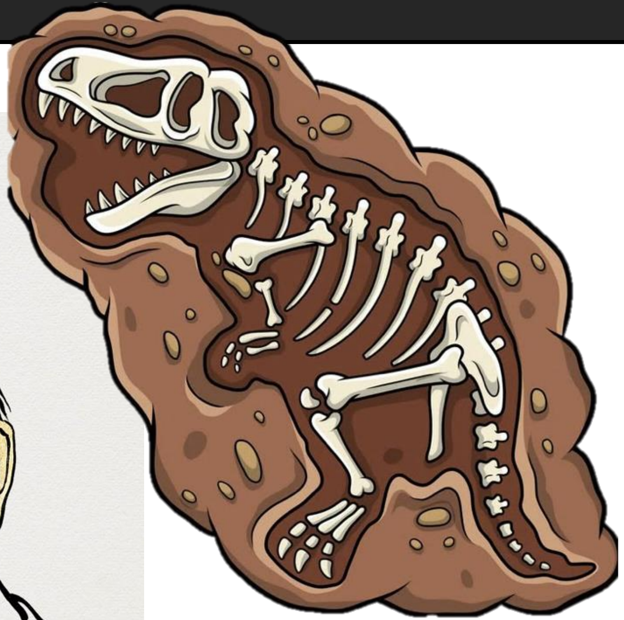
John Hopfield Geoffrey Hinton

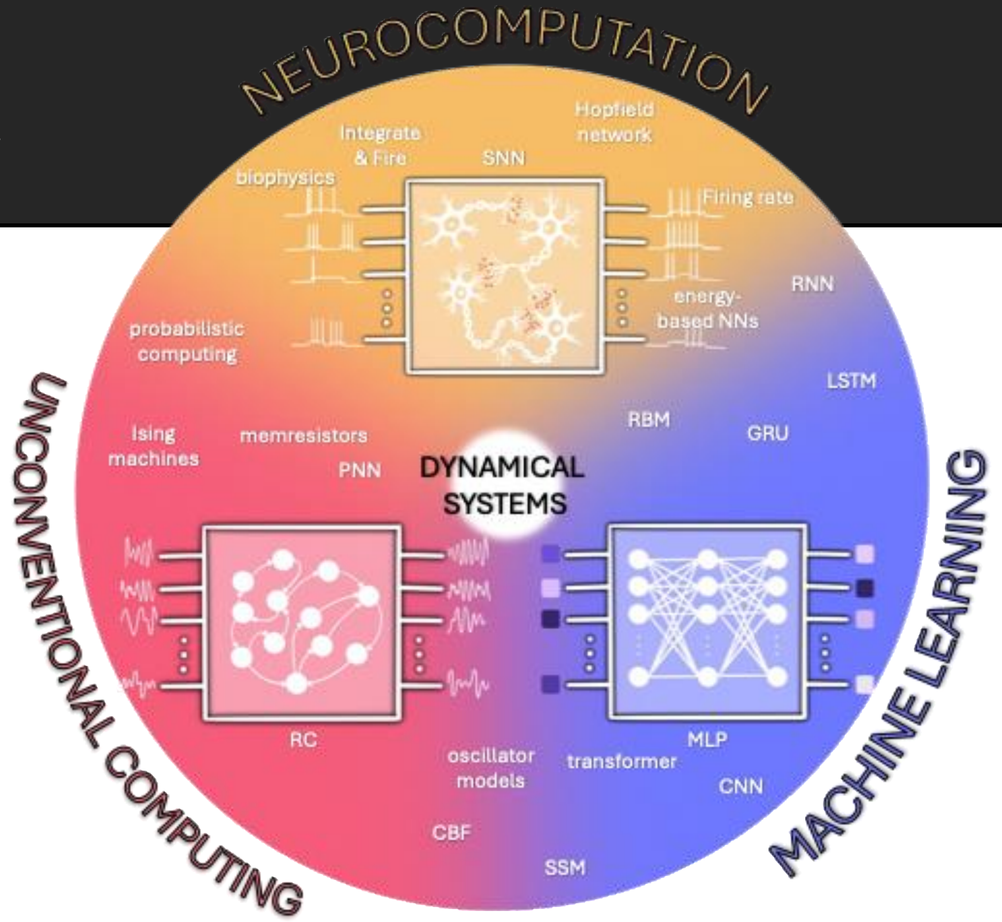
Philip Anderson, Daniel Thouless, David Amit (spin glasses)
Yoshua Bengio, Yan LeCunn (deep learning)
David Rumelhart (backpropagation)
Terrence Sejnowski (Boltzmann machines)
Alex Krizhevsky, Ilya Sutskever (AlexNet)
et al…

Insights from classical work



John Hopfield   Geoffrey Hinton





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et al...

How we can design/apply these models for...

- combinatorial optimization (Ising machines)
  Ermin Wei & Ahmed Allibhoy
- neuro-inspired / data-driven control
  Dmitri Chklovskii, Enrique Mallada & Jorge Cortés