55th Winter Colloquium on the Physics of Quantum Electronics

# Disorder-promoted synchronization and coherence in coupled laser networks

**Arthur Montanari**

Center for Network Dynamics
Department of Physics and Astronomy
Northwestern University

January 9, 2026

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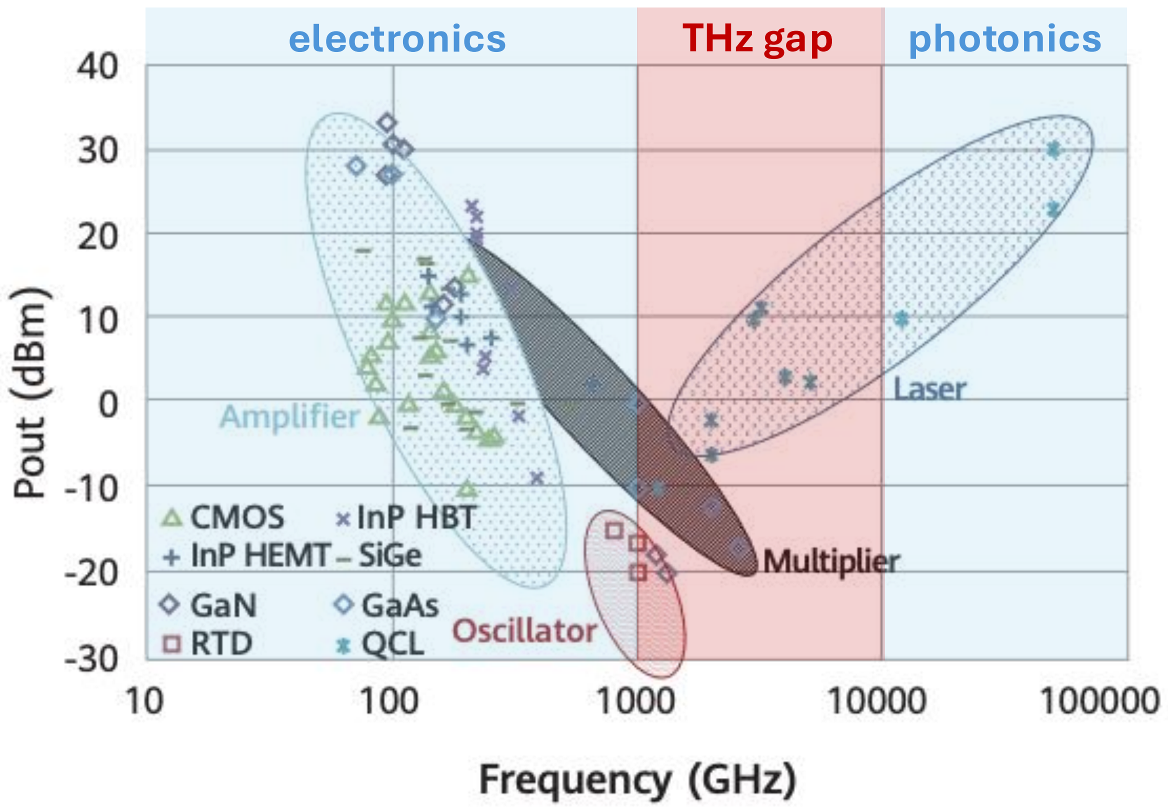
# The THz Gap




Potential applications of THz gap:

- imaging, spectroscopy, sensing (ideal penetration)
- high-speed, free-space communication (wireless communication, LIDAR)
- THz computing (analog, neuromorphic computing, Ising machines)

G Wang, et al. Terahertz sensing and communication towards future intelligence connected networks. *Huawei Research Report* (2022).

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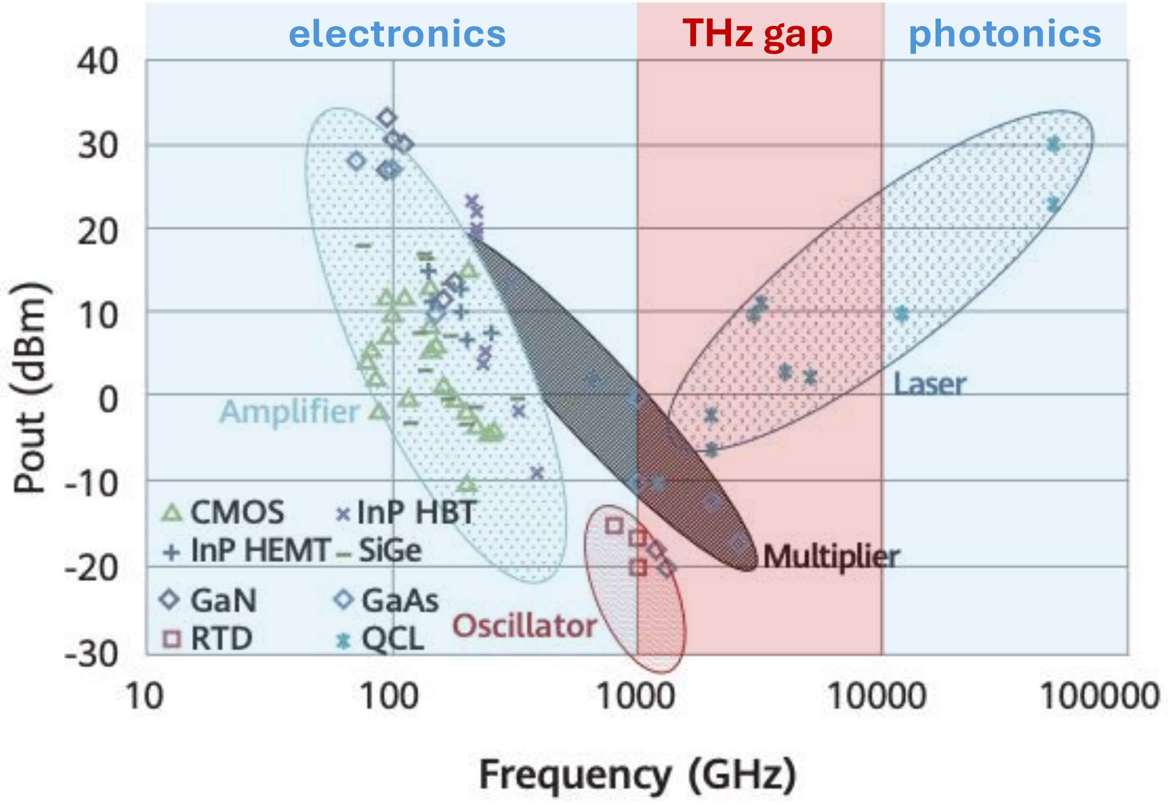


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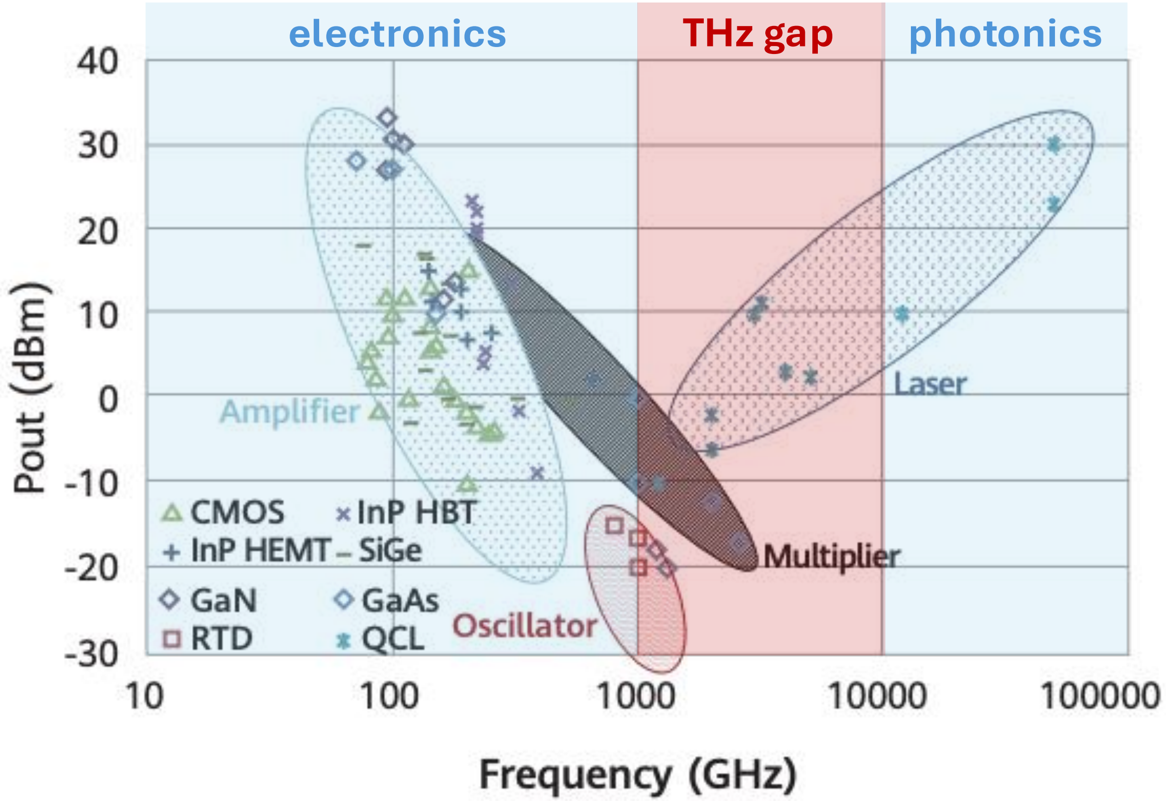


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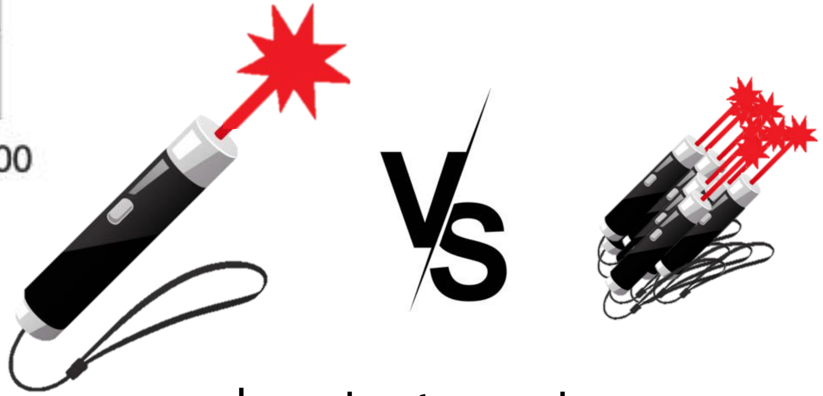


how to generate coherent beams?

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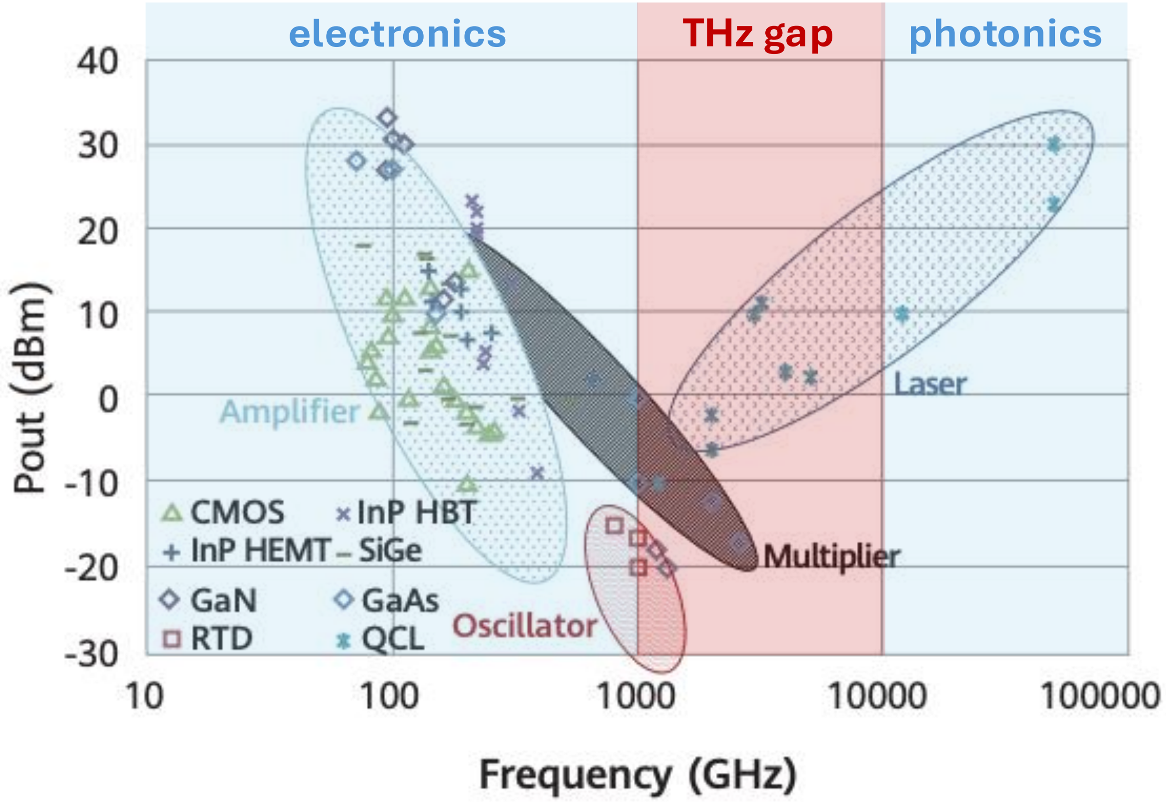


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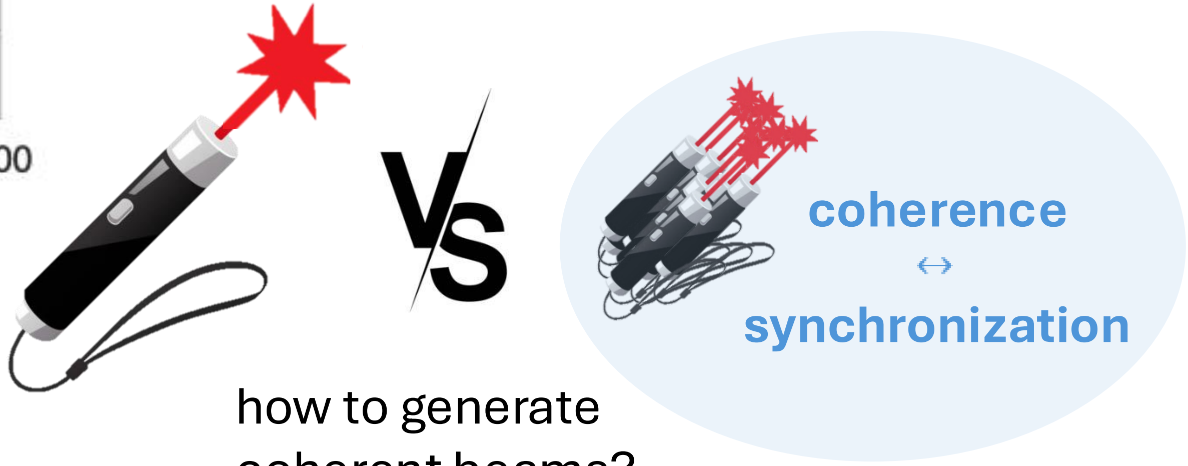


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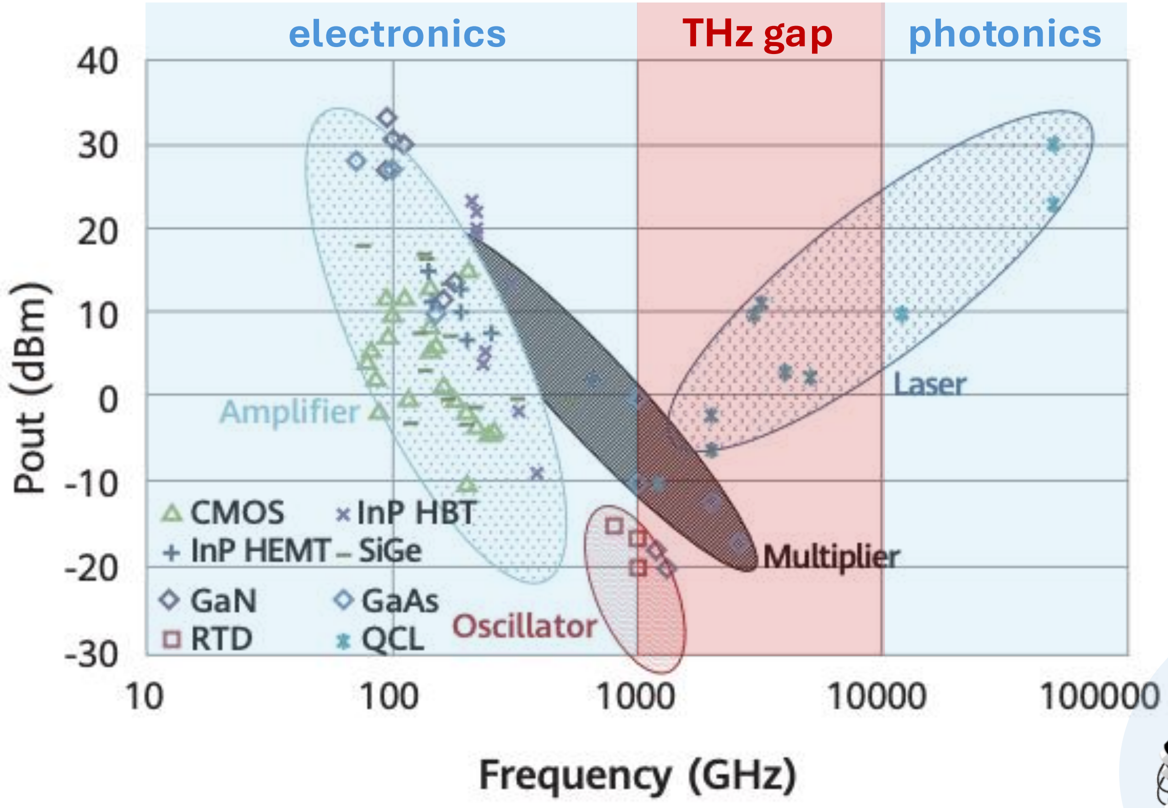


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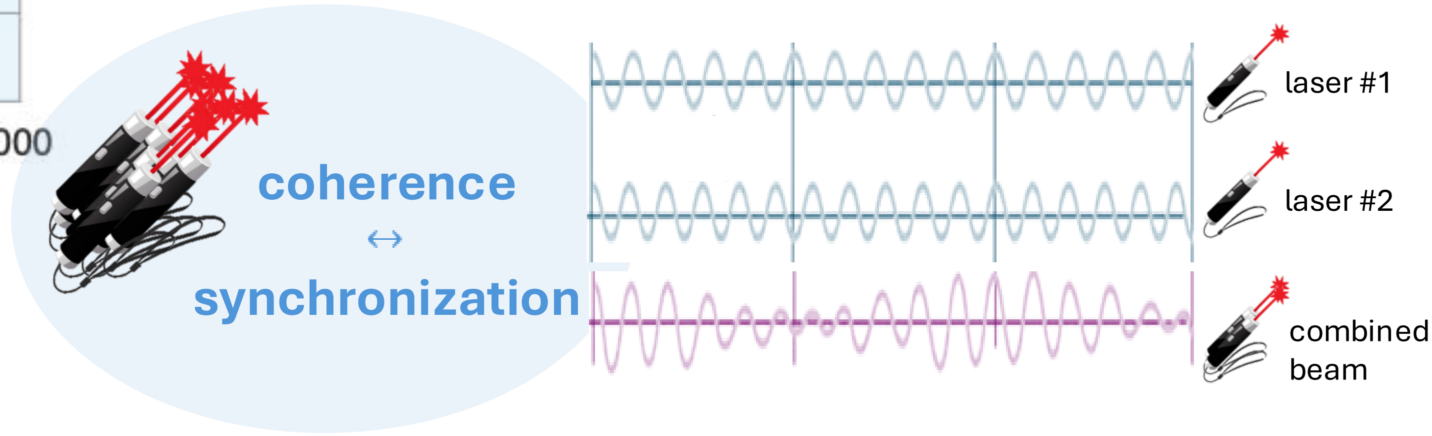

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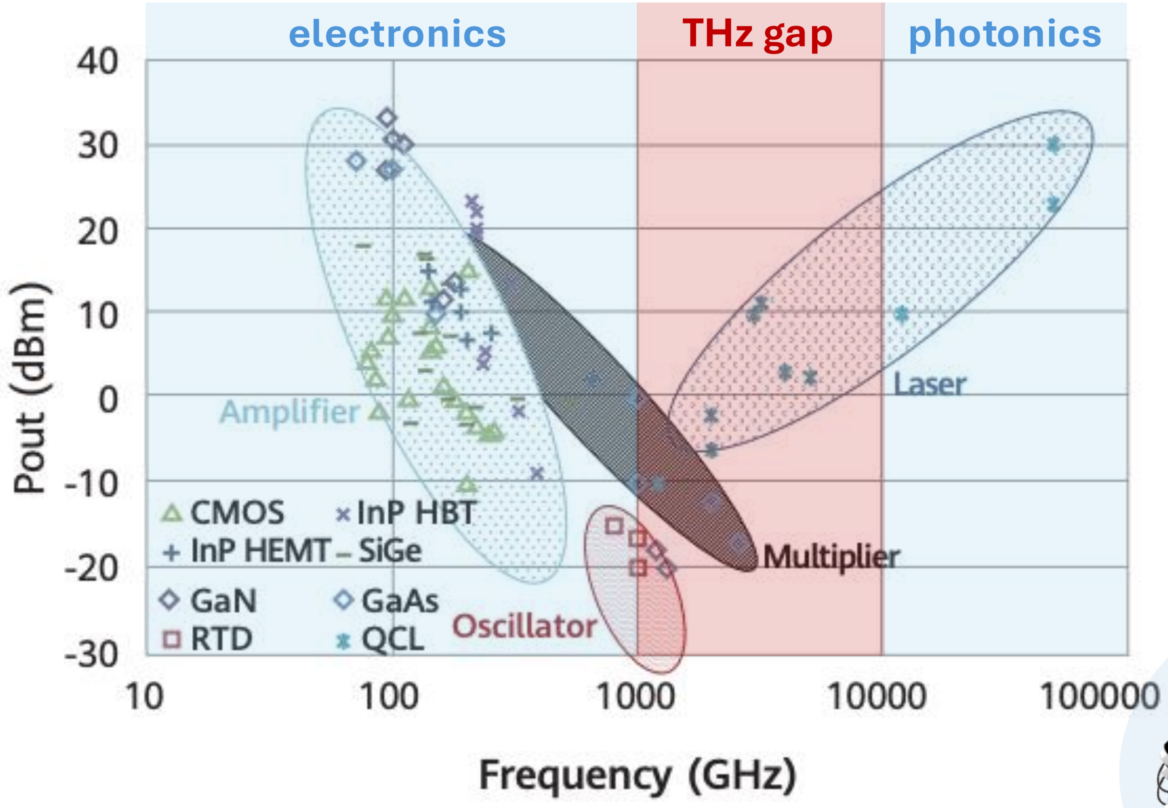


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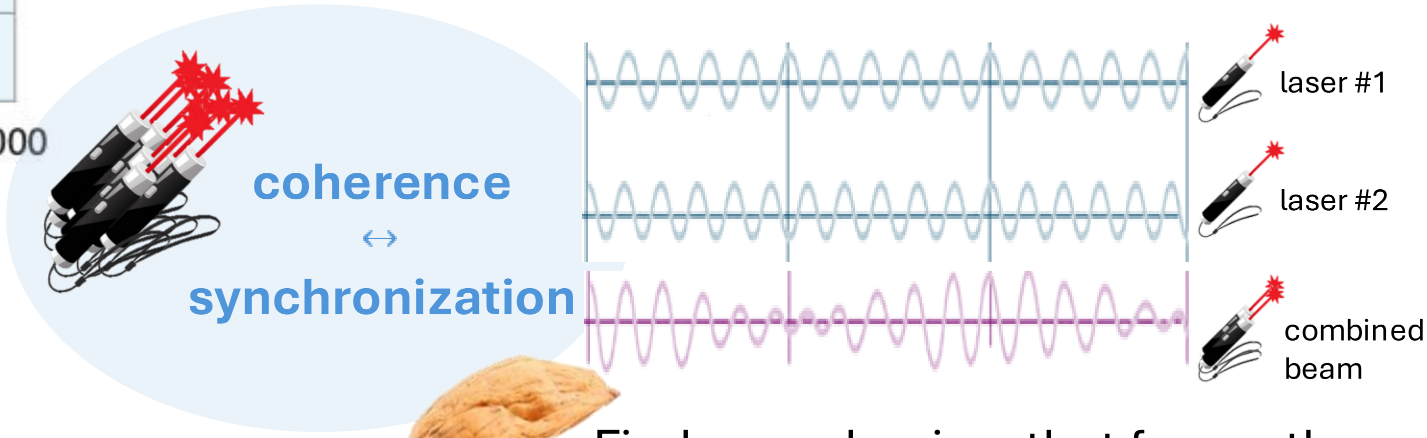


Find a mechanism that forces the laser beams to interact & sync

# Laser sync

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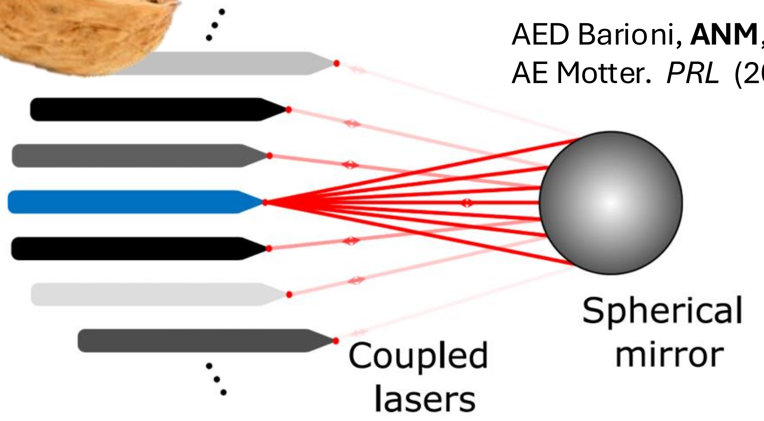
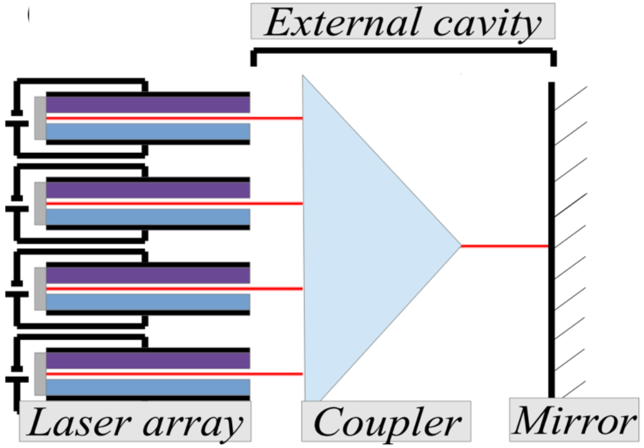



AED Barioni, **ANM**,
AE Motter. *PRL* (2025).

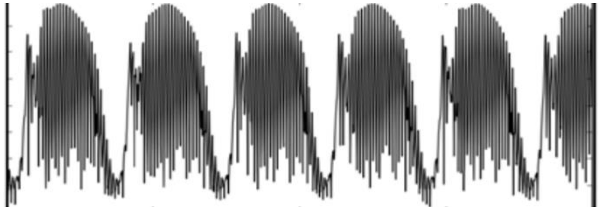



F Bohm, et al.
*PRE* (2015).

## Low-frequency fluctuations

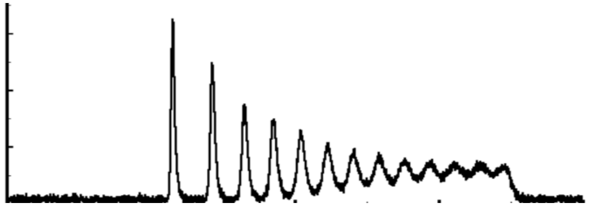
A Torcini, et al. *PRA* (2006)

## Regular pulse packages



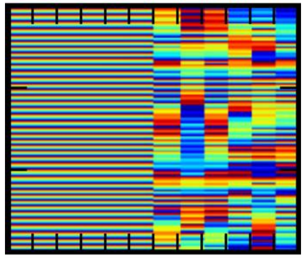
S Ruschel, S Yanchuk. *Chaos* (2017)

## Relaxation oscillations



B Liu, et al. *Opt Express* (2021)

## Chimera states



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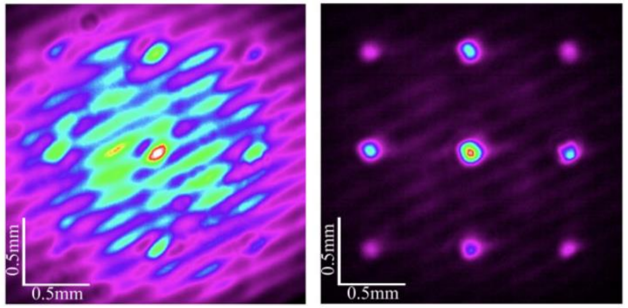


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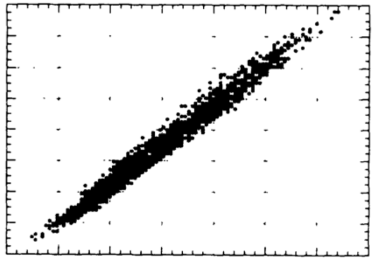

R Roy, KS Thornburg.
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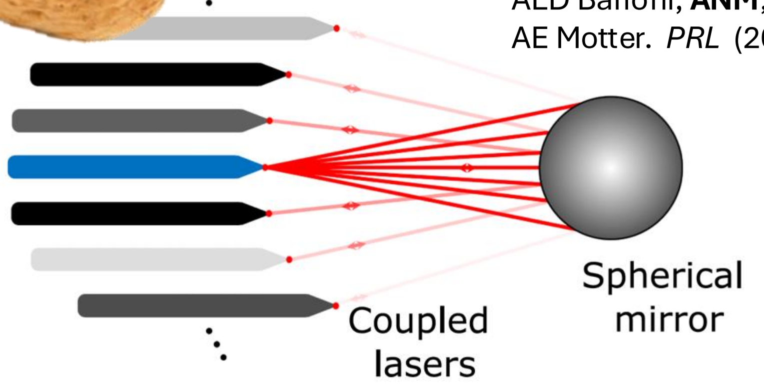


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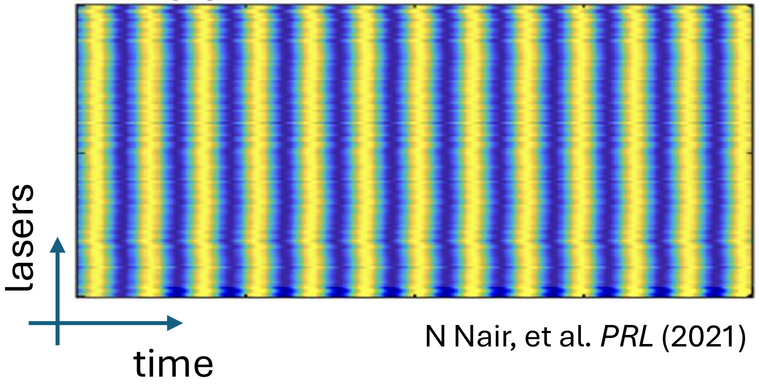
## Frequency synchronization




N Nair, et al. *PRL* (2021)

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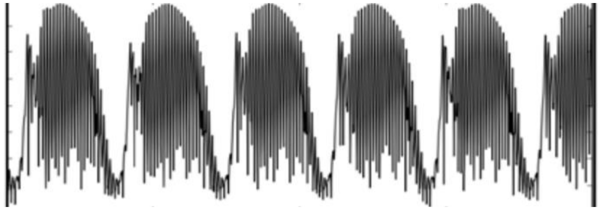

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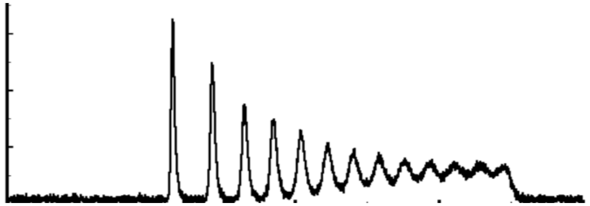

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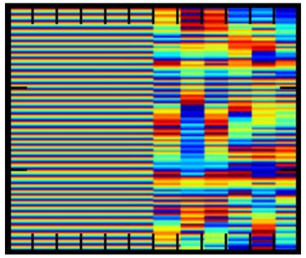

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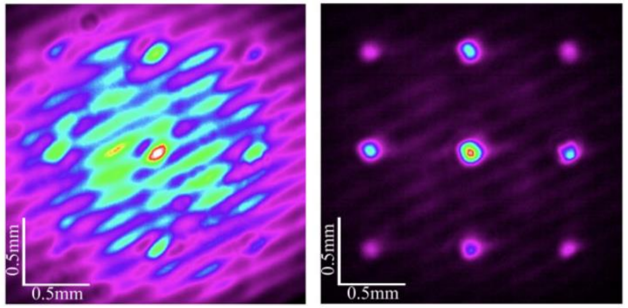


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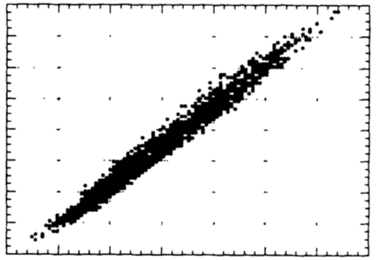
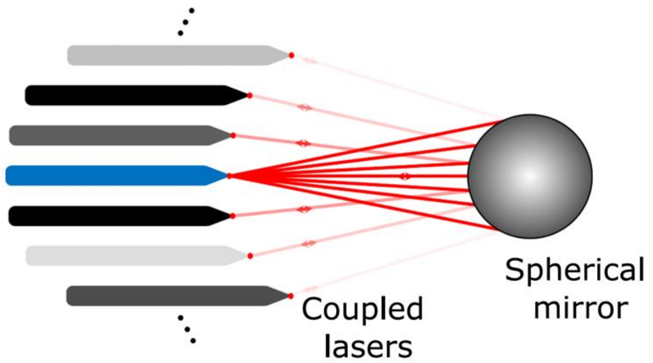


R Roy, KS Thornburg. *PRL* (1994).

# Lang-Kobayashi model

for single-mode semiconductor diode lasers (e.g., GaAs, InP, GaN)

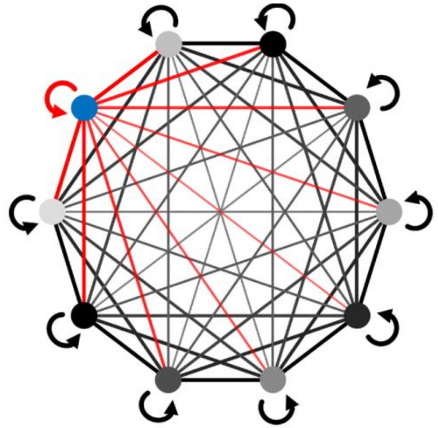



Electric field dynamics

$$\dot{E}_j(t) = \underbrace{\frac{1+i\alpha_j}{2}\left(G_j-\gamma\right)E_j(t)}_{\text{phase-amplitude coupling}} + \underbrace{i\omega_j E_j(t)}_{\text{natural frequency}} + \underbrace{\kappa_j \sum_{k=1}^{M} A_{jk} E_k(t-\tau_{jk})}_{\text{delayed-coupling (external cavity)}},$$

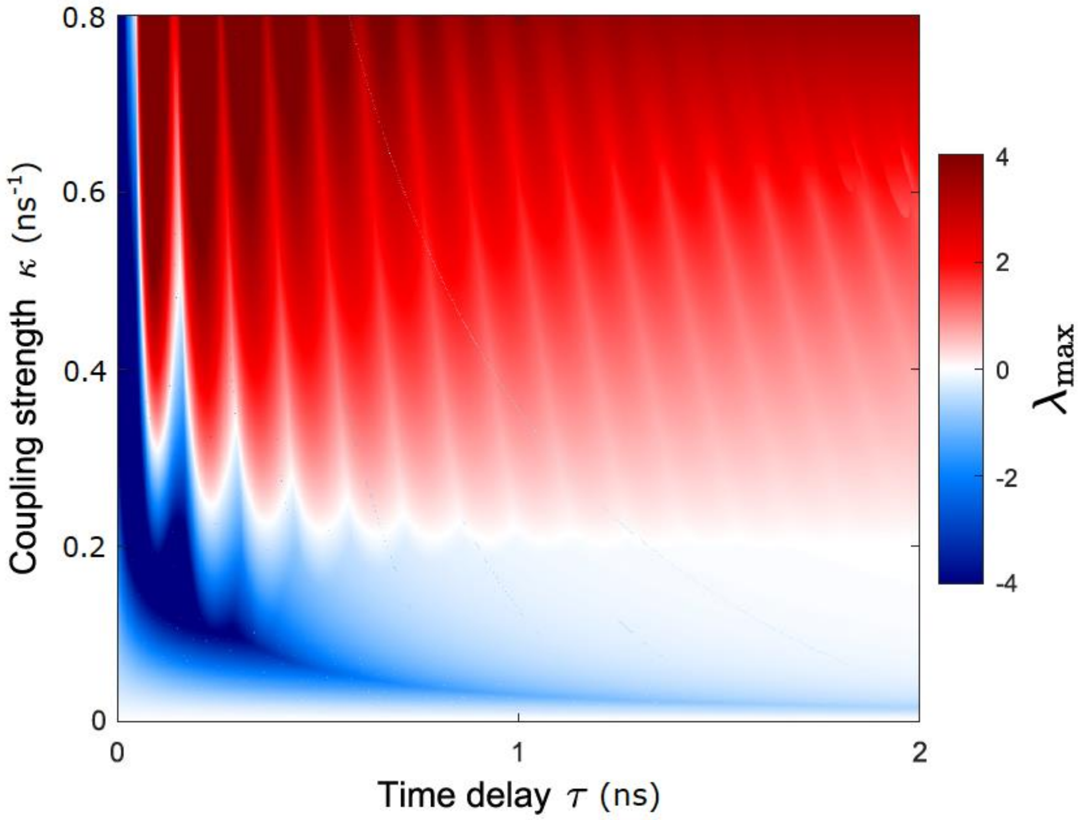
Carrier number dynamics

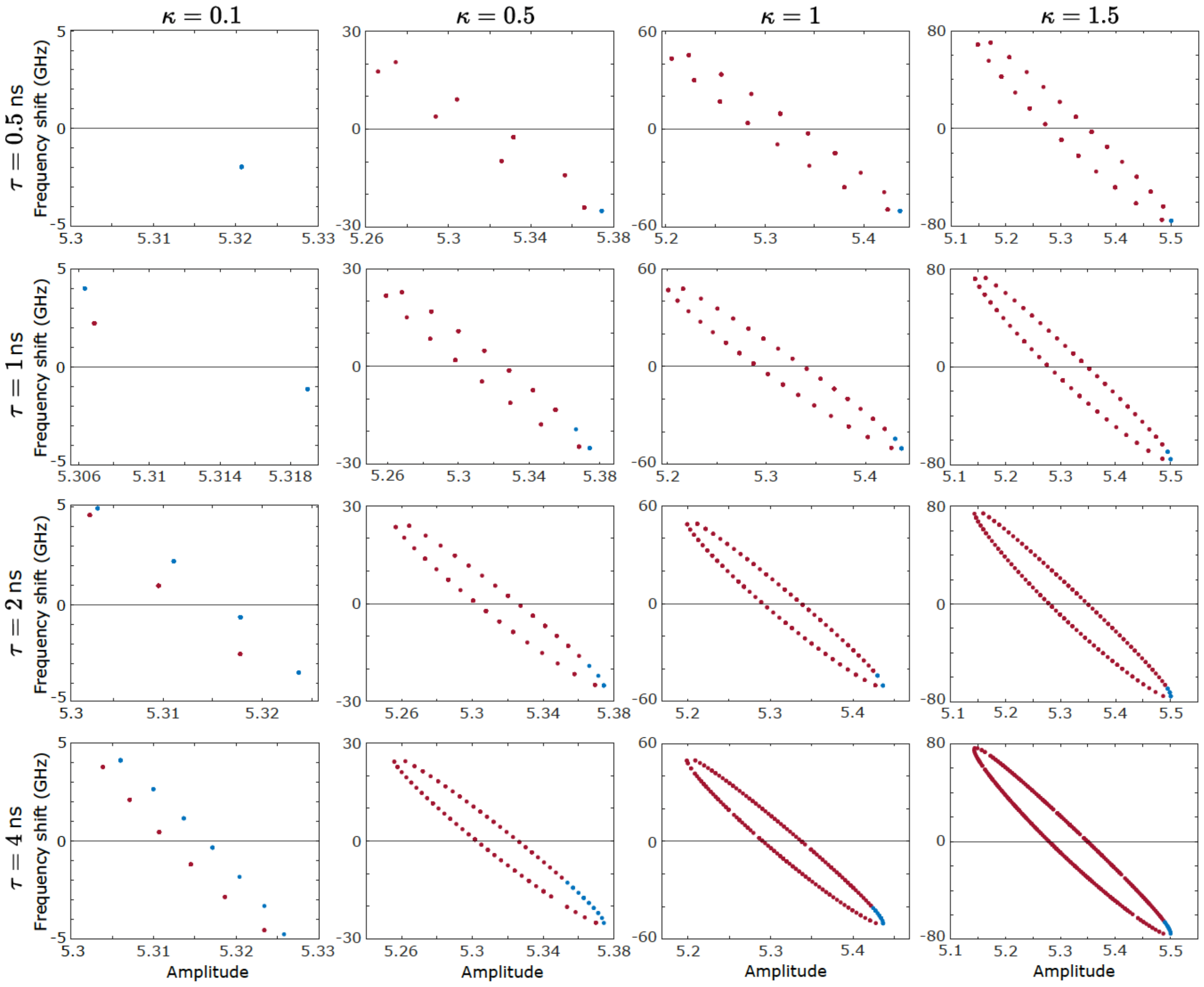
$$\dot{N}_j(t) = \underbrace{J_0}_{\text{current source}} - \underbrace{\gamma_n N_j(t)}_{\text{damping}} - \underbrace{G_j\left|E_j(t)\right|^2}_{\text{media gain}},$$



R Lang, K Kobayashi.
*IEEE J. Quantum Electronics* (1980).

# Multistability of the LK model



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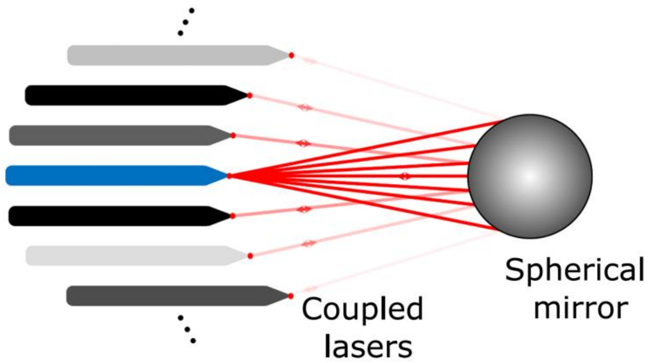
phase-amplitude coupling

natural frequency

delayed-coupling (external cavity)

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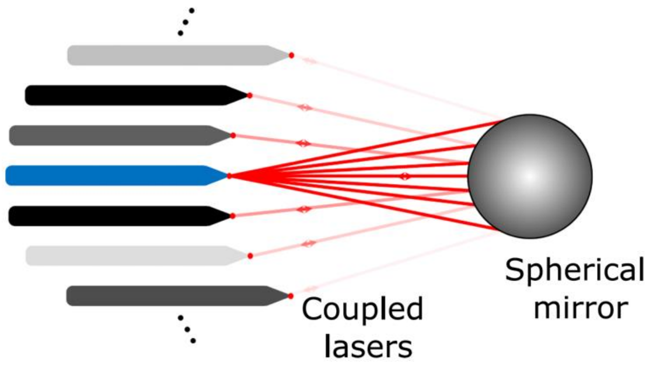


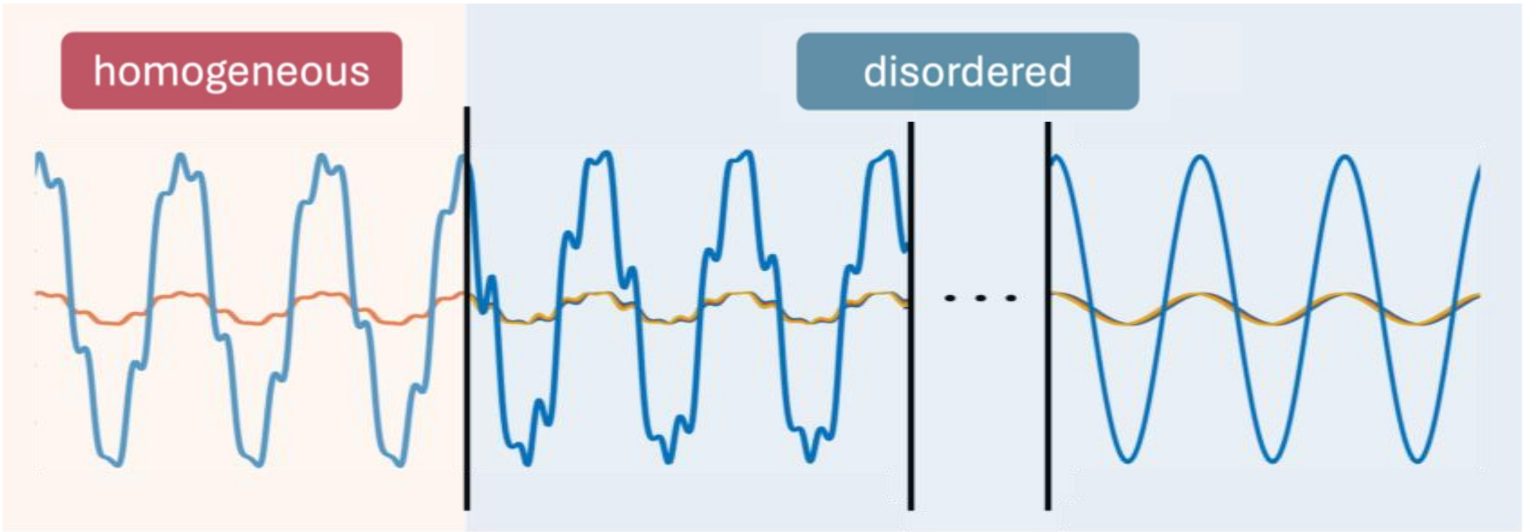


$\omega_j \sim \mathcal{N}(0,1)$

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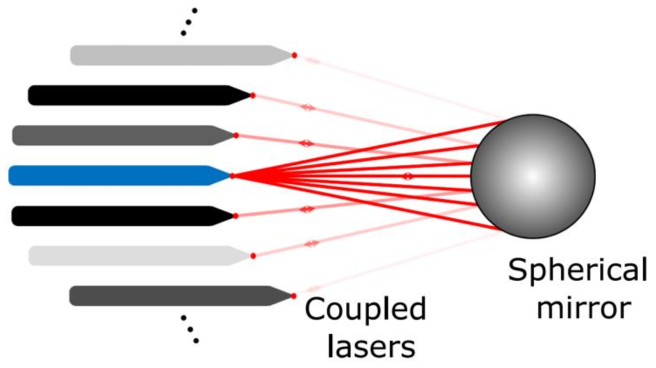


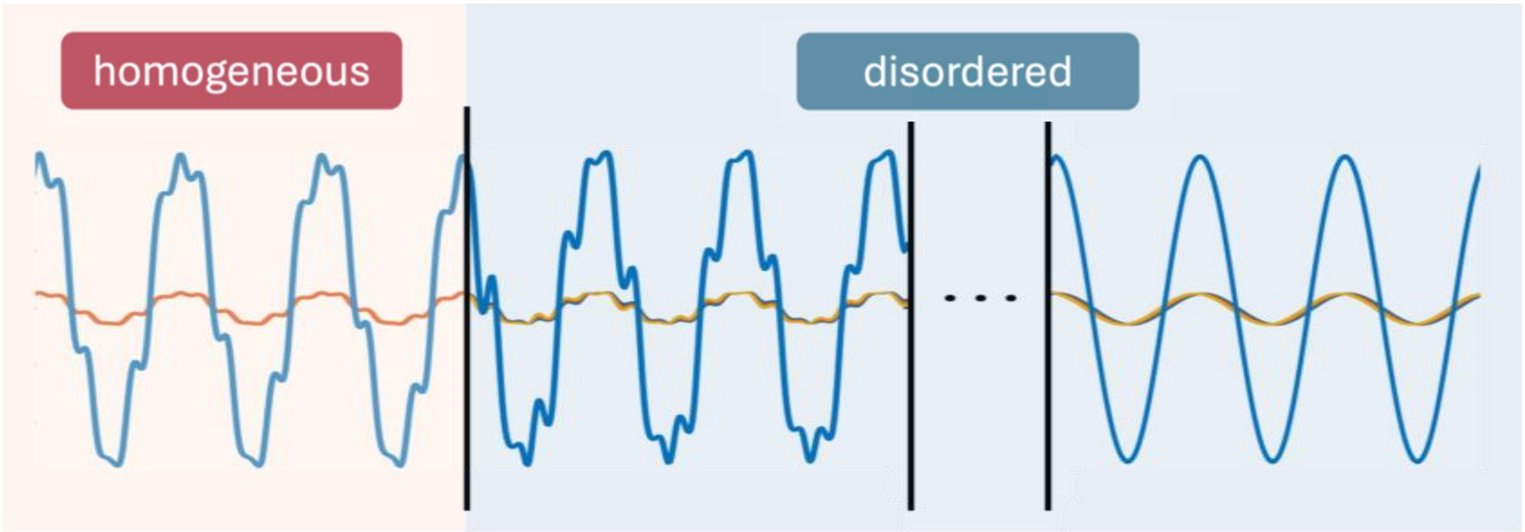


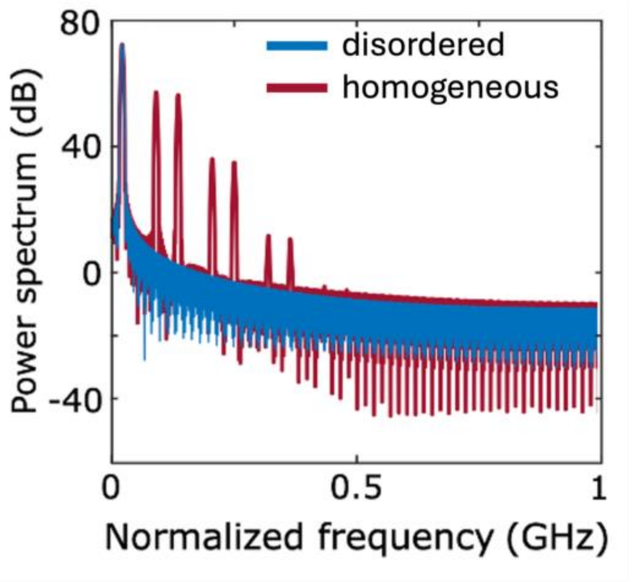

# Some stability analysis to study it

Nonlinear time-delay system (LK model)

$$\dot{\mathbf{x}}_j(t) = \underbrace{\mathbf{f}_j(\mathbf{x}_j(t))}_{\text{laser dynamics}} + \underbrace{\kappa_j \sum_{k=1}^{M} A_{jk}\mathbf{h}\big(\mathbf{x}_j(t), \mathbf{x}_k(t-\tau)\big)}_{\text{delayed coupling}}$$

Linearization around the desired synchronous state $E_j(t) = r_j^* e^{i(\Omega t + \delta_j^*)}$

$$\dot{\boldsymbol{\eta}}(t) = J_1 \boldsymbol{\eta}(t) + J_2 \boldsymbol{\eta}(t-\tau)$$

# Stability analysis

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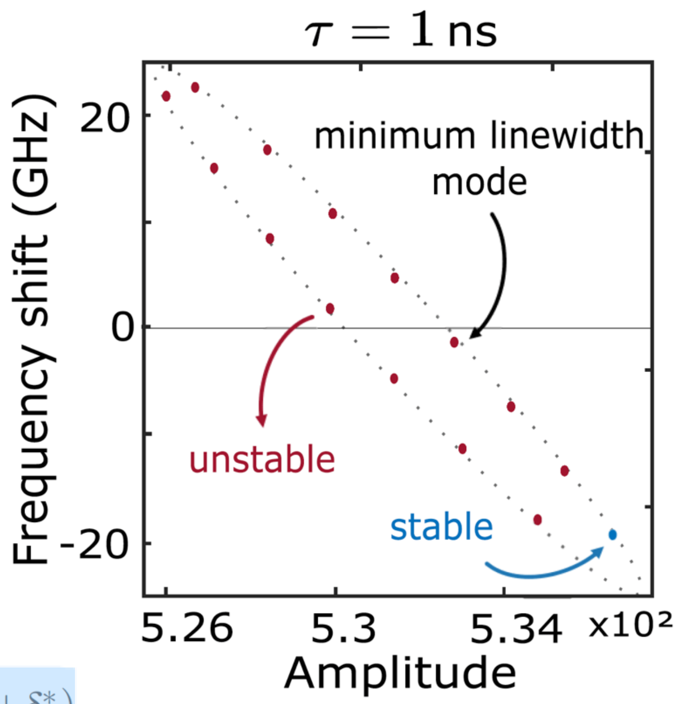
laser dynamics — delayed coupling




ideal lasing frequency

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coherent if $\delta_j^* \approx 0, \ \forall j$

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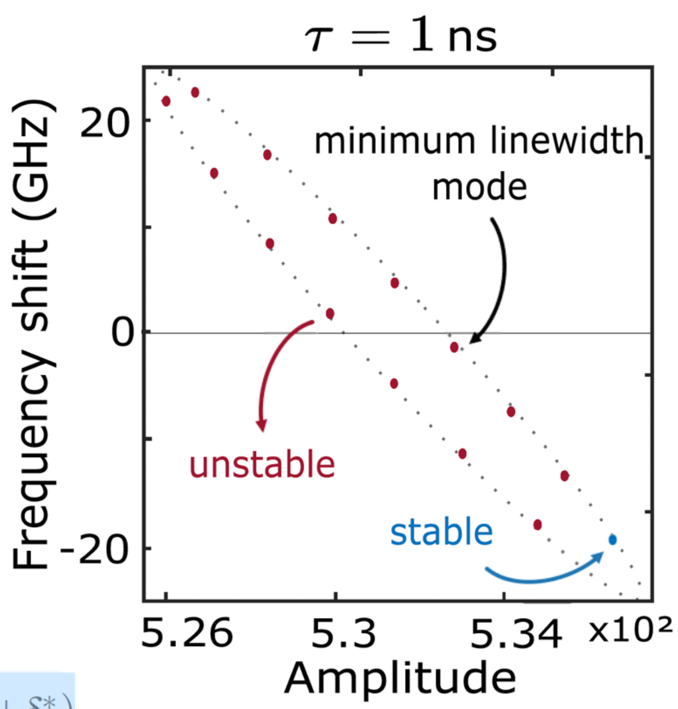


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Solve characteristic equation to find the (generalized) eigenvalues $\lambda_\ell$

$$\det\left(J_1 + J_2 e^{-\lambda_\ell \tau} - \lambda_\ell I_{3M}\right) = 0$$

using MATLAB package DDE-BIFTOOL

Synchronous state is stable iff $\lambda_{\max} = \max \mathrm{Re}\{\lambda_\ell\} < 0$

# Main result

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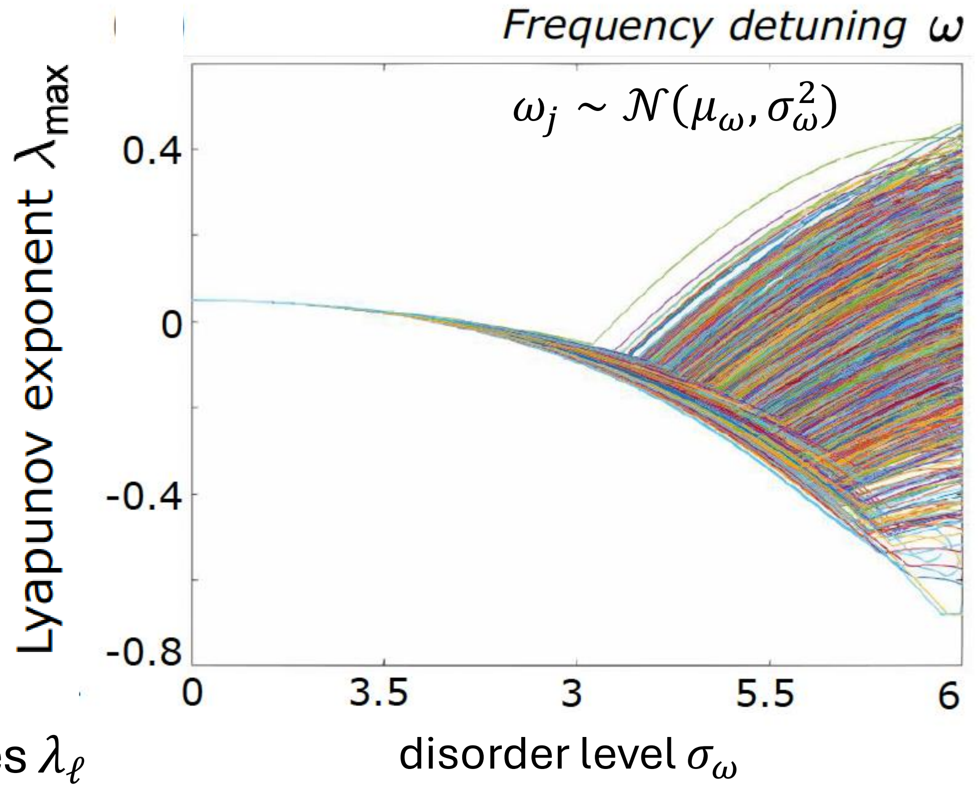

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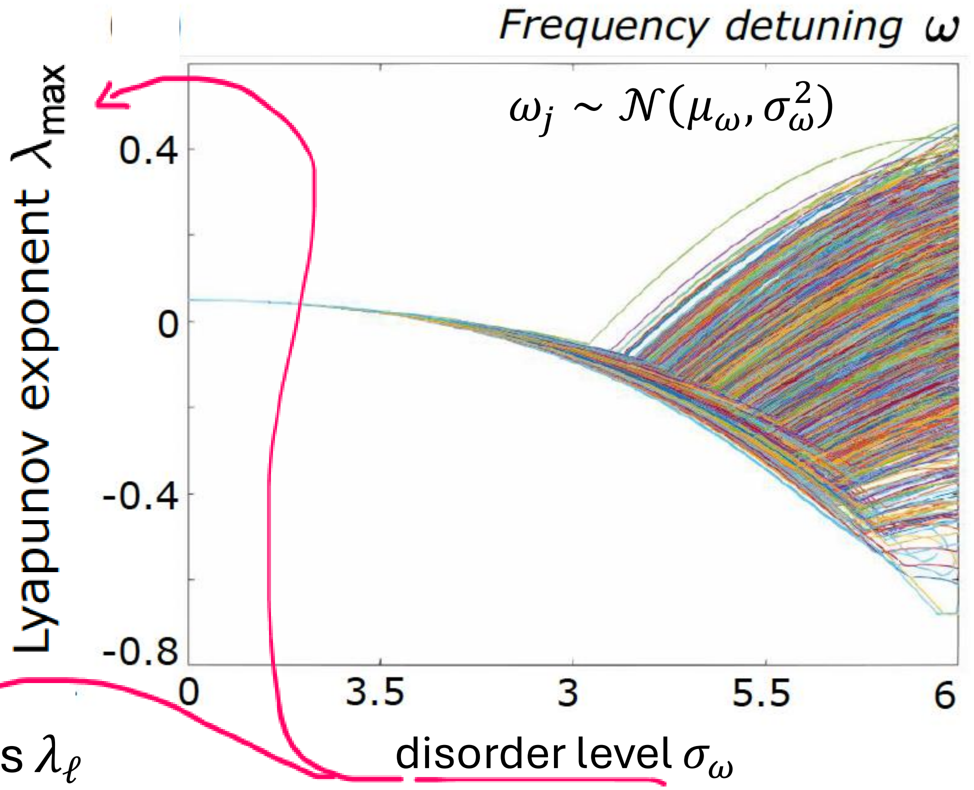


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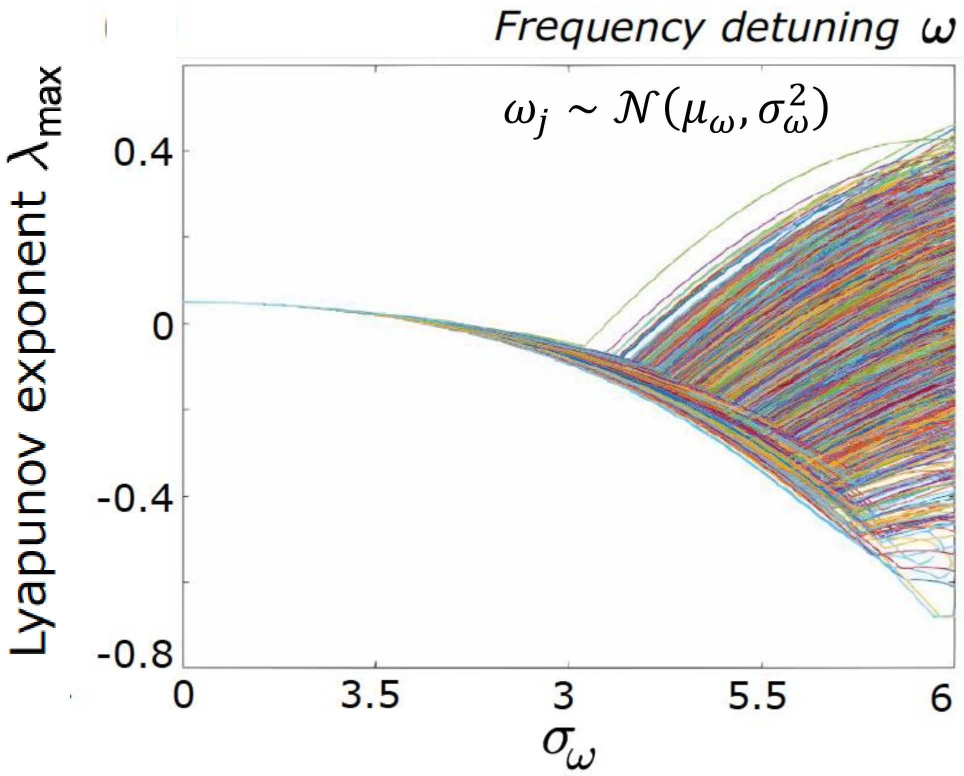


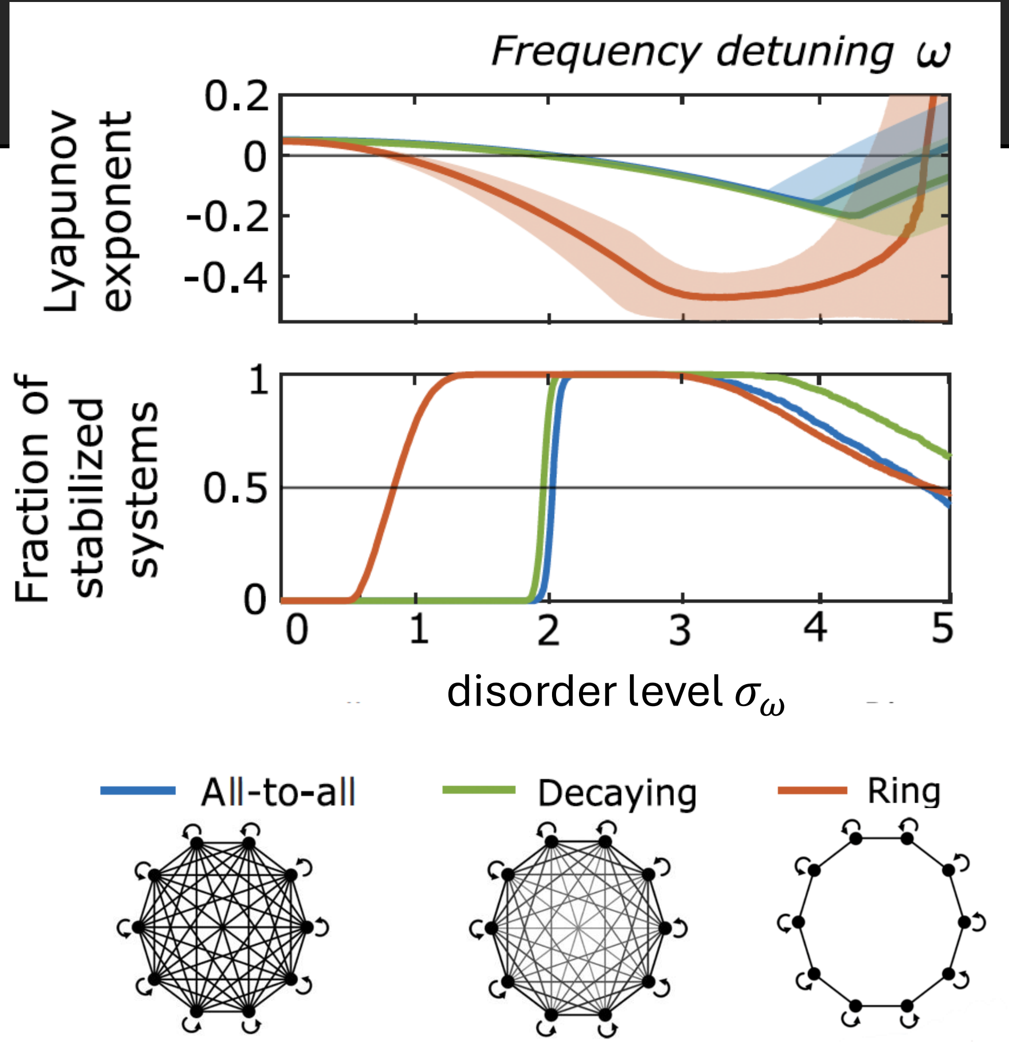

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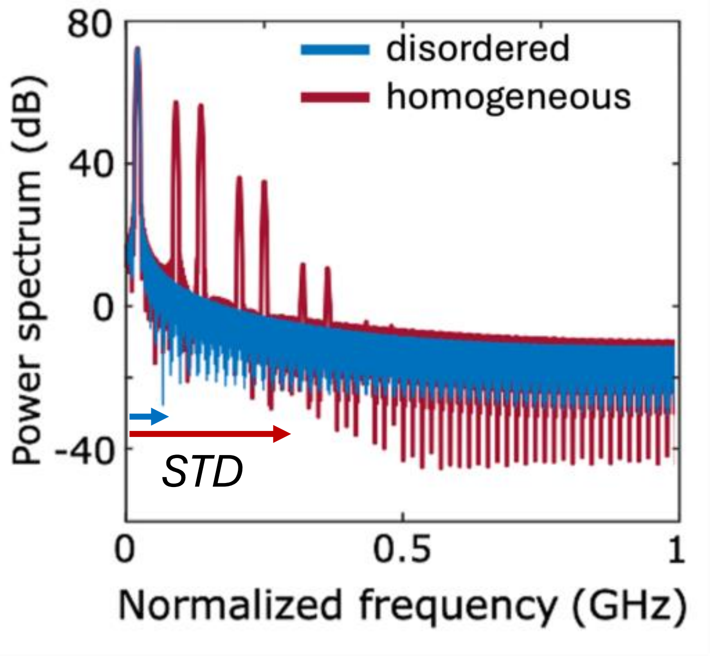
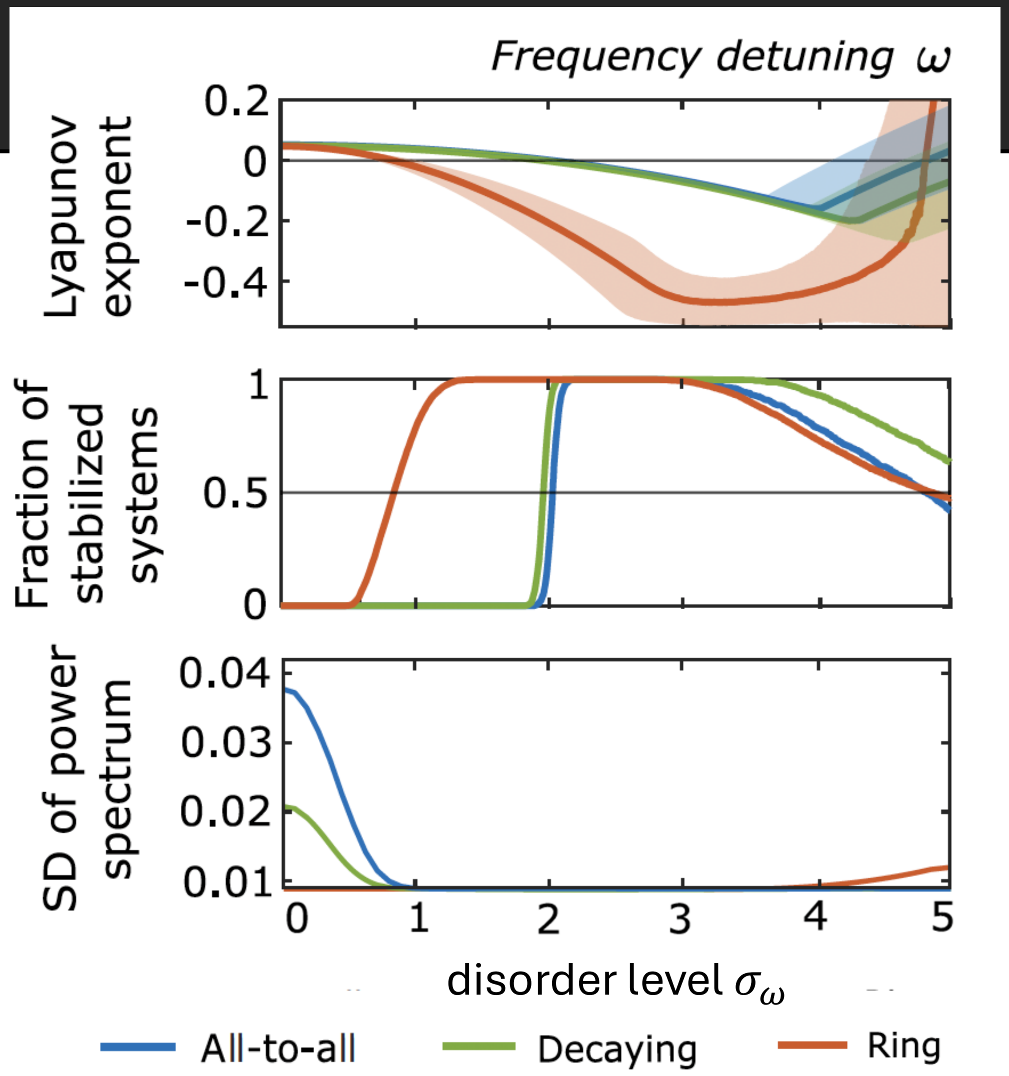



the synchronous state is stable

$\lambda_{\max} < 0$

the synchronous state is coherent

$\delta_j^* \approx 0, \forall j$



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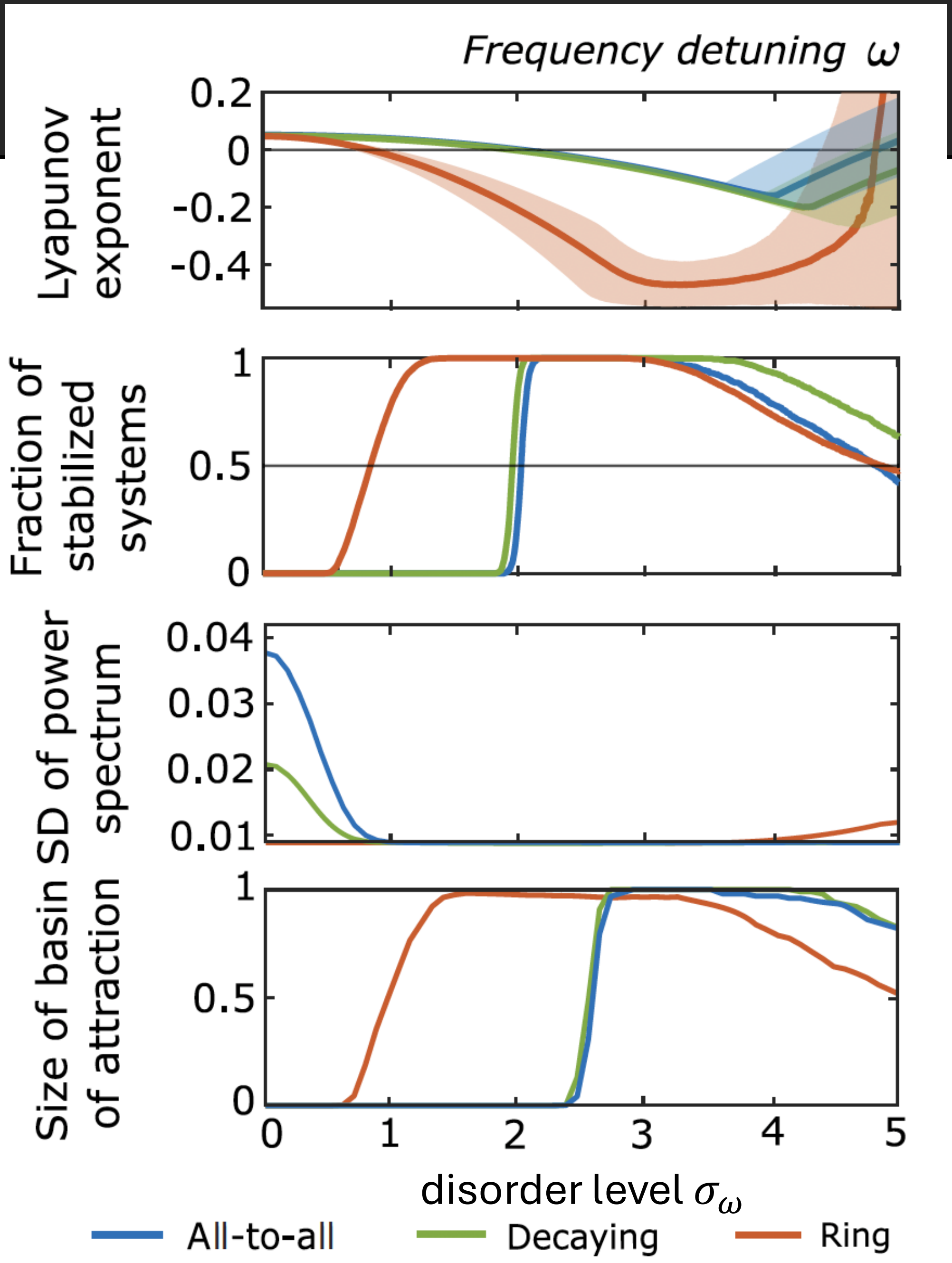

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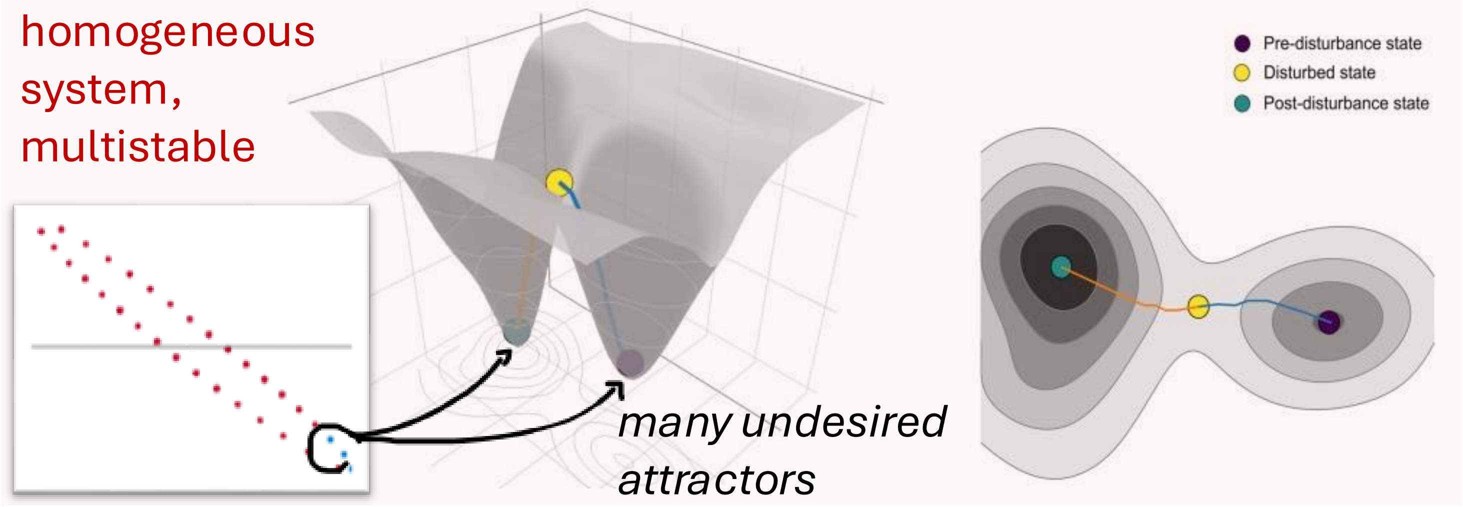


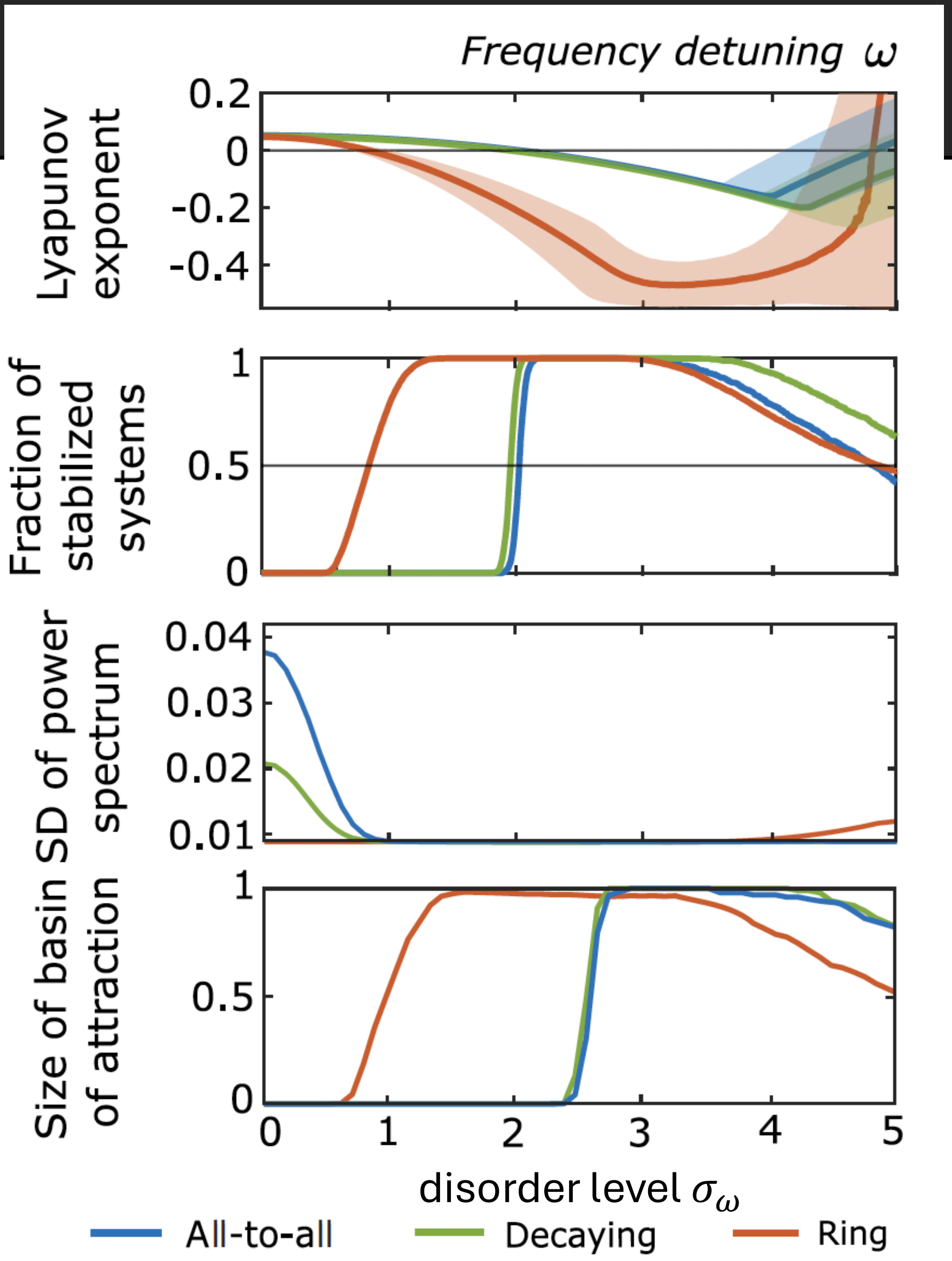

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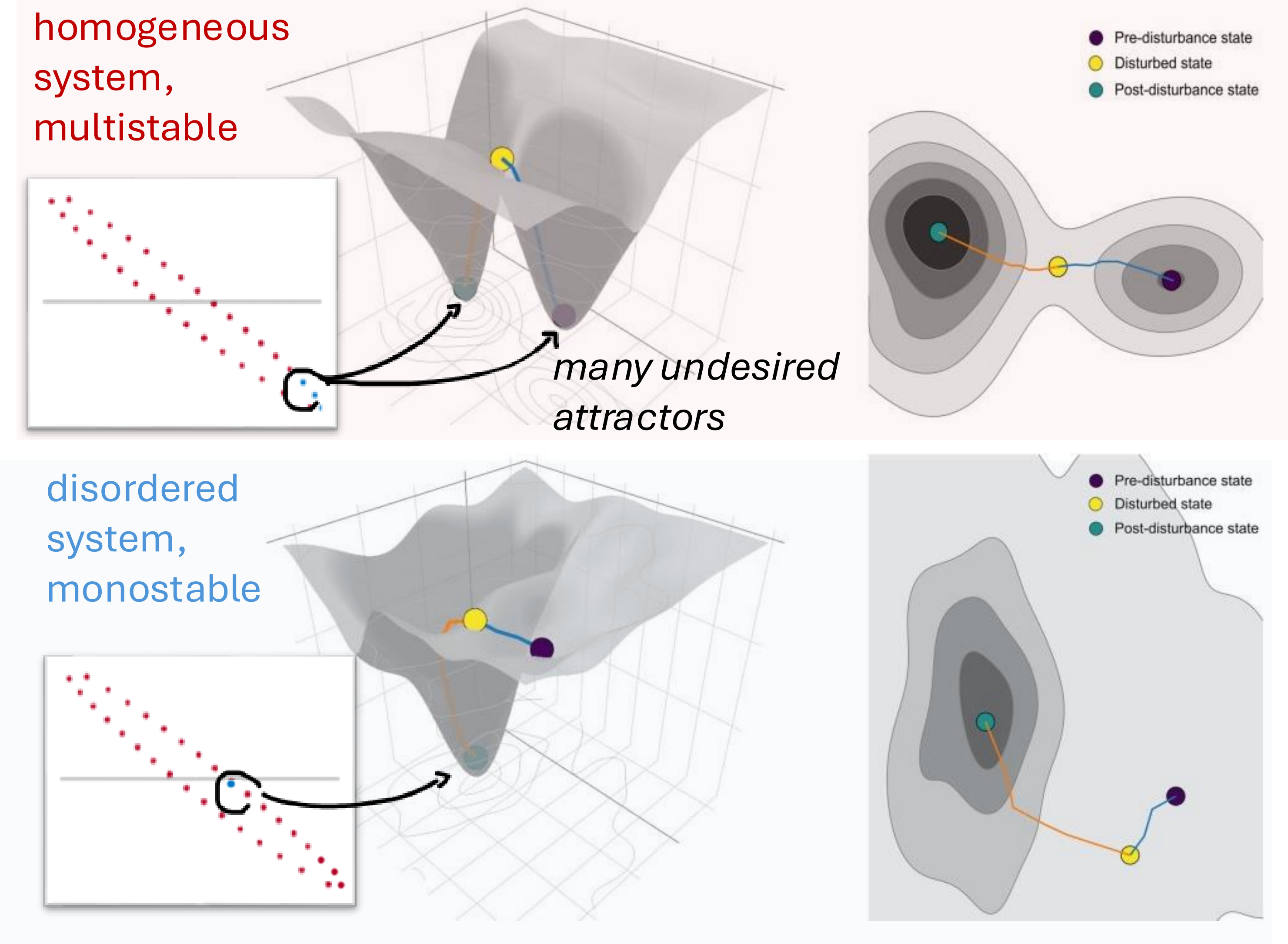
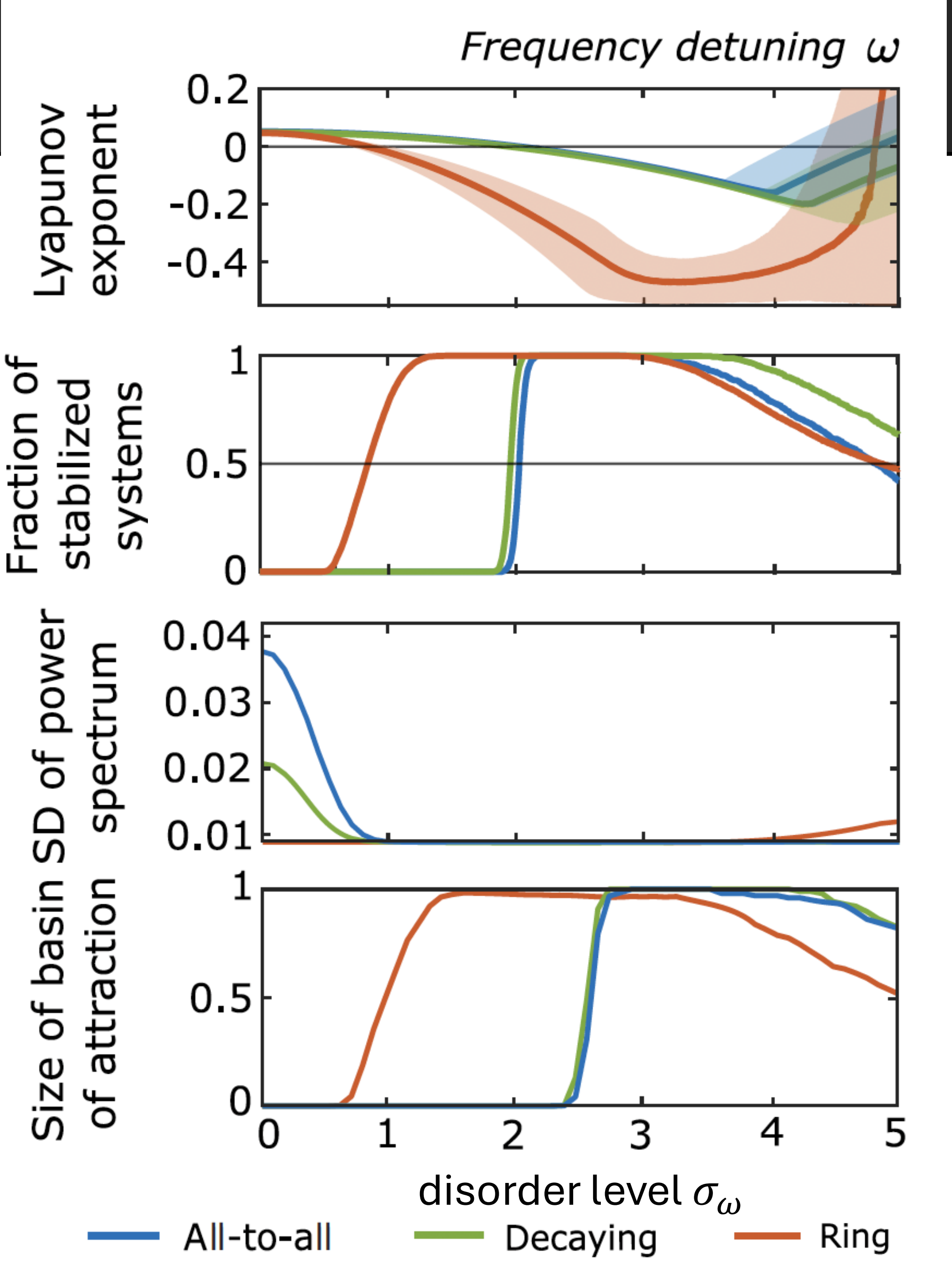
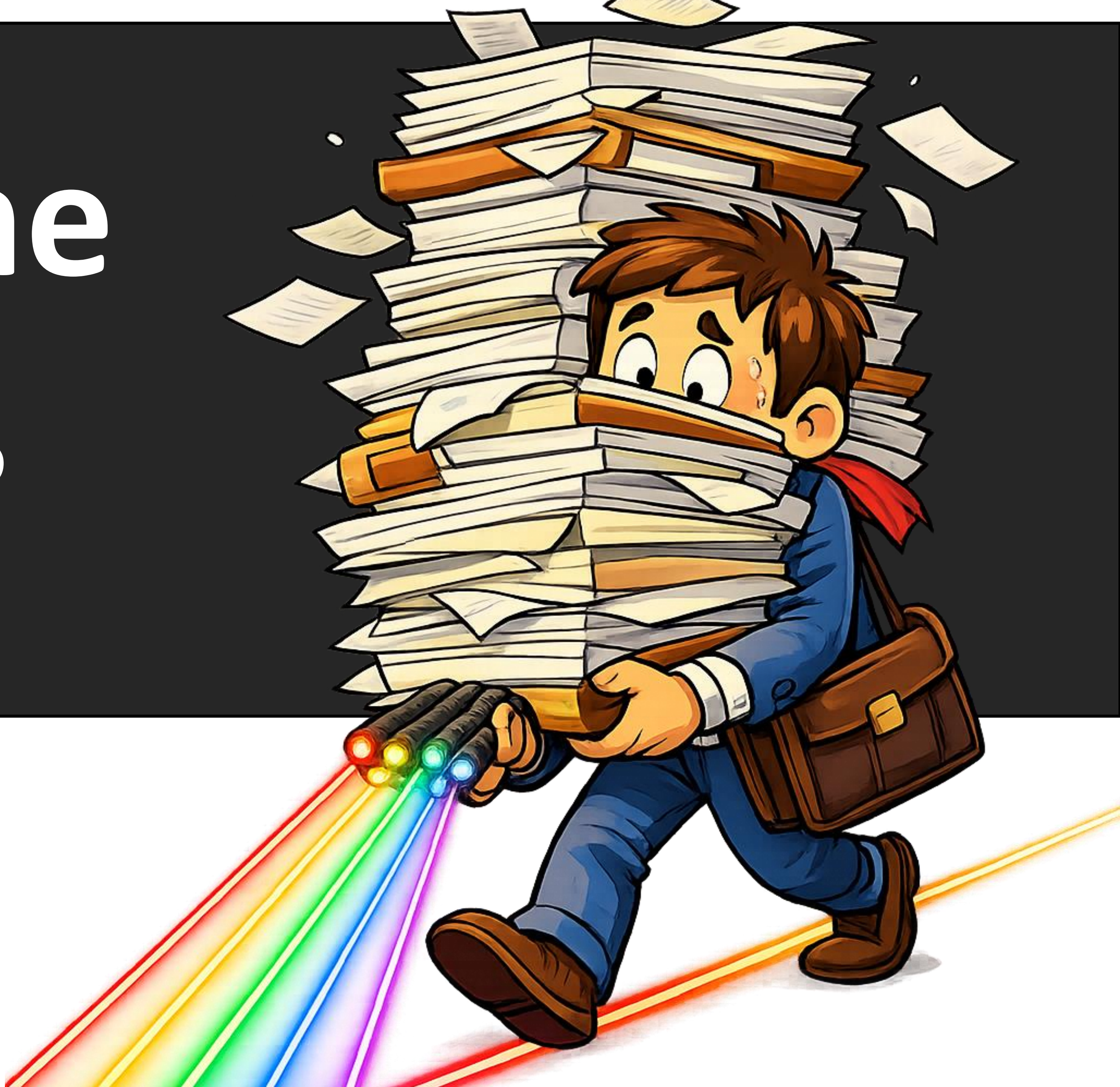
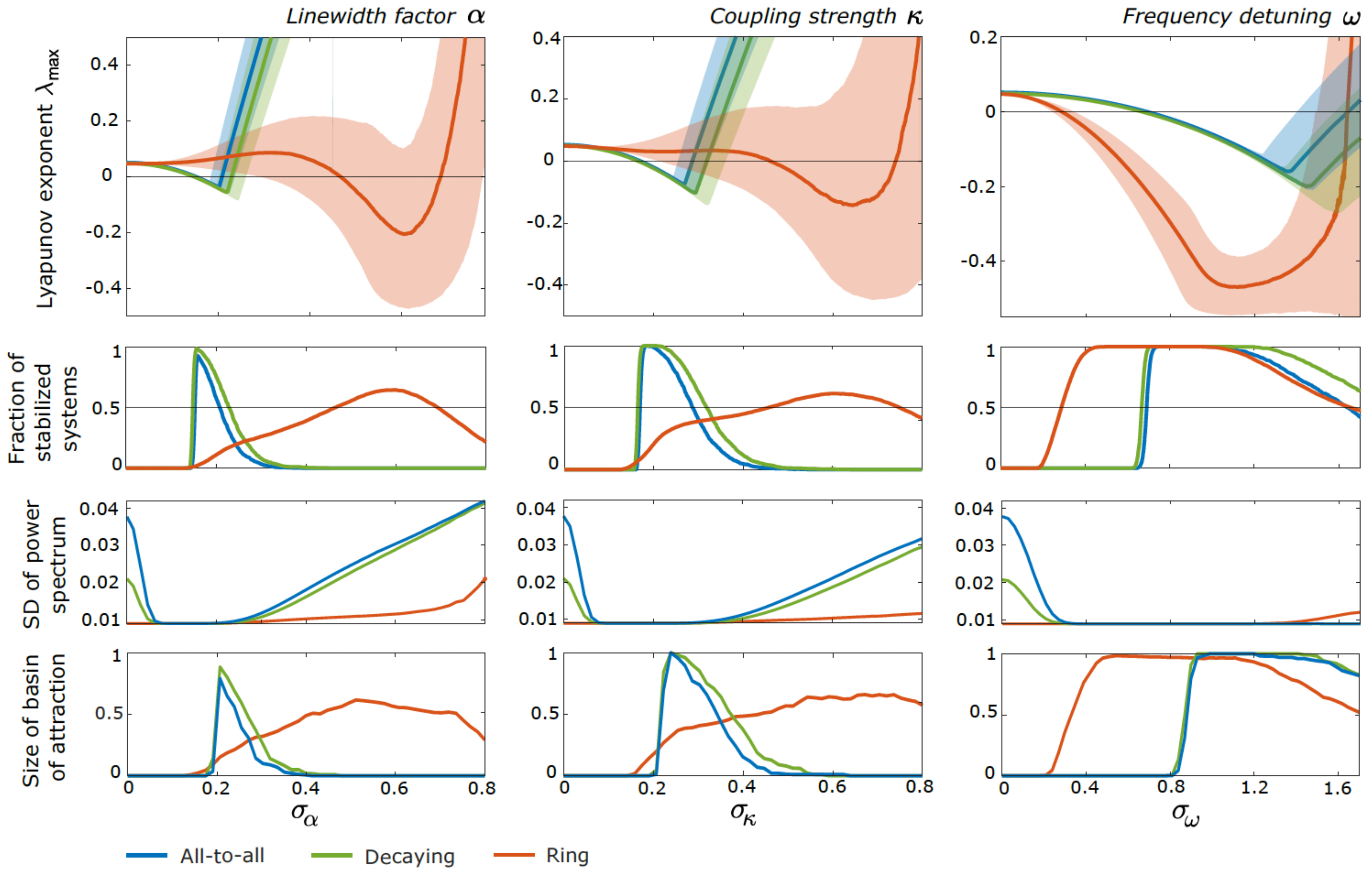


# Take-home messages



# Take-home message #1




*Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of* ***parameter****, …*

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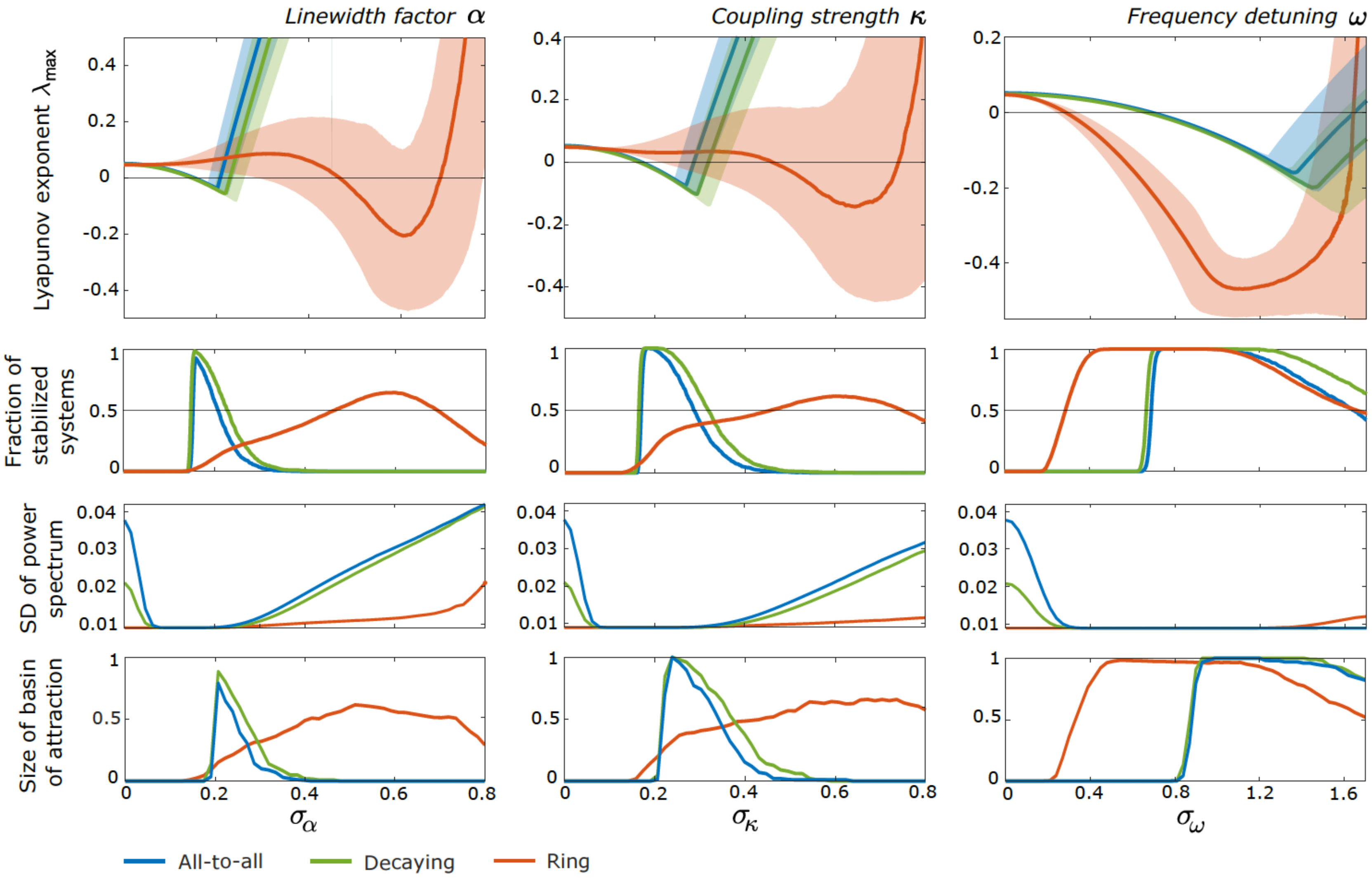
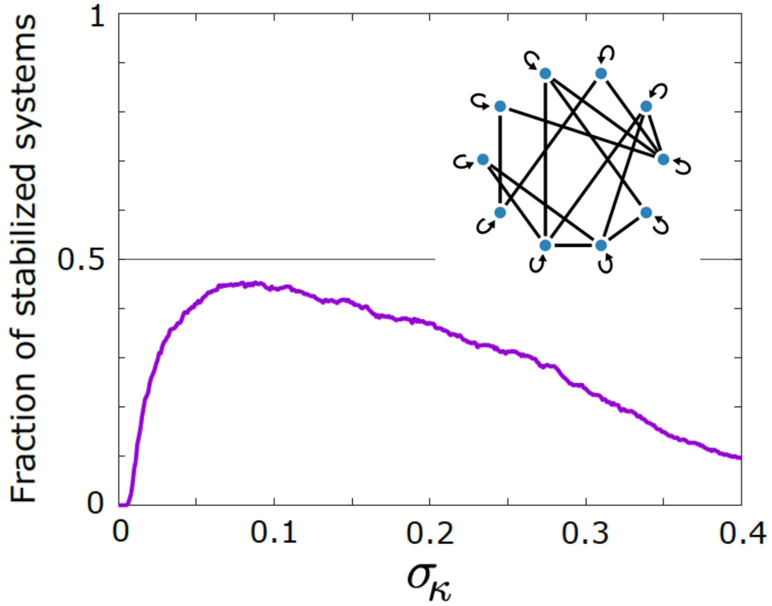



*Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of parameter,* ***network structure****,*
*…*



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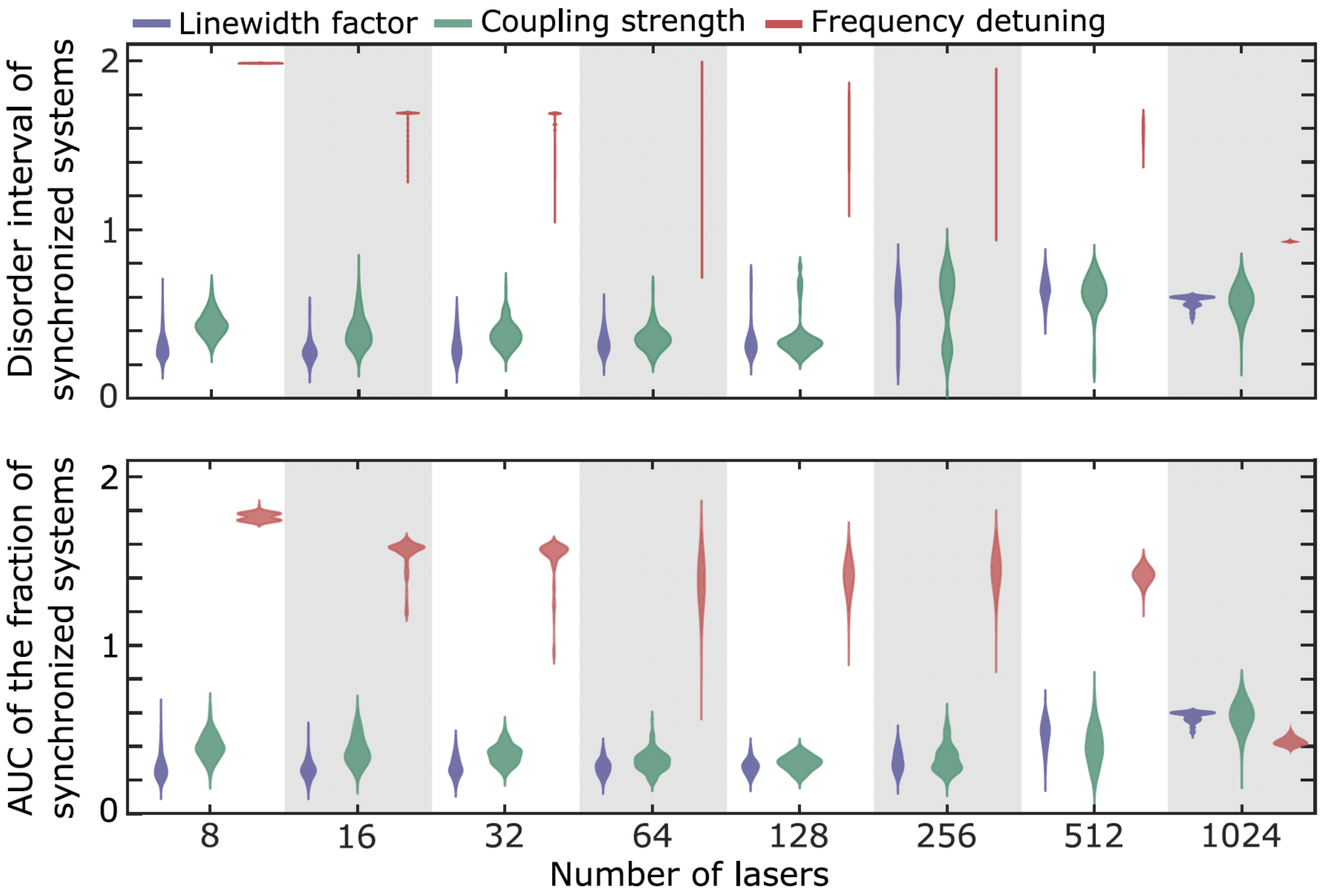
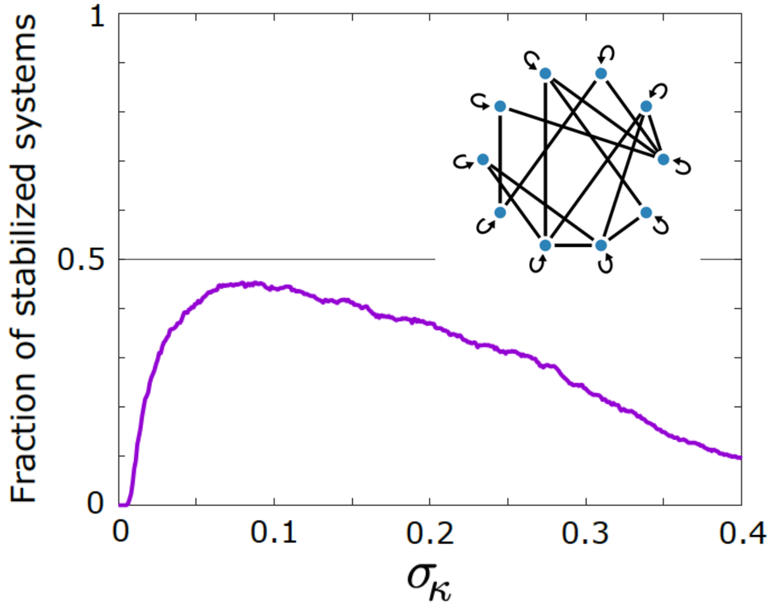
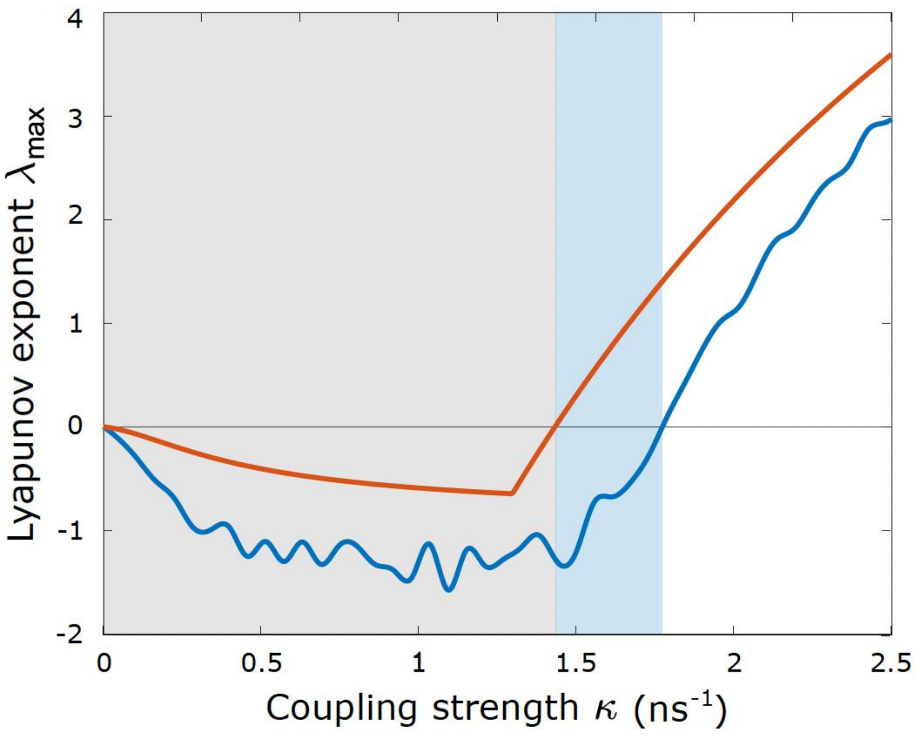
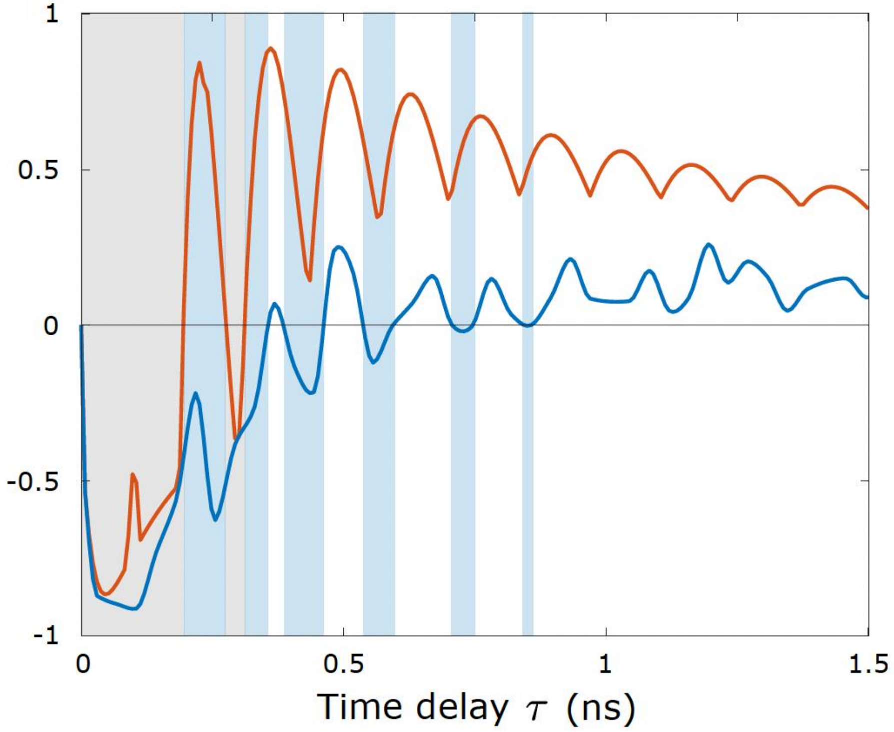



*Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of parameter, network structure, and* ***number of lasers****.*



# Take-home message #2

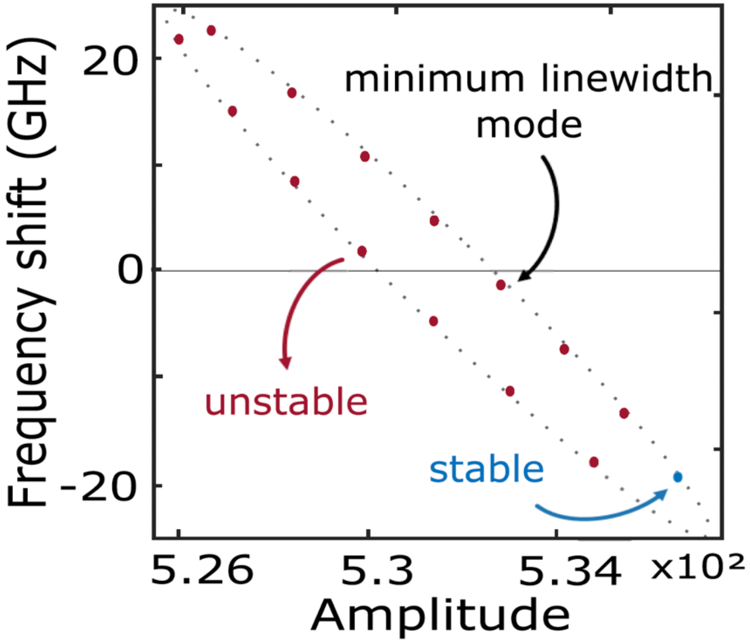



*Disordered systems exhibit larger stability margins, outperforming homogeneous ones*

# Interpretability

Pre-specified synchronous state: $E_j(t) = r_j^* e^{i(\Omega t + \delta_j^*)}$




disorder: $\omega_j \sim \mathcal{N}(0, \sigma^2)$

Why does disorder drive the stability of this state?

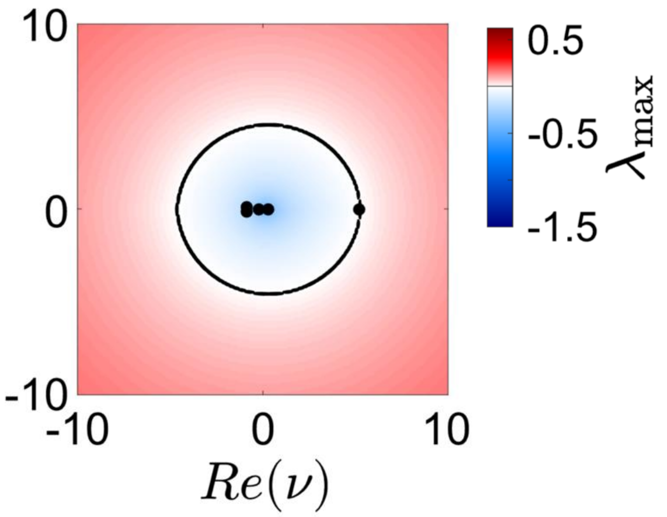
# Interpretability

For non-delayed systems: Pecora, Carroll. *PRL* (1998).
For delayed systems: Choe, Dahms, Hövel, Schöll. *PRE* (2010).

Master stability function analysis (identical lasers)

$$\dot{\boldsymbol{\xi}}_j(t) = [\mathrm{D}^{(0)}\mathbf{f} + \kappa d \mathrm{D}^{(0)}\mathbf{h}]\boldsymbol{\xi}_j(t) + \kappa \nu \mathrm{D}^{(\tau)}\mathbf{h}\boldsymbol{\xi}_j(t-\tau)$$

3-dimensional vector (mode) | instantaneous dynamics | network eigenvalue | delayed dynamics



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Master stability function analysis (non-identical lasers)

For non-delayed systems: Sugitani, Zhang, Motter. *PRL* (2021).
For delayed systems: Barioni, Montanari, Motter. *PRL* (2025).

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# Disorder: Good or Bad?

| | | |
|---|---|---|
| Nair, Hu, Berrill, Wiesenfeld, Braiman. *PRL* (2021) | → Lang-Kobayashi model | misaligned time delays |
| Zhang, Ocampo-Espindola, Kiss, Motter. *PNAS* (2021) | → Stuart-Landau model | frequency detuning |
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| Laser class | Laser rate equations (weak phase-amplitude coupling) | Lang-Kobayashi model (strong phase-amplitude coupling) |
|---|---|---|
| **Class A** (reduction) | $\dot{E}_j = \frac{1}{\tau_c}(G_j - \gamma)E_j + i\omega_j E_j + \frac{\kappa_j}{\tau_c}\sum_k A_{jk}E_k(t-\tau)$ | $\dot{E}_j = \frac{1+i\alpha_j}{2}(G_j(t) - \gamma)E_j + i\omega_j E_j + k_j \sum_k A_{jk}E_k(t-\tau)$ |
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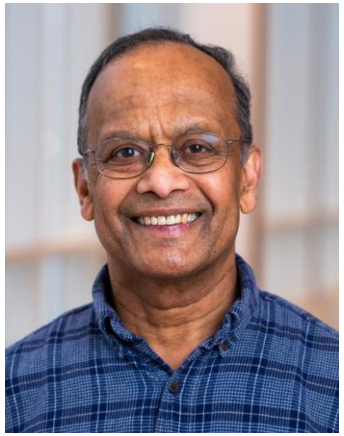
| **Class C** (extension) | $\dot{E}_j = -\frac{\gamma}{\tau_c}E_j + \frac{1}{\tau_c}P_j + i\omega_j E_j + \frac{\kappa}{\tau_c}\sum_k A_{jk}E_k(t-\tau)$ <br> $\dot{P}_j = \gamma_\perp(-P_j + G_j E_j)$ <br> $\dot{G}_j = \frac{1}{\tau_f}\left(J_0 - G_j\left(s\lvert E_j\rvert^2 + 1\right)\right)$ | $\dot{E}_j = -\frac{\gamma}{2}E_j - P_j + i\omega_j E_j + \kappa_j \sum_k A_{jk}E_k(t-\tau)$ <br> $\dot{P}_j = -(\gamma_\perp + i\Delta)P_j + G_j(t)E_j$ <br> $\dot{N}_j = J_0 - \gamma_n N_j - G_j(t)\lvert E_j\rvert^2$ |



Rajarshi Roy (UMD)

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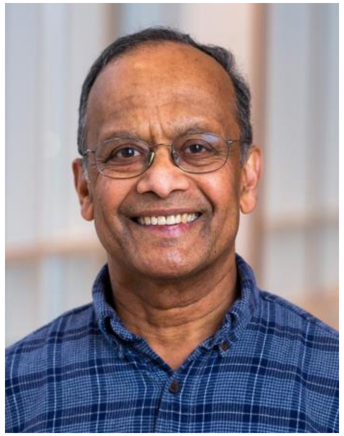

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Lasers sync because of (not despite!) heterogeneity when… time delays are significant and there is strong phase-amplitude coupling in gain media (e.g., semiconductor lasers)


# What's next?

## *Disorder for physical computing*

A Allibhoy, **AN Montanari**, F Pasqualetti, AE Motter.
Global optimization through heterogeneous oscillator Ising machines.
*Proceedings of the IEEE Conference on Decision and Control* (2025).
arXiv:2505.17027

# Acknowledgments

**montanariarthur.com**

slides available at my website

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**Interpretable Disorder-Promoted Synchronization and Coherence in Coupled Laser Networks**

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