Disordered systems for neurocomputation

Arthur Montanari

Center for Network Dynamics

Department of Physics and Astronomy

Northwestern University

May 19, 2026

Disordered systems for stability & neurocomputation

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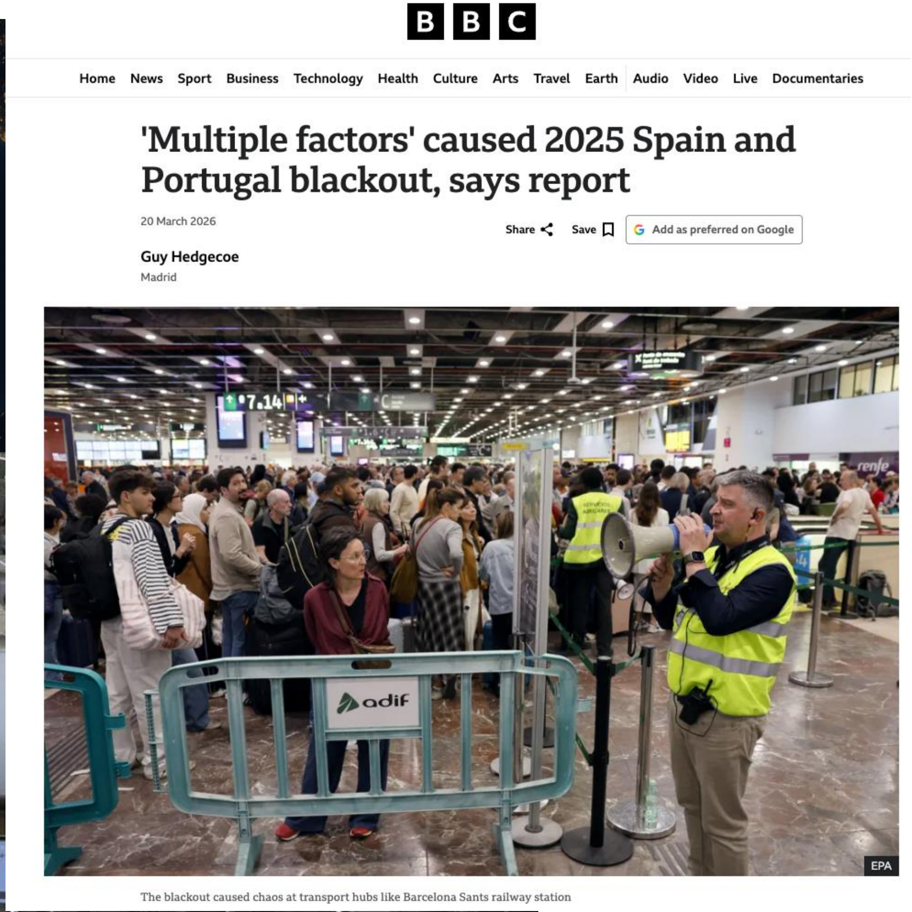
A short story on power grid stability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'Multiple factors' caused 2025 Spain and Portugal blackout, says report

20 March 2026

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Guy Hedgecoe
Madrid

EPA

The blackout caused chaos at transport hubs like Barcelona Sants railway station

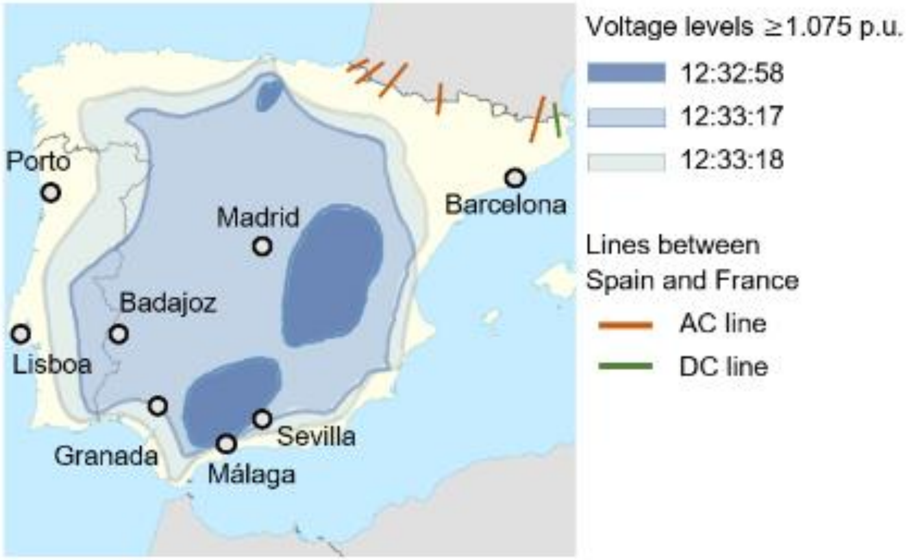

» Grid Incident in Spain and Portugal on 28 April 2025

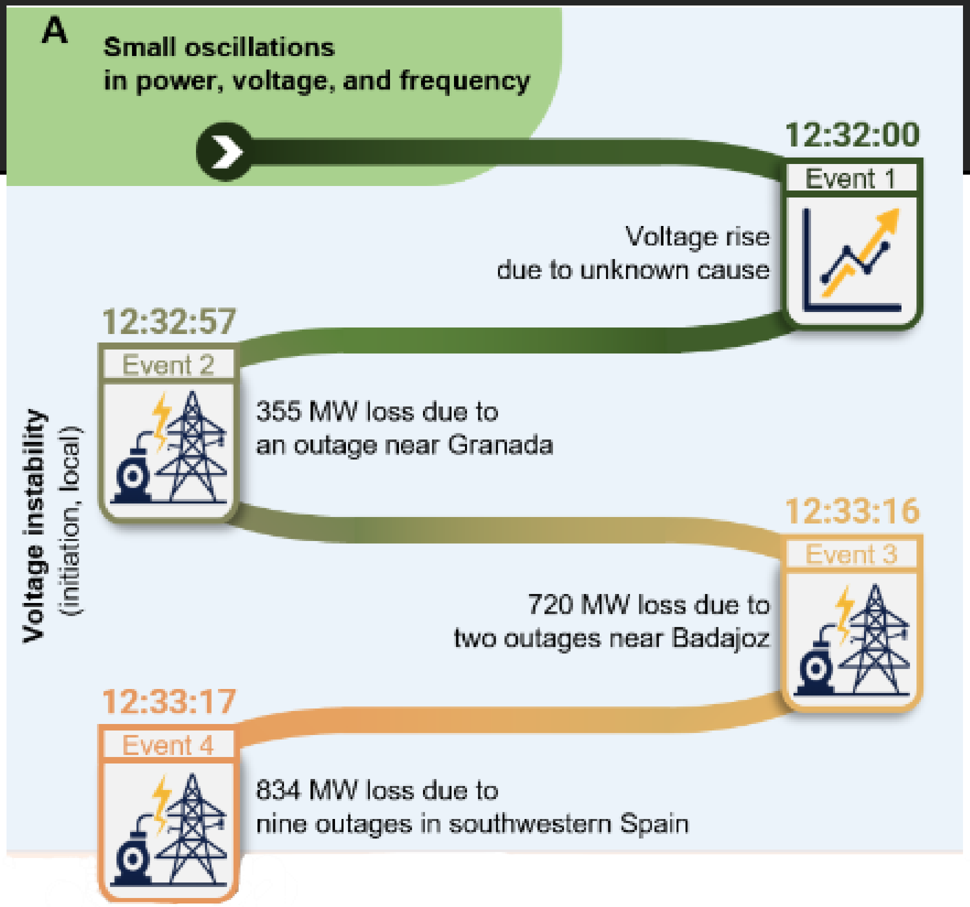
ICS Investigation Expert Panel
Final Report

entsoe

20 March 2026

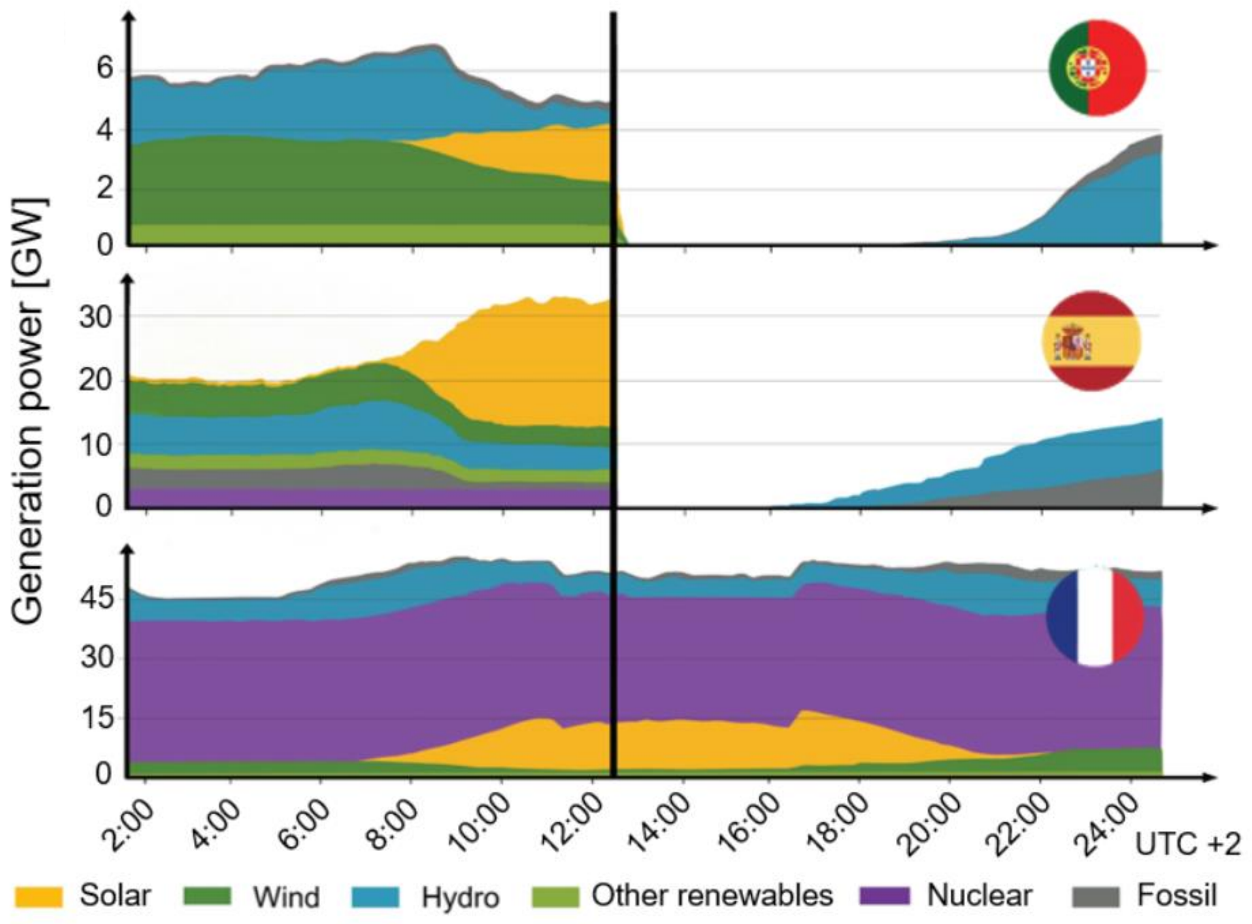
The Iberian blackout

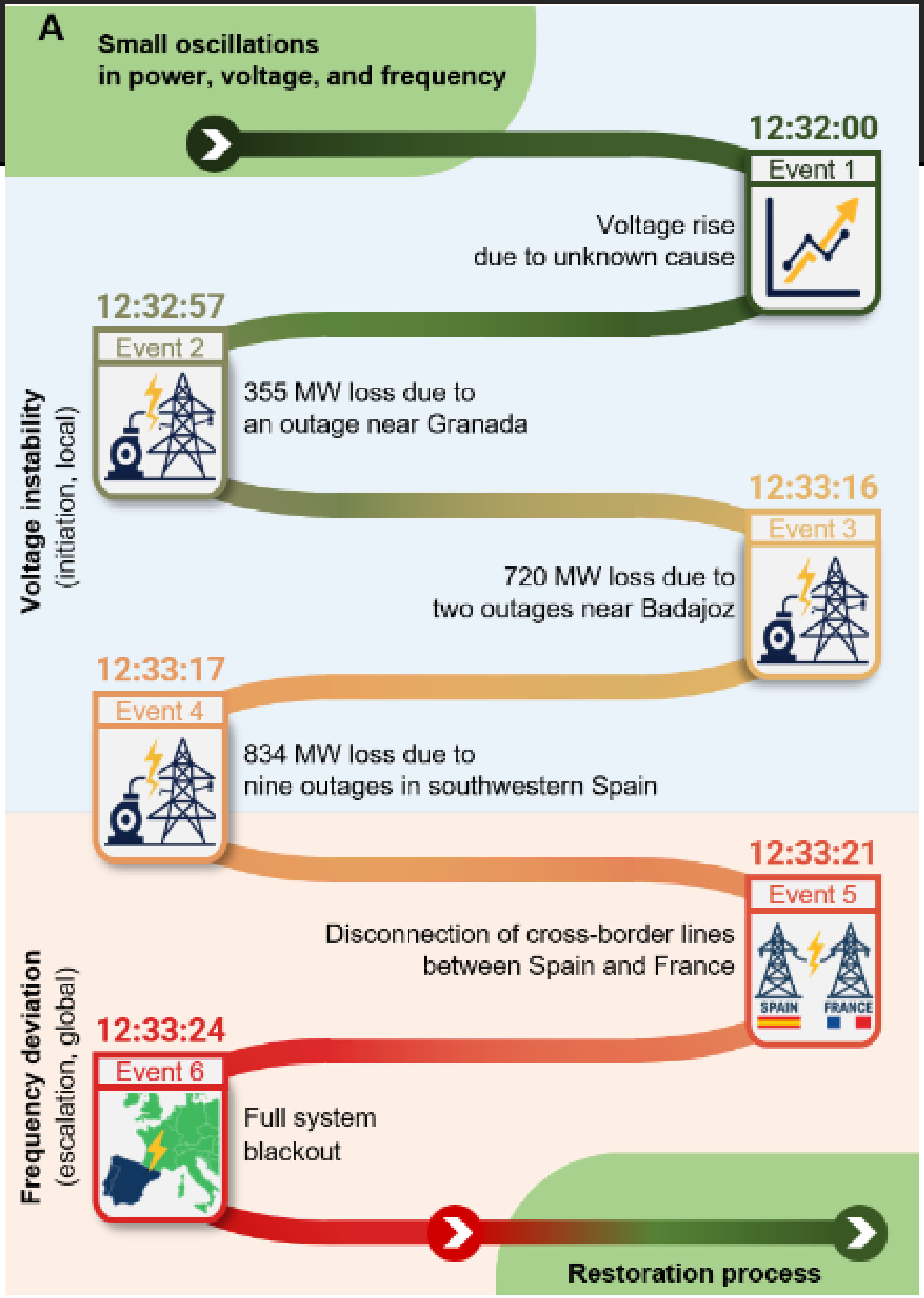




Wang, **ANM**, Motter.
Joule, accepted (2026).

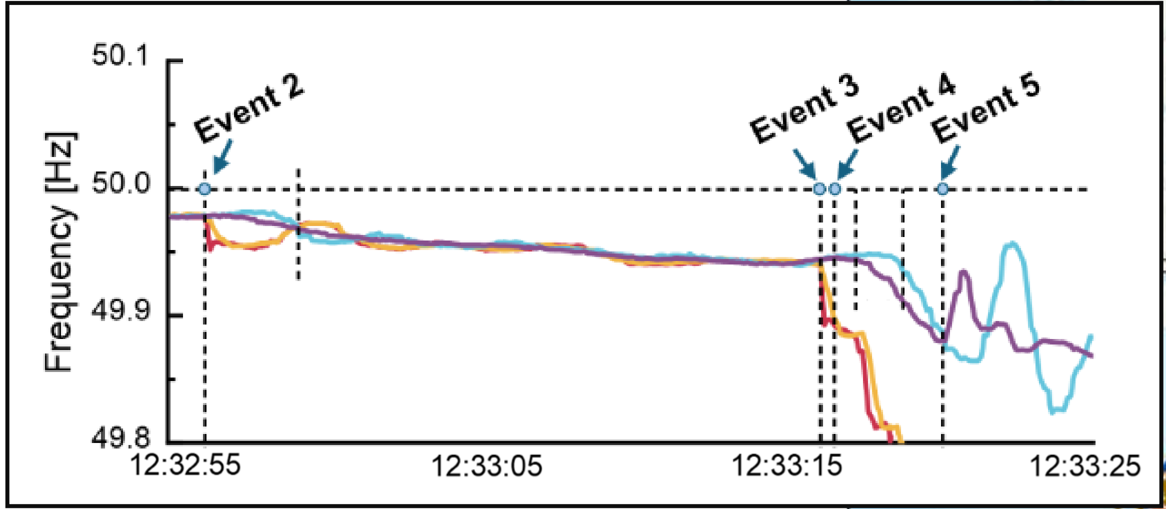
The Iberian blackout





Wang, **ANM**, Motter.
Joule, accepted (2026).

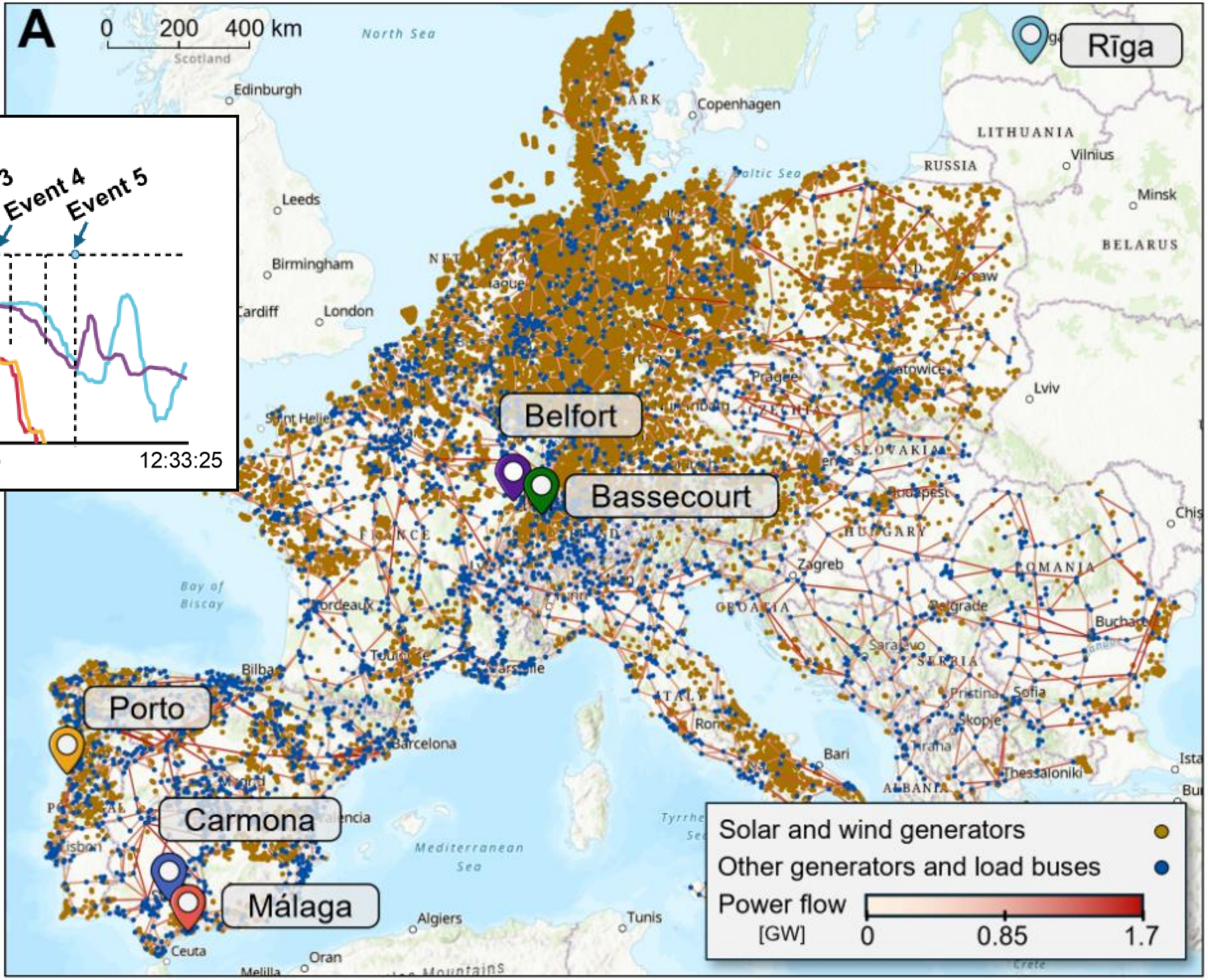
Data



Gridradar. Blackout – Iberian Peninsula (2025).

ENTSO-E. Grid Incident in Spain and Portugal on 28 April 2025. Tech. Rep. (2026).

Málaga, ES
Porto, PT
Belfort, FR
Rīga, LV



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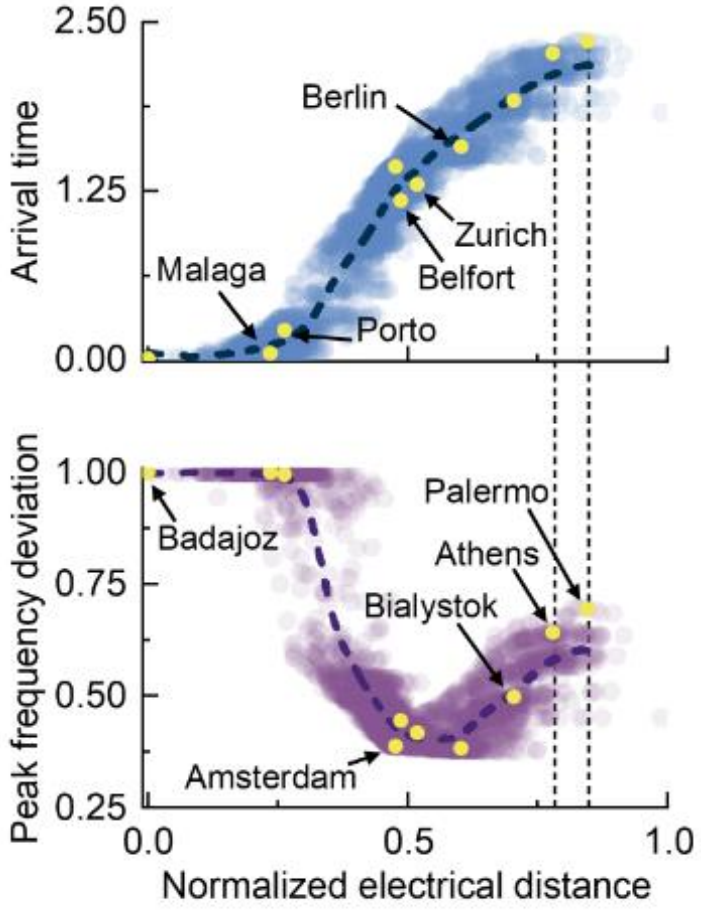
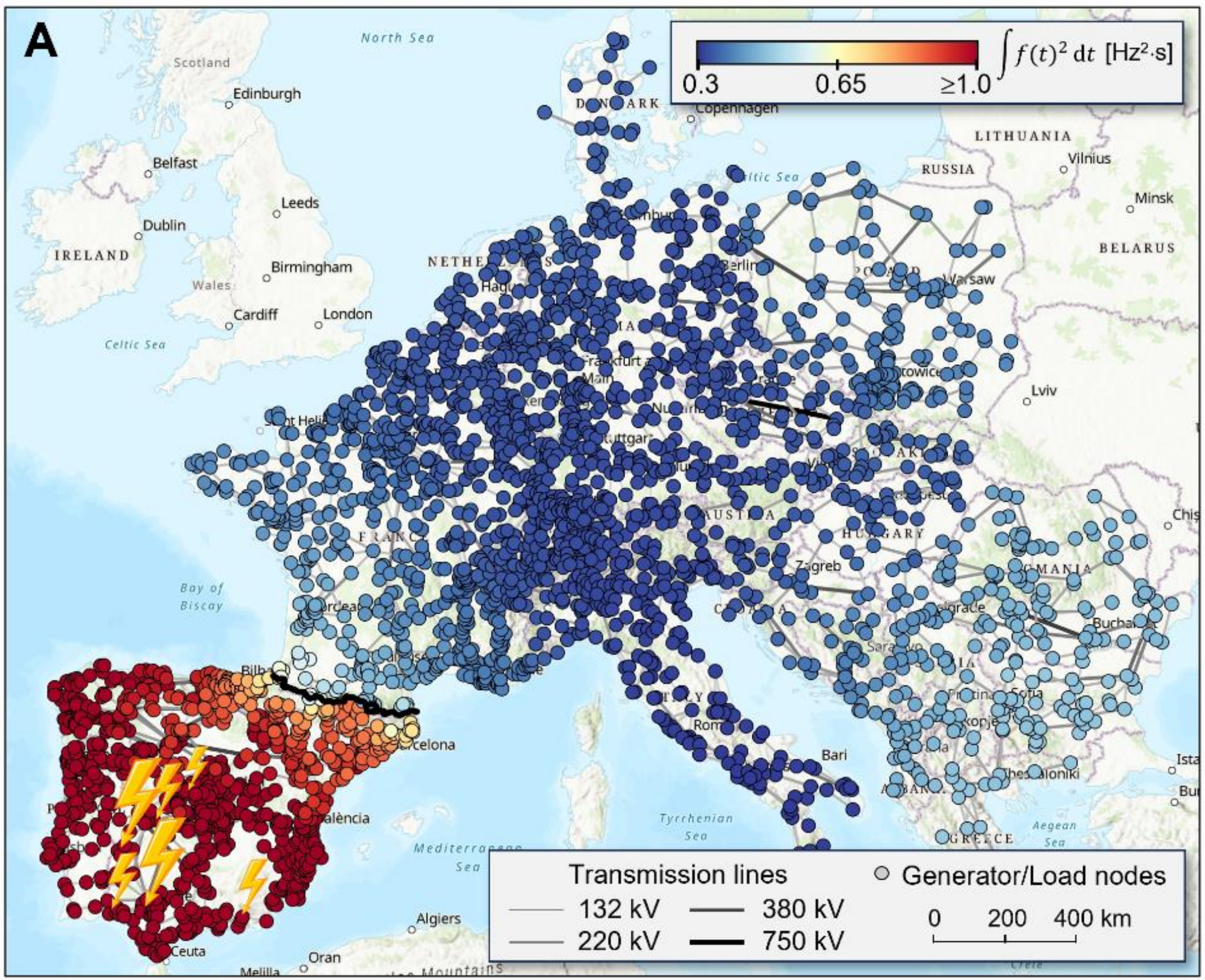
Simulations





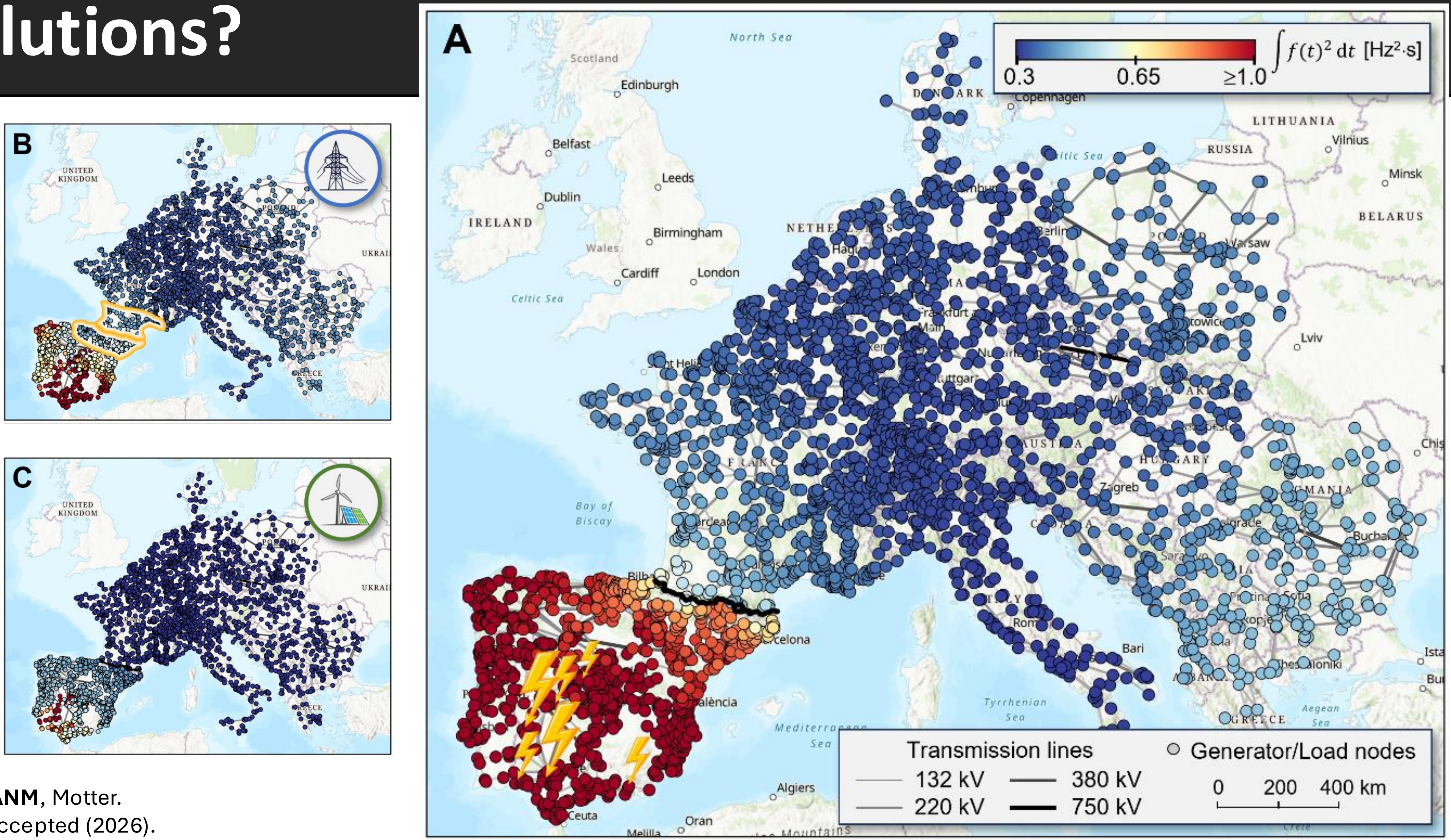
Wang, **ANM**, Motter.
Joule, accepted (2026).

Solutions?

rethink design of...

network coupling

dynamical parameters

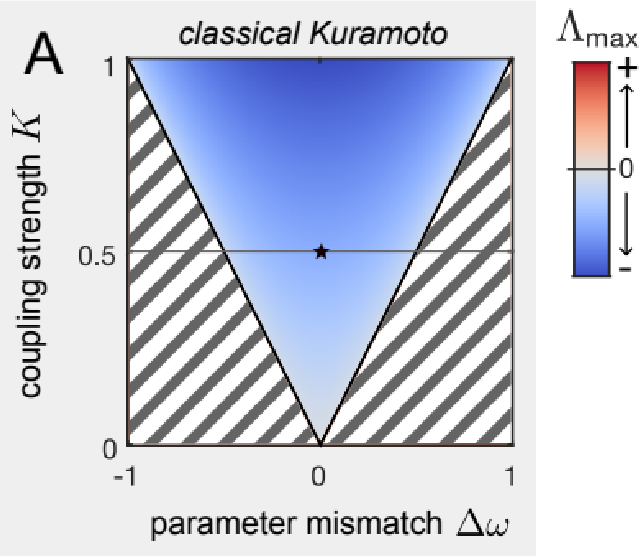


Wang, **ANM**, Motter.
Joule, accepted (2026).

Part I: Disorder & Stability

Part II: Disorder & Computation

Complex systems perspective



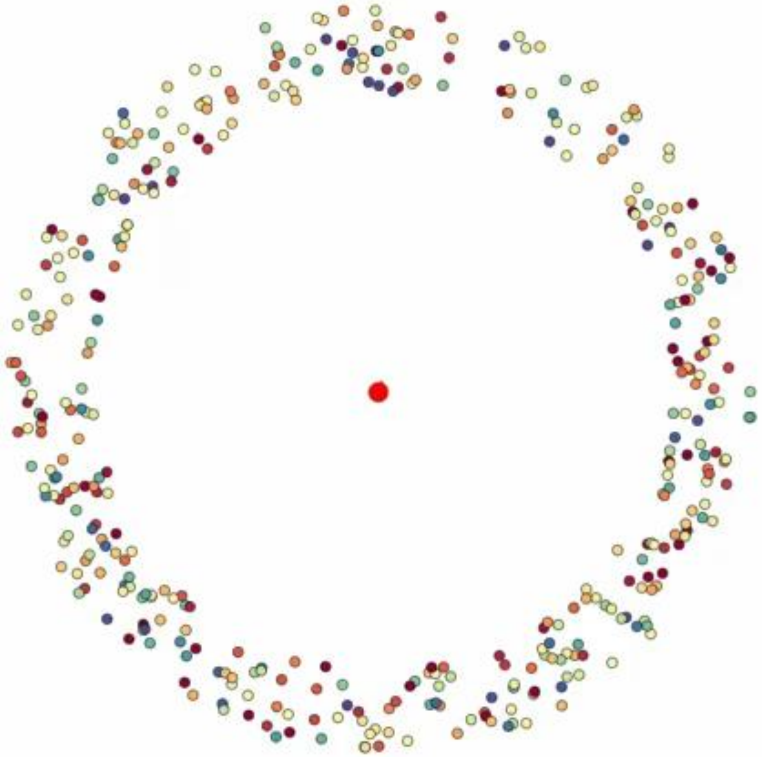
$$\dot{\phi}_i = \omega_i + K \sum_{j=1}^{N} A_{ij} \sin(\phi_j - \phi_i)$$

$N = 2 \rightarrow \left|\frac{\Delta\omega}{2K}\right| < 1$

$N > 2,$ Strogatz. *Physica D* (2000).

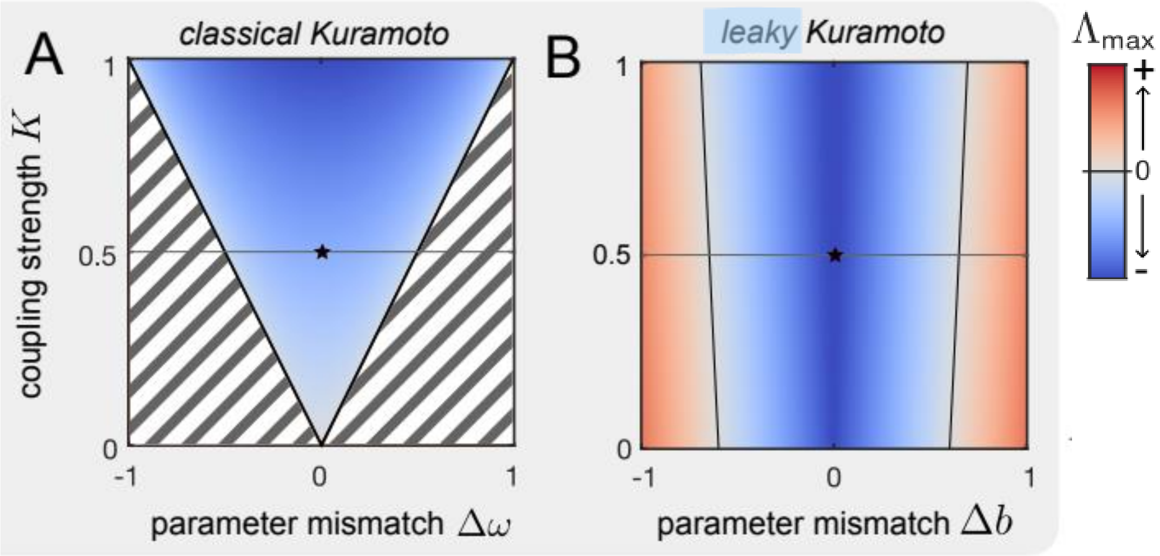
Rodrigues, et al.
Physics Reports (2016).

Dorfler, Chertkov, Bullo.
PNAS (2013).
$\|L^{\dagger}\omega\|_{\mathcal{E},\infty} < 1$



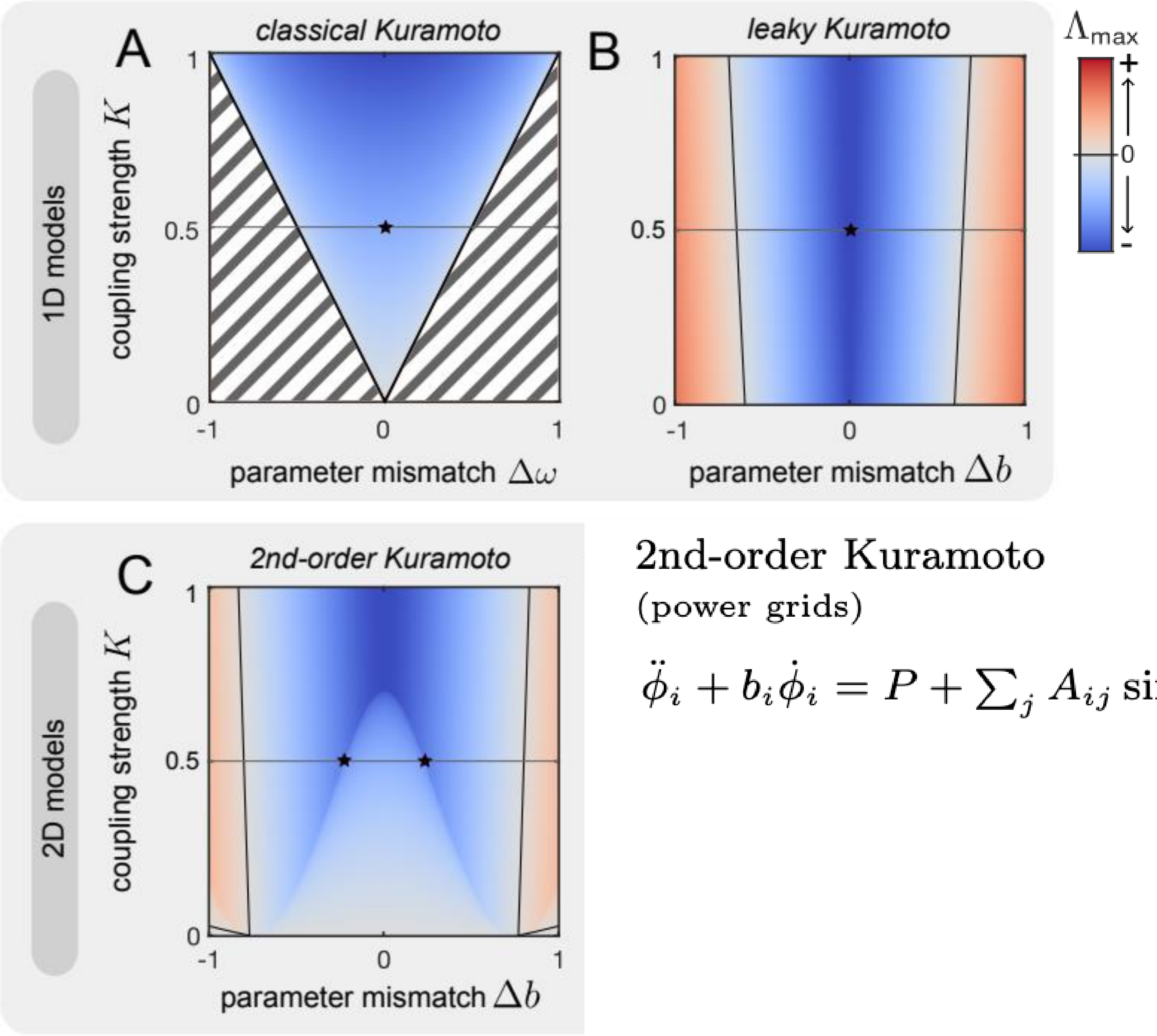
www.complexity-explorables.org/slides/ride-my-kuramotocycle/

Complex systems perspective



Leaky Kuramoto (synchronization)	$\dot{\phi}_i + b_i\phi_i = \omega + \sum_j A_{ij}\sin(\phi_j - \phi_i)$	$-(B+L)$
Driven Kuramoto (pacemaker)	$\dot{\phi}_i + b_i\sin(\phi_i - \psi) = \omega + \sum_j A_{ij}\sin(\phi_j - \phi_i)$	$-(B+L)$
Consensus (opinion, coordination)	$\dot{x}_i + b_i x_i = \sum_j A_{ij}(x_j - x_i)$	$-(B+L)$

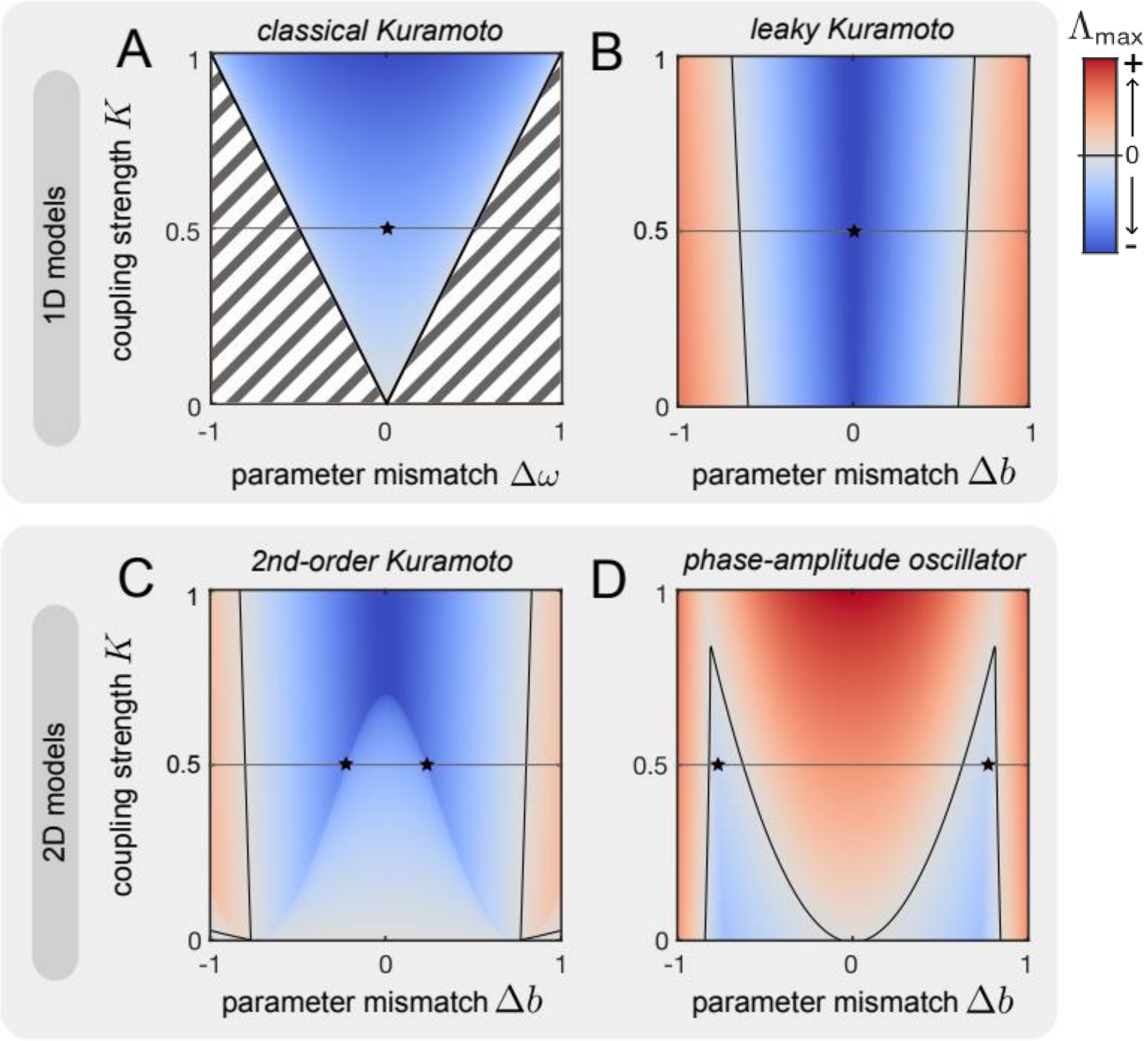
Disorder-promoted stability



2nd-order Kuramoto
(power grids)

$$\ddot{\phi}_i + b_i\dot{\phi}_i = P + \sum_j A_{ij}\sin(\phi_j - \phi_i)$$

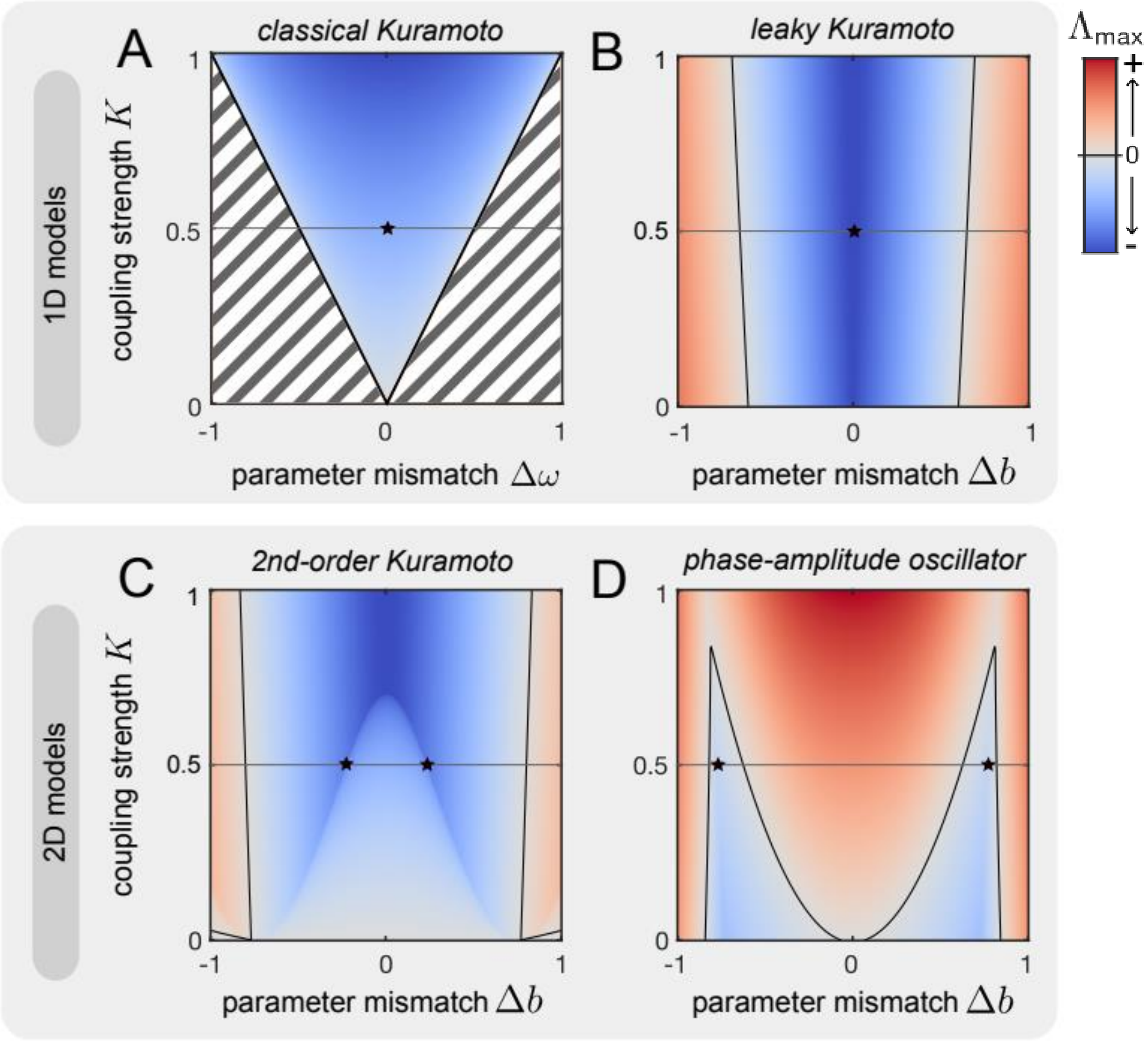
Disorder-promoted stability



Phase-amplitude oscillator

$$\dot{r}_i = b_i r_i (1 - r_i) + \varepsilon r_i \textstyle\sum_j A_{ij} \cos(\phi_j - \phi_i)$$
$$\dot{\phi}_i = \omega + r_i - 1 + r_i \textstyle\sum_j A_{ij} \sin(\phi_j - \phi_i)$$

Disorder-promoted stability: condition?



$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

Given a network structure, what is the optimal parameter assignment that maximizes stability ?

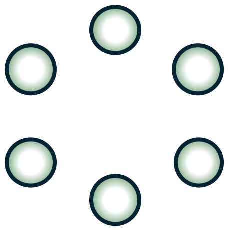
ANM, Zanin, Motter. Under review (2026).

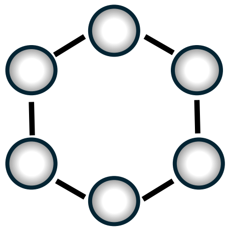
Disorder-promoted stability: condition?

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

nodal symmetries

network symmetries





$P\boldsymbol{b} = \boldsymbol{b}$

$P^T A P = A$

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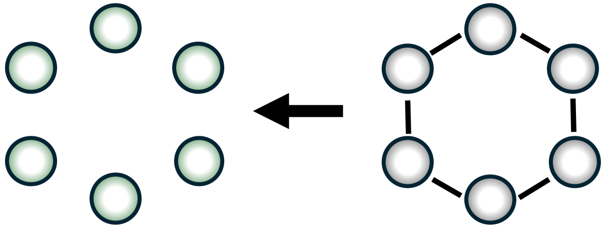
nodal symmetries

network symmetries

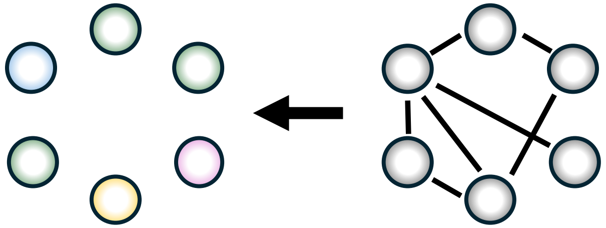
homog. parameters



homog. degrees

$P\boldsymbol{b} = \boldsymbol{b}$ $\quad$ $P^T A P = A$

heterog. parameters

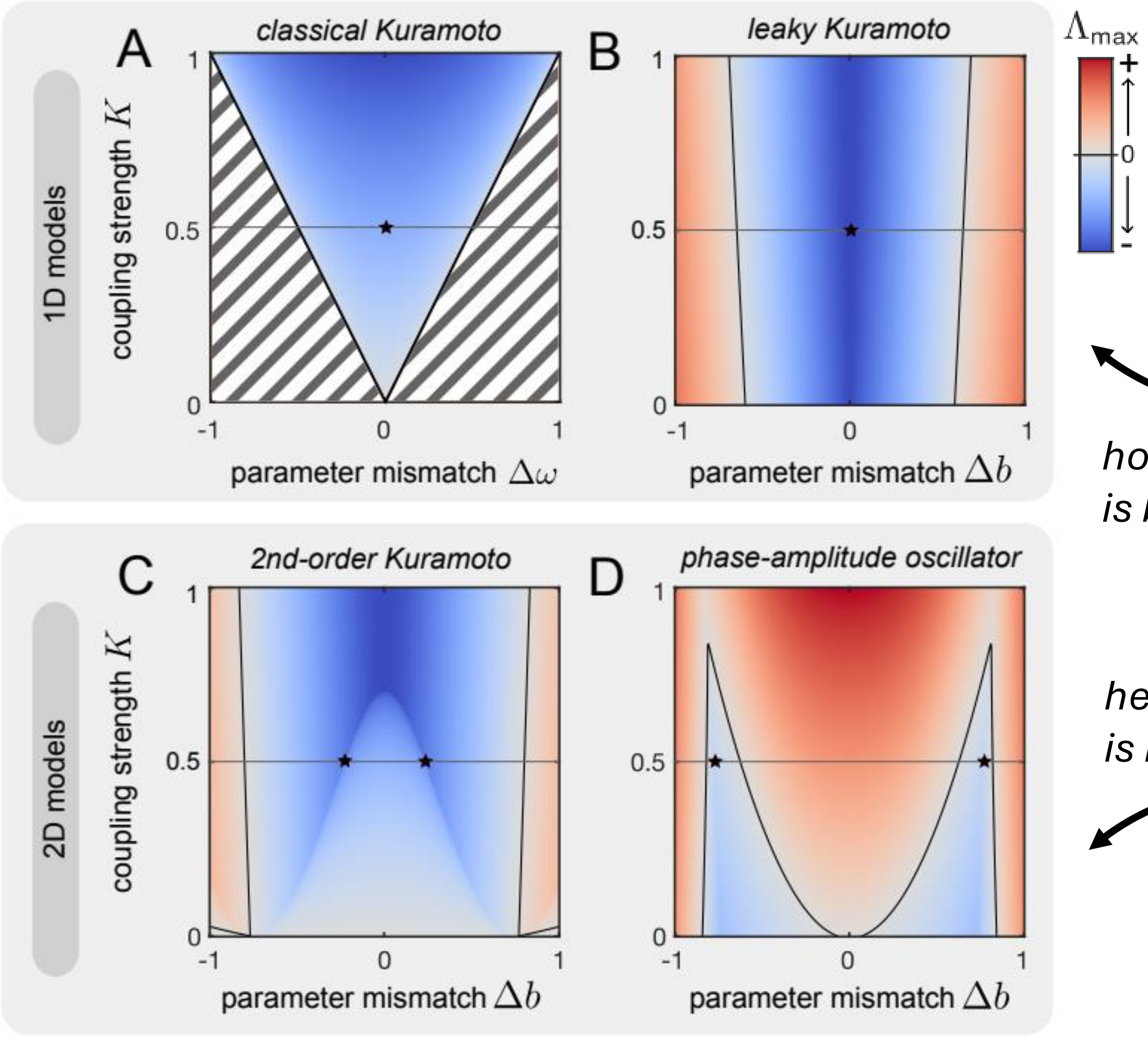


heterog. degrees

$P\boldsymbol{b} \neq \boldsymbol{b}$ $\quad$ $P^T A P \neq A$

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?



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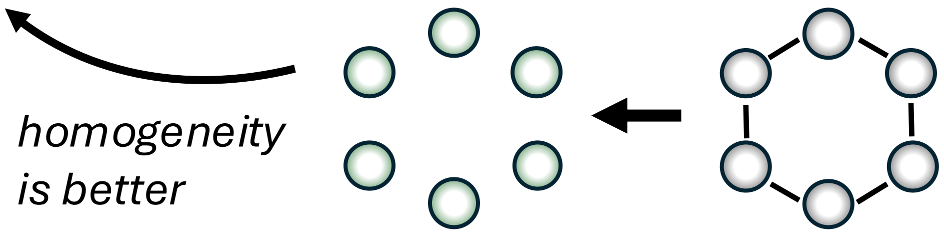
nodal symmetries

network symmetries

homogeneity is better



homog. degrees

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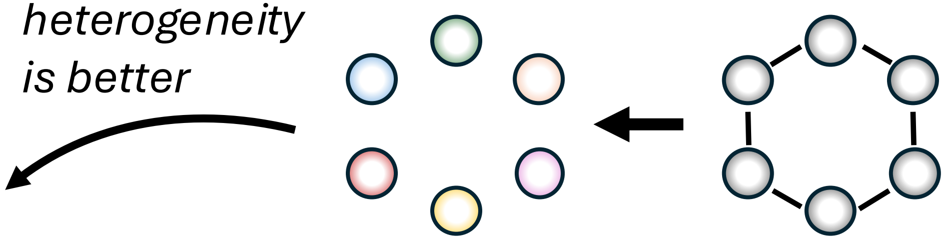
heterogeneity is better



homog. degrees

$P\boldsymbol{b} \neq \boldsymbol{b}$ $\quad$ $P^T A P = A$

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?

$$\delta\dot{\boldsymbol{x}} = J(\boldsymbol{b}; A)\Big|_{\boldsymbol{x}^{\mathrm{eq}}} \delta\boldsymbol{x}$$

J is an affine function of $\boldsymbol{b}$

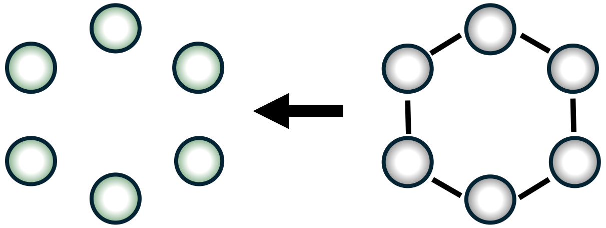
$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

nodal symmetries

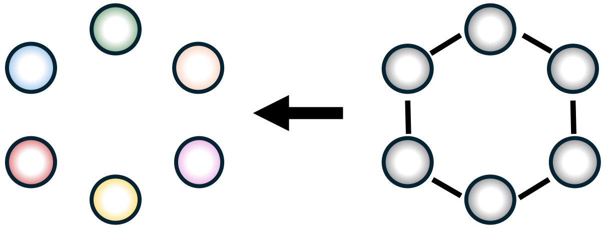
network symmetries

$$\boldsymbol{b}^* = \arg\min_{\boldsymbol{b}} \Lambda_{\max}\big(J(\boldsymbol{b}; A)\big)$$

"stability enhancement"



$P\boldsymbol{b} = \boldsymbol{b}$ $\qquad$ $P^T A P = A$



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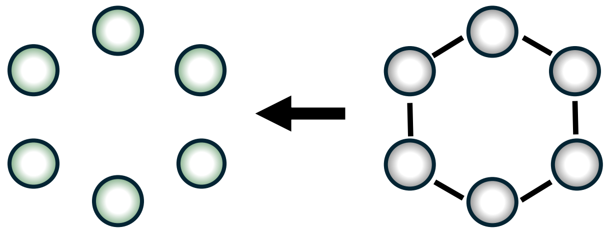
Theorem. *If* the optimization problem $\min_{\boldsymbol{b}} \Lambda_{\max}\big(J(b; A)\big)$ is convex,

then $\boldsymbol{b}^*$ must satisfy the network symmetries.

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

nodal symmetries

network symmetries



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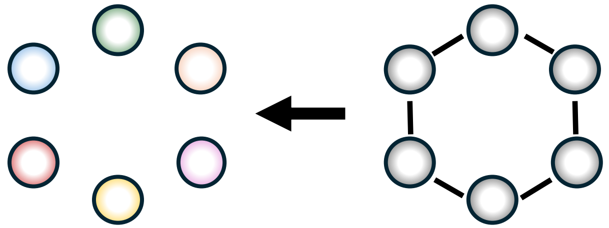

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ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?

$$\delta \dot{\boldsymbol{x}} = \underbrace{J(\boldsymbol{b}; A)}_{} \Big|_{\boldsymbol{x}^{\text{eq}}} \delta \boldsymbol{x} \longleftarrow \dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij} \boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

J is an affine function of $\boldsymbol{b}$

nodal symmetries

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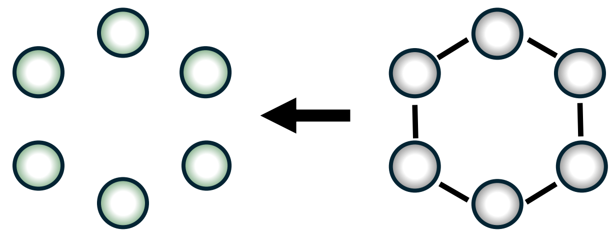
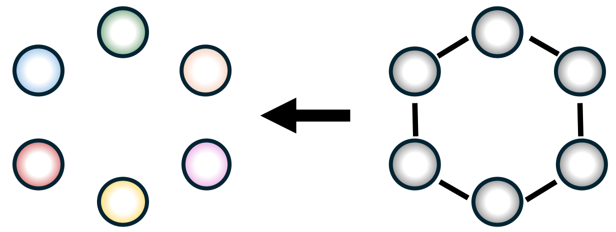
Corollary. *If* $J(\boldsymbol{b}; A)$ is non-Hermitian,

then the optimization problem is non-convex for *almost all* networks.



$P\boldsymbol{b} = \boldsymbol{b}$ $\qquad$ $P^T A P = A$



$P\boldsymbol{b} \neq \boldsymbol{b}$ $\qquad$ $P^T A P = A$

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?

$$\delta\dot{\boldsymbol{x}} = \underbrace{J(\boldsymbol{b};A)}_{J \text{ is an affine function of } \boldsymbol{b}}\Big|_{\boldsymbol{x}^{\text{eq}}}\delta\boldsymbol{x} \longleftarrow \dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i,\boldsymbol{x}_j)$$

nodal symmetries

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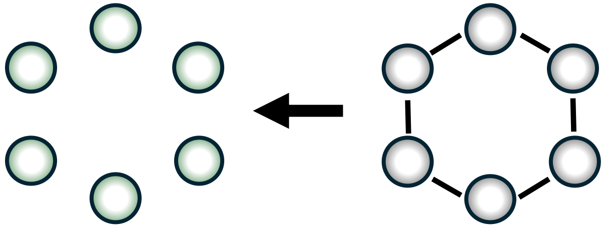
Corollary. *If* $J(\boldsymbol{b};A)$ is non-Hermitian,

then the optimization problem is non-convex for *almost all* networks.

<u>sufficiency</u>

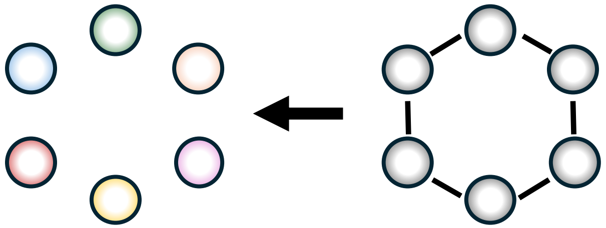
$\boldsymbol{b}^*$ is homogeneous *if* $J(\boldsymbol{b};A)$ is Hermitian.



$P\boldsymbol{b} = \boldsymbol{b}$ $\quad$ $P^TAP = A$

<u>necessity</u>

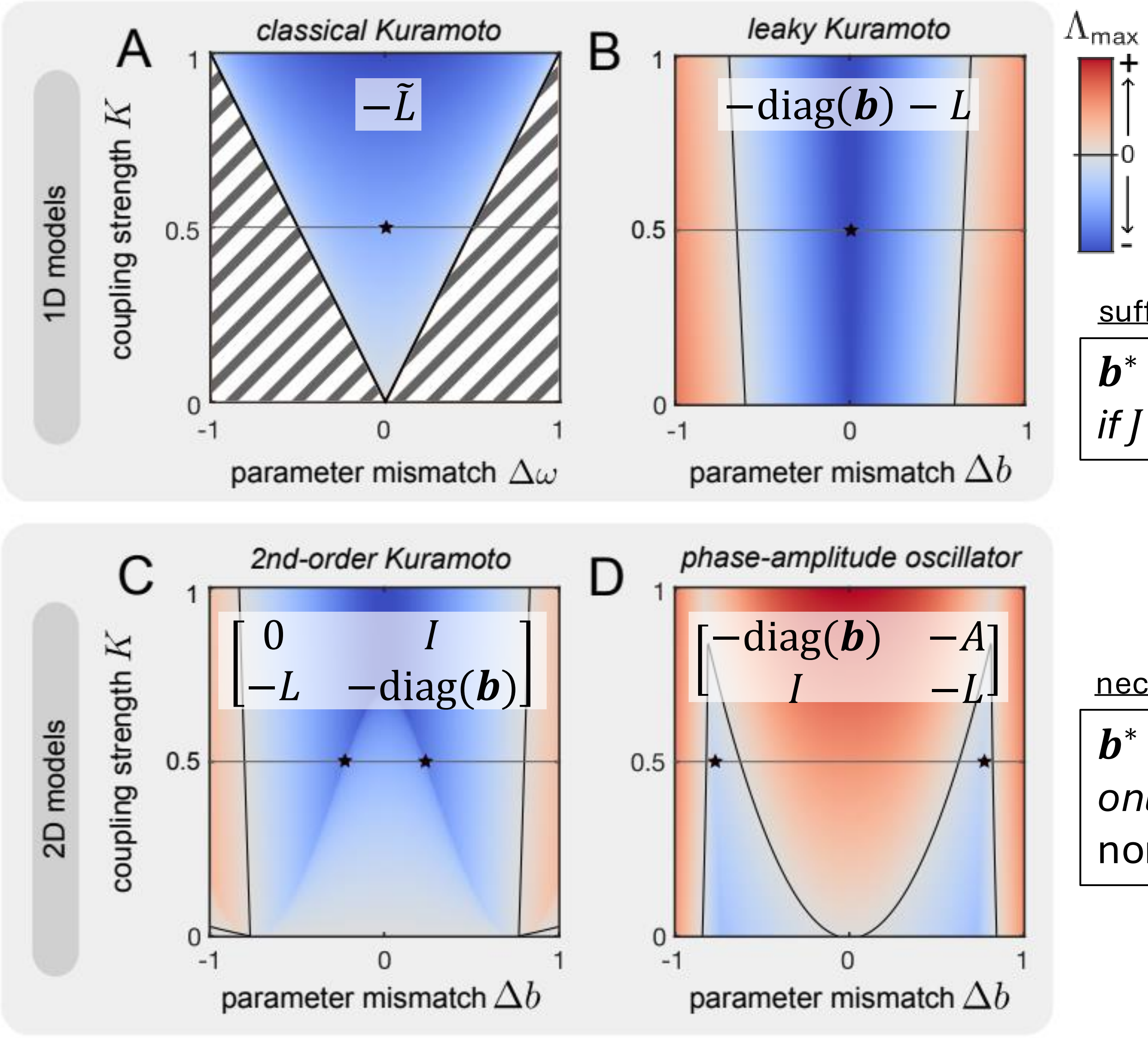
$\boldsymbol{b}^*$ is heterogeneous *only if* $J(\boldsymbol{b};A)$ is non-Hermitian.



$P\boldsymbol{b} \neq \boldsymbol{b}$ $\quad$ $P^TAP = A$

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?



<u>sufficiency</u>

$\boldsymbol{b}^*$ is homogeneous *if* $J(\boldsymbol{b}; A)$ is Hermitian.

<u>necessity</u>

$\boldsymbol{b}^*$ is heterogeneous *only if* $J(\boldsymbol{b}; A)$ is non-Hermitian.

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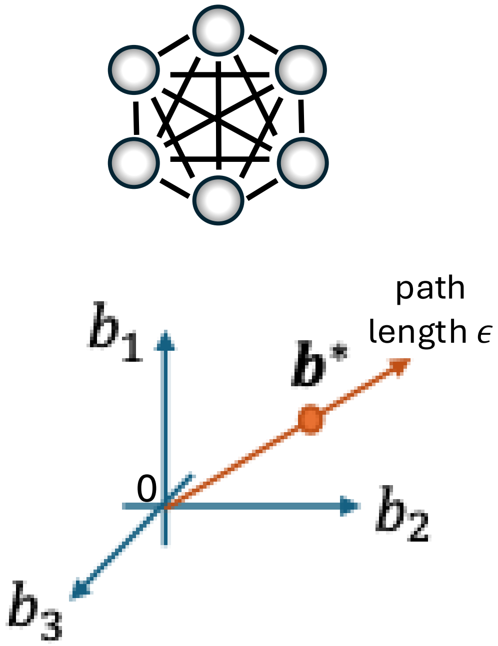
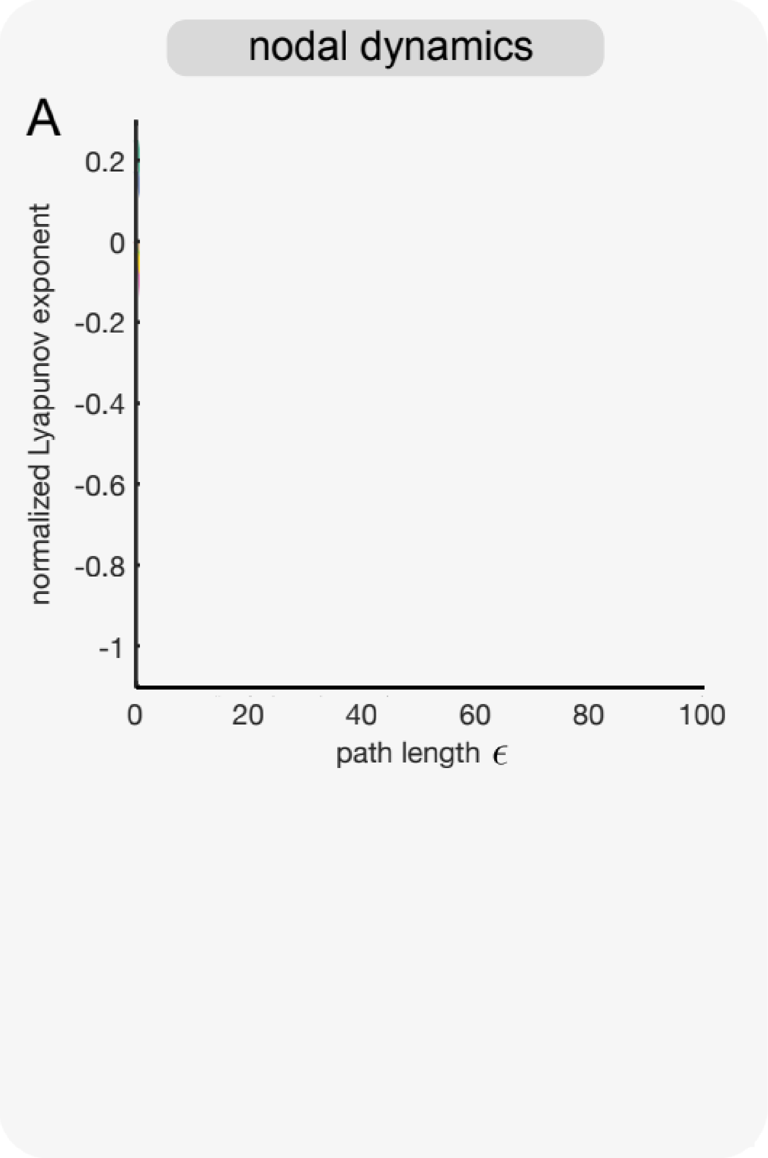
Optimizing heterogeneity





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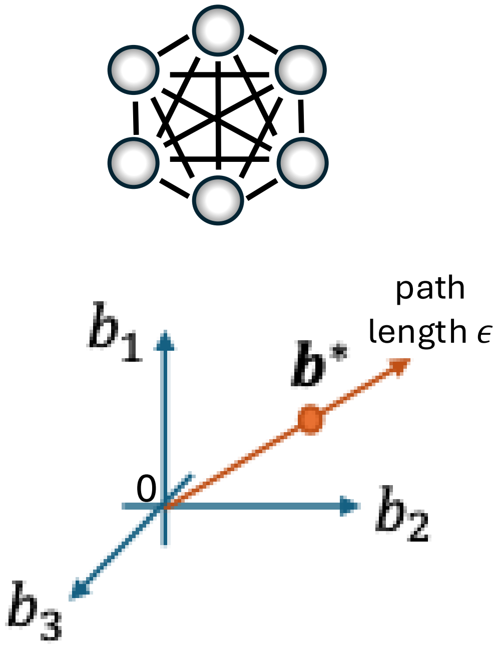
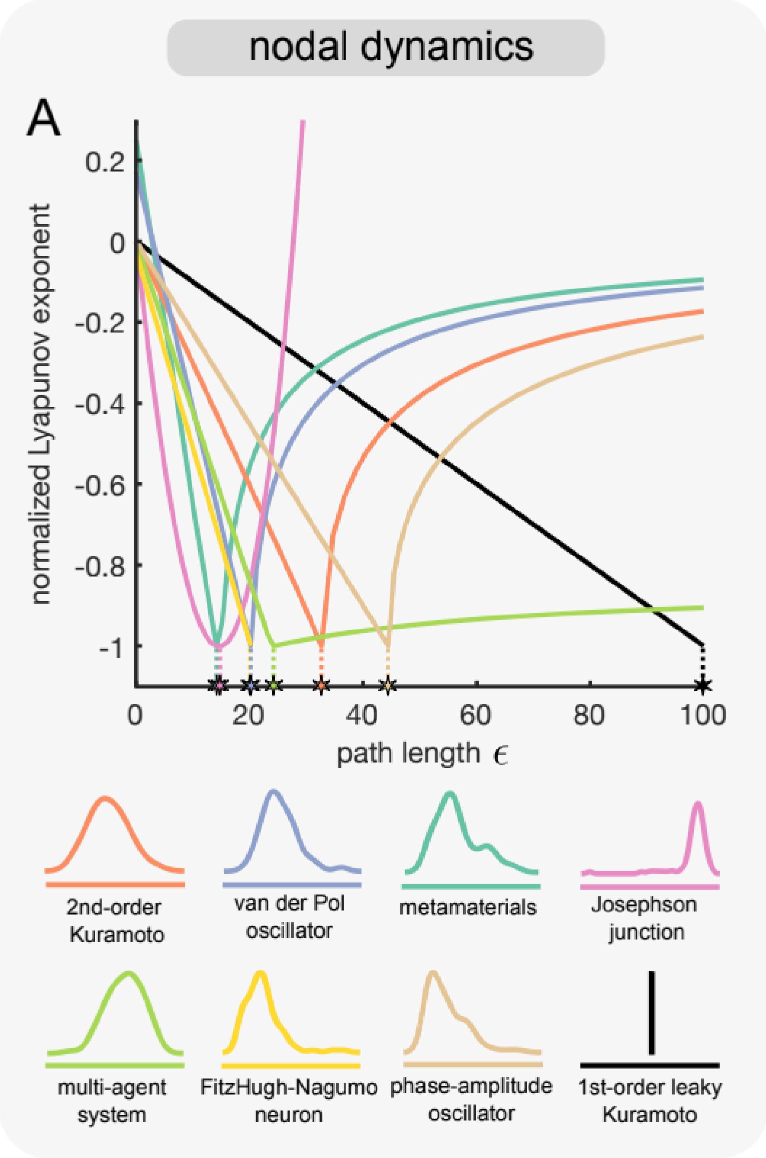
Optimizing heterogeneity



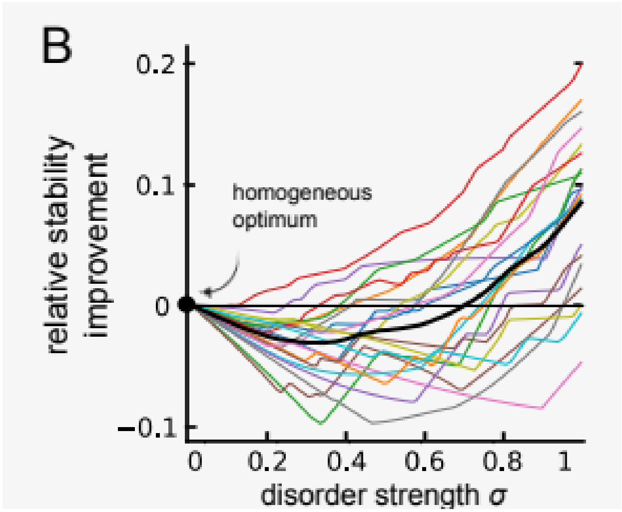


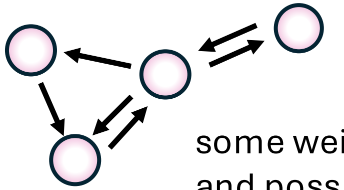
	System	Equations of motion	Jacobian matrix
first-order models	Leaky Kuramoto (synchronization)	$\dot{\phi}_i + b_i\phi_i = \omega + \sum_j A_{ij}\sin(\phi_j - \phi_i)$	$-(B+L)$
	Driven Kuramoto (pacemaker)	$\dot{\phi}_i + b_i\sin(\phi_i - \psi) = \omega + \sum_j A_{ij}\sin(\phi_j - \phi_i)$	$-(B+L)$
	Consensus (opinion, coordination)	$\dot{x}_i + b_i x_i = \sum_j A_{ij}(x_j - x_i)$	$-(B+L)$
second-order models	2nd-order Kuramoto (power grids)	$\ddot{\phi}_i + b_i\dot{\phi}_i = P + \sum_j A_{ij}\sin(\phi_j - \phi_i)$	$\begin{bmatrix} 0_N & I_N \\ -L & -B \end{bmatrix}$
	van der Pol (circuits, vibration)	$\ddot{x}_i + b_i(1 - x_i^2)\dot{x}_i + x_i = \sum_j A_{ij}(x_j - x_i)$	$\begin{bmatrix} 0_N & I_N \\ -(L+I_N) & -B \end{bmatrix}$
	Spring-mass system (materials)	$\ddot{x}_i + b_i\dot{x}_i + kx_i = \sum_j A_{ij}(x_j - x_i)$	$\begin{bmatrix} 0_N & I_N \\ -(L+kI_N) & -B \end{bmatrix}$
	Josephson junction (superconductors)	$\ddot{\phi}_i + b_i\dot{\phi}_i + I_c\sin(\phi_i) = I_b + \sum_j A_{ij}(\dot{\phi}_j - \dot{\phi}_i)$	$\begin{bmatrix} 0_N & I_N \\ -\sqrt{I_c^2 - I_b^2}\,I_N & -(L+B) \end{bmatrix}$
	Multi-agent system (flocking, navigation)	$\ddot{x}_i + b_i(\dot{x}_i - \dot{x}_t) + b_i(x_i - x_t) = \sum_j A_{ij}[(x_j - x_i) + (\dot{x}_j - \dot{x}_i)]$	$\begin{bmatrix} 0_N & I_N \\ -(L+B) & -(L+B) \end{bmatrix}$
2-dimensional models	FitzHugh-Nagumo (neurons)	$\dot{v}_i = v_i - \frac{v_i^3}{3} - w_i + \sum_j A_{ij}(v_j - v_i)$ $\tau\dot{w}_i = v_i + a - b_i w_i$	$\begin{bmatrix} -L + I_N - V^2 & -I_N \\ \tau^{-1}I_N & -\tau^{-1}B \end{bmatrix}$
	Phase-amplitude oscillator (lasers, circuits)	$\dot{r}_i = b_i r_i(1 - r_i) + \varepsilon r_i\sum_j A_{ij}\cos(\phi_j - \phi_i)$ $\dot{\phi}_i = \omega + r_i - 1 + r_i\sum_j A_{ij}\sin(\phi_j - \phi_i)$	$\begin{bmatrix} -B & -\varepsilon A \\ I_N & -L \end{bmatrix}$

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability

*now the analysis is for disordered parameters, so that $b_i \sim \mathcal{N}(\mu, \sigma^2)$



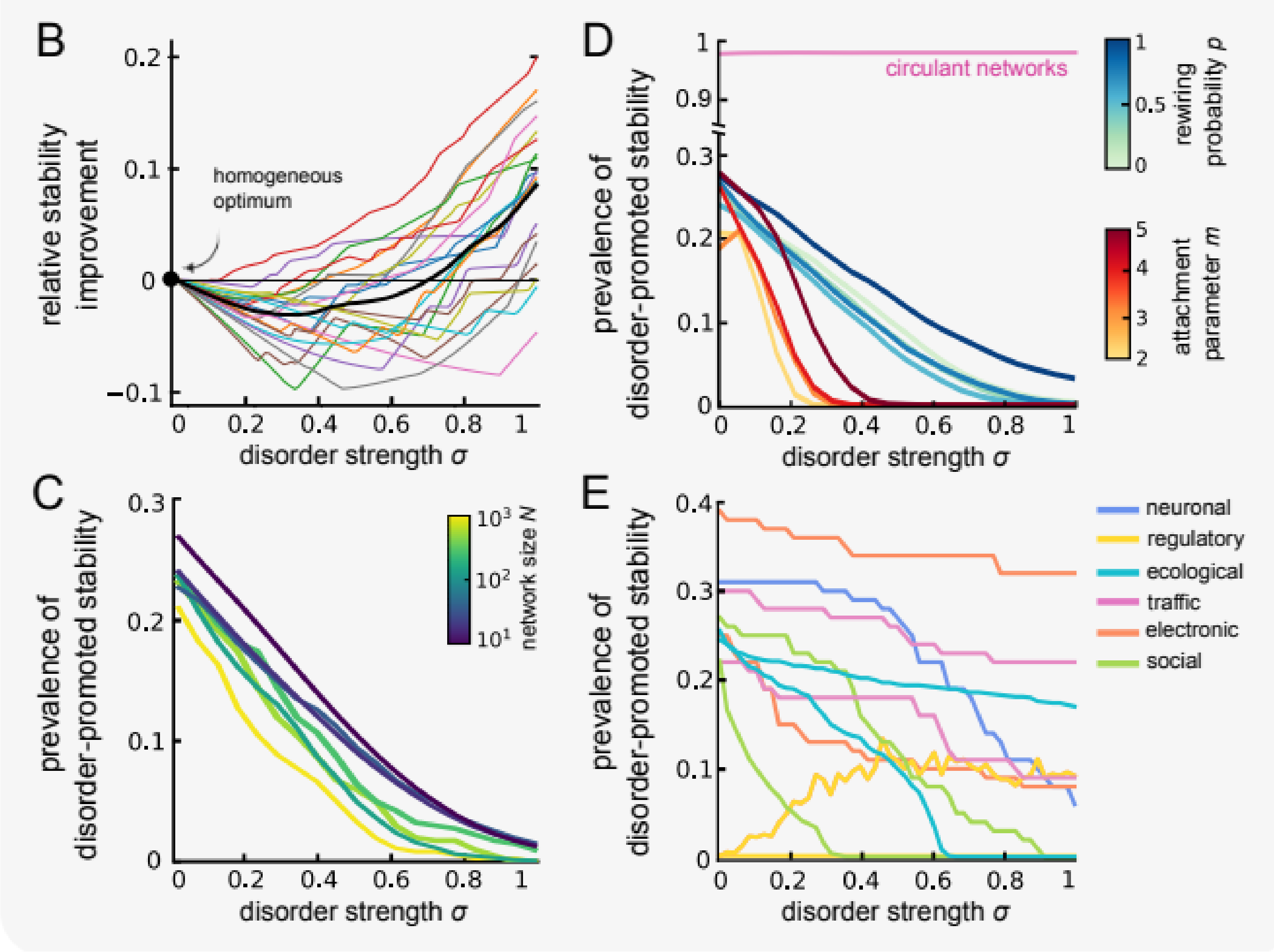


some weighted, directed,
and possibly sparse network

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability

*now the analysis is for disordered parameters, so that $b_i \sim \mathcal{N}(\mu, \sigma^2)$



directed small-world networks

directed scale-free networks

directed empirical networks

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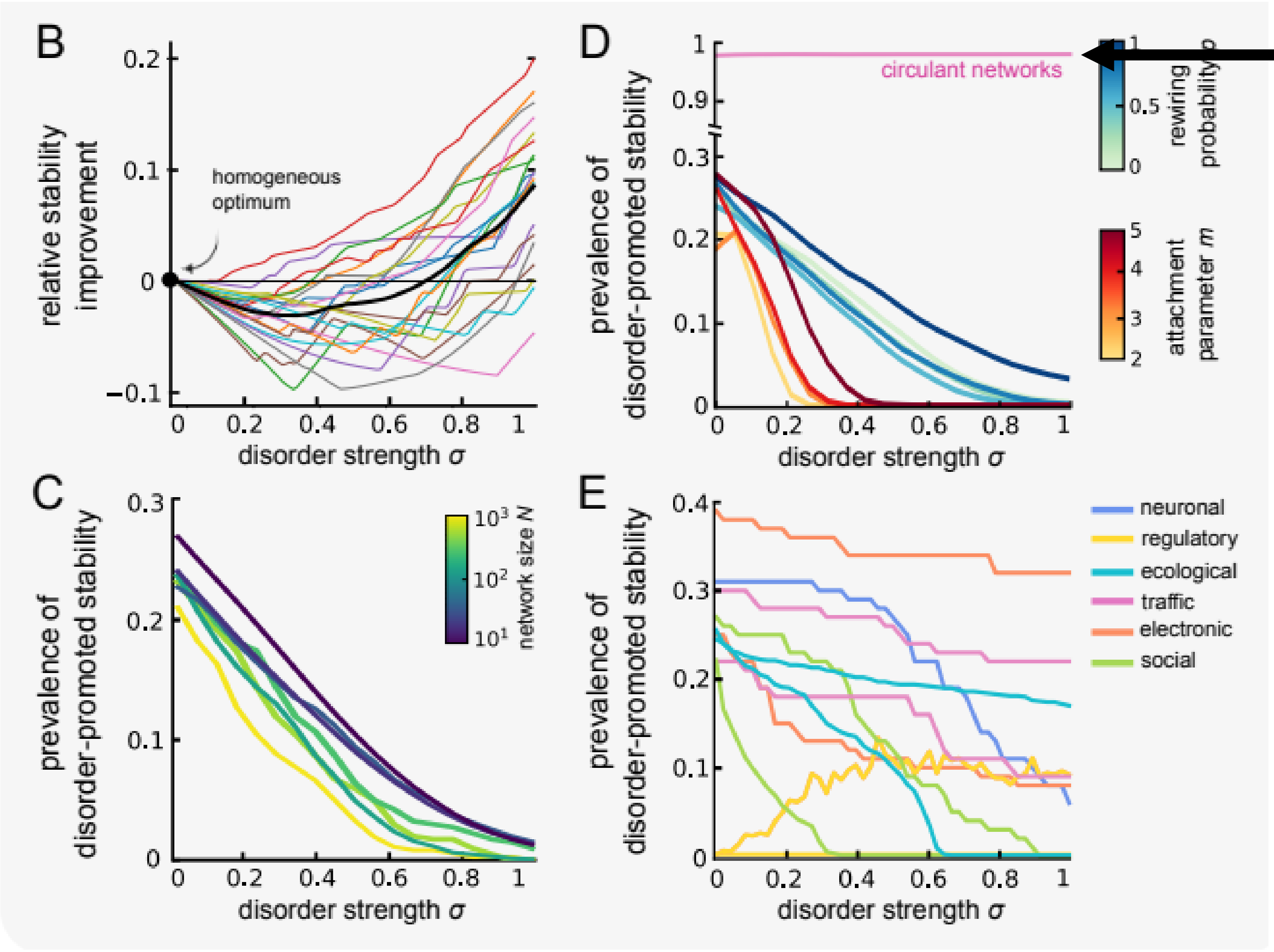
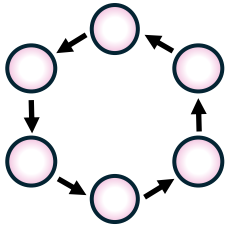
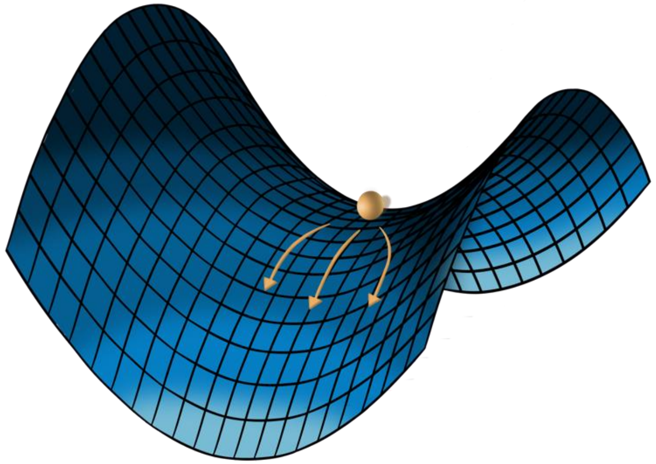
Disorder-promoted stability

*now the analysis is for disordered parameters, so that $b_i \sim \mathcal{N}(\mu, \sigma^2)$



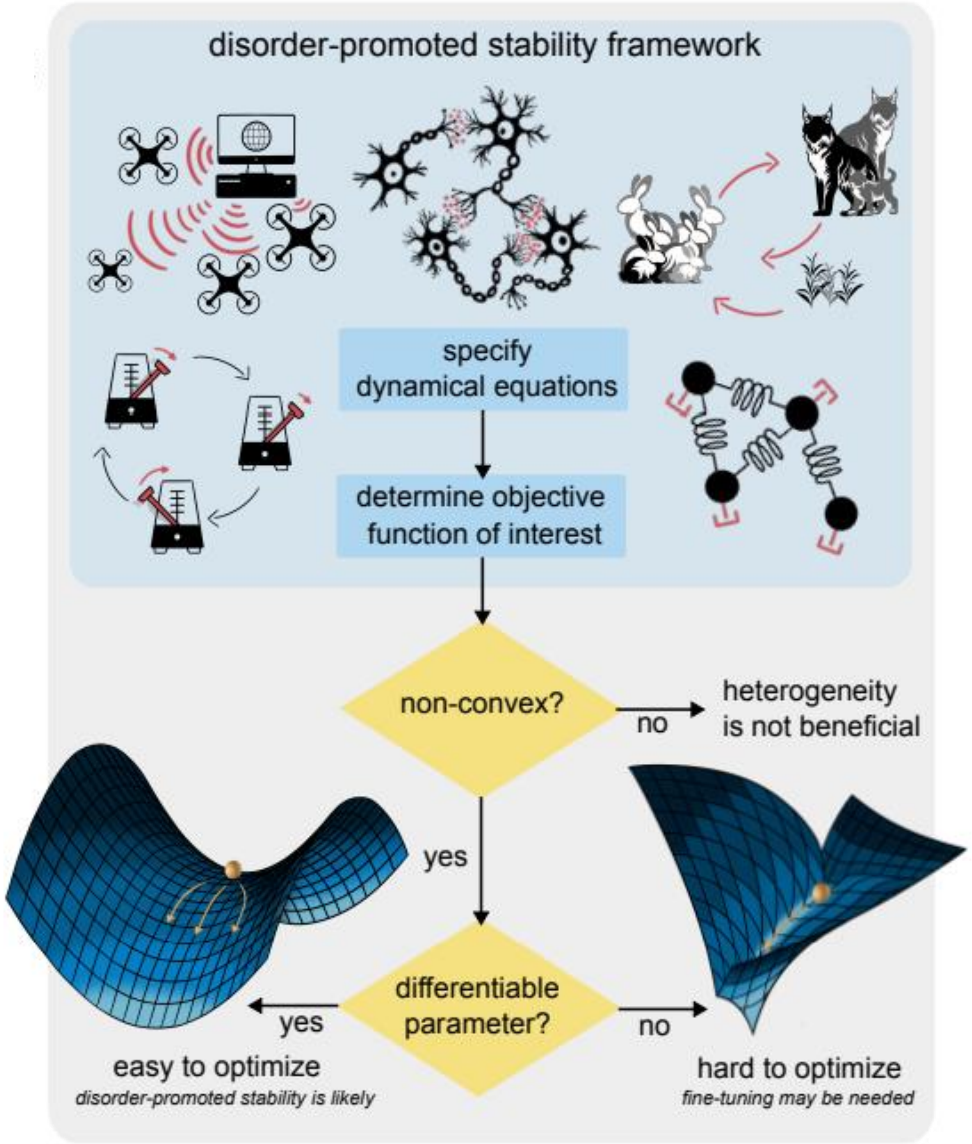


homogeneous
optimal parameter $\boldsymbol{b}^*$
is differentiable
and a saddle point



ANM, Zanin, Motter. Under review (2026).

The framework



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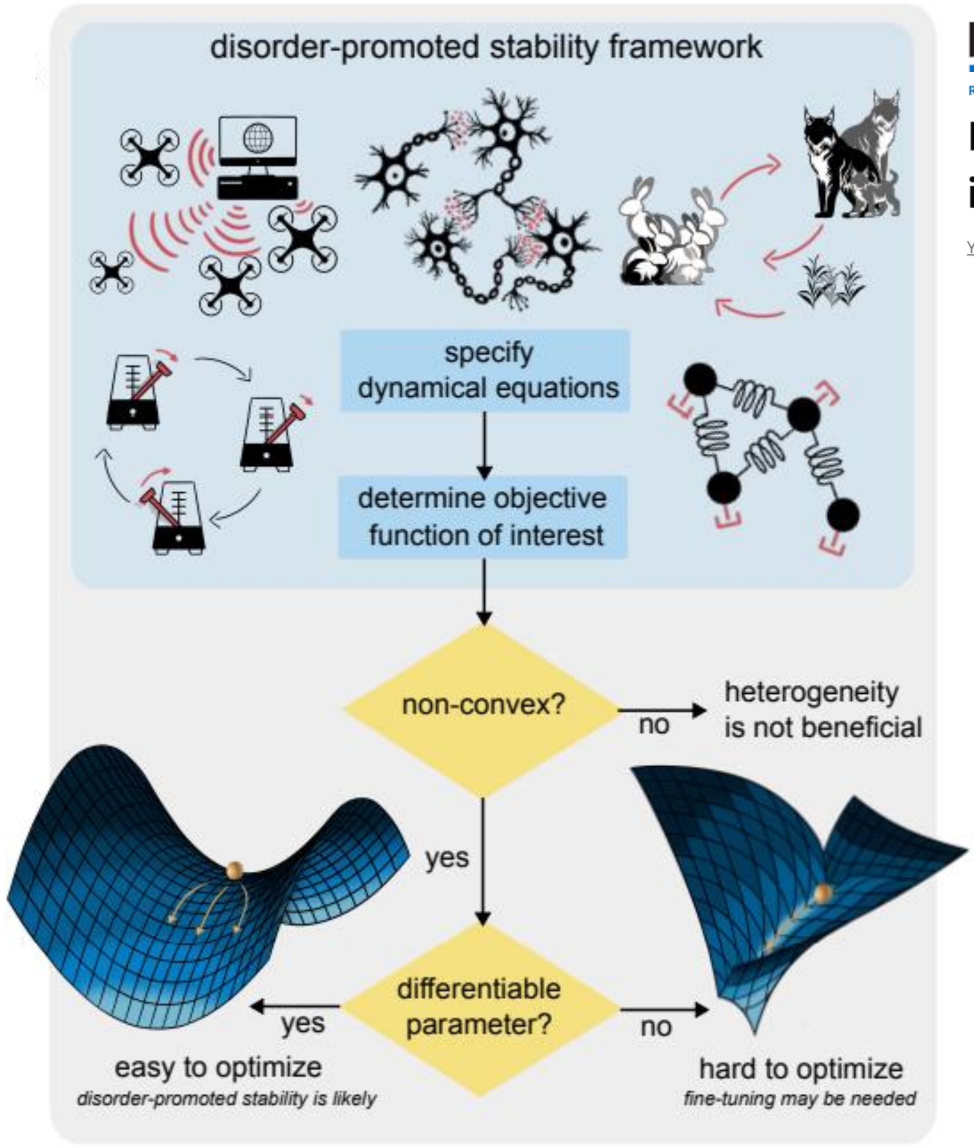


PNAS
RESEARCH ARTICLE | APPLIED PHYSICAL SCIENCES

Random heterogeneity outperforms design in network synchronization

Yuanzhao Zhang, Jorge L. Ocampo-Espindola, István Z. Kiss, and Adilson E. Motter Authors Info & Affiliations

LETTERS TO NATURE

Taming spatiotemporal chaos with disorder

Y. Braiman*†, John F. Lindner*‡ & William L. Ditto*

* Applied Chaos Laboratory, School of Physics, Georgia Institute of Technology, Atlanta, Georgia 30332, USA
‡ The College of Wooster, Wooster, Ohio 44691, USA

PNAS
RESEARCH ARTICLE | NEUROSCIENCE
OPEN ACCESS

Neural heterogeneity controls computations in spiking neural networks

Richard Gast[a,b,1], Sara A. Solla[a], and Ann Kennedy[a,b]

nature communications

Article

Emergent microrobotic oscillators via asymmetry-induced order

PHYSICAL REVIEW RESEARCH 7, 013207 (2025)

Social network heterogeneity promotes depolarization of multidimensional correlated opinions

Jaume Ojer,[1] Michele Starnini,[2,3,*] and Romualdo Pastor-Satorras[1,†]

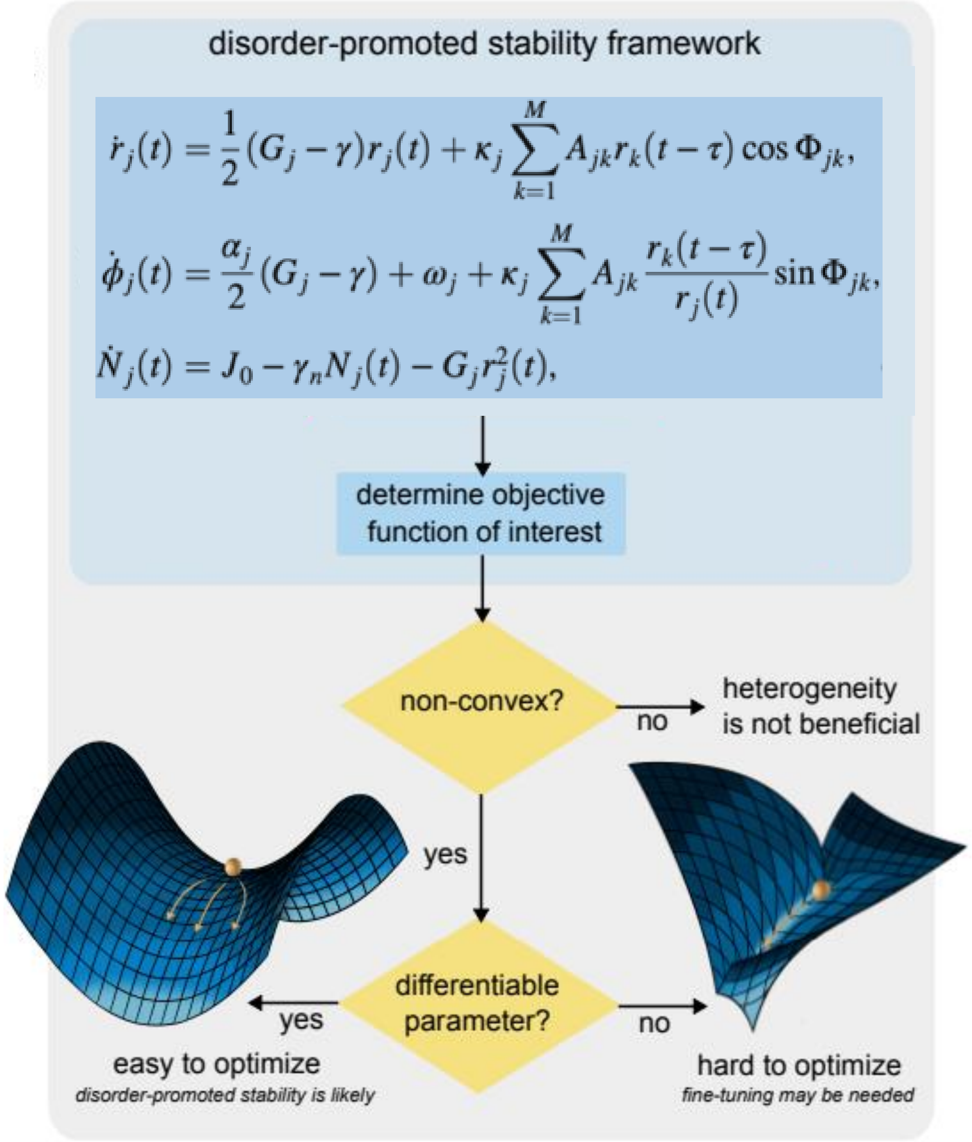
PRX ENERGY 3, 033003 (2024)

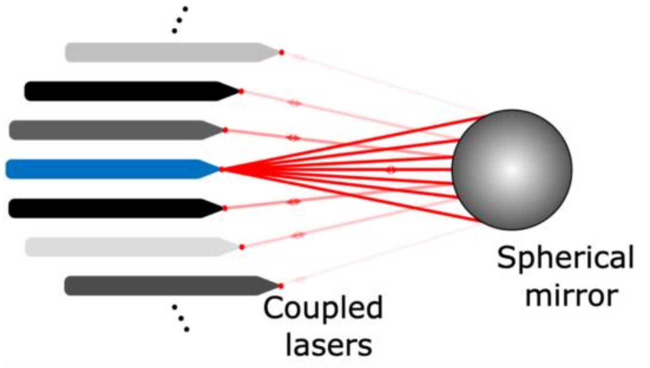
Stabilizing Large-Scale Electric Power Grids with Adaptive Inertia

Julian Fritzsch[1,2,*] and Philippe Jacquod[1,2,†]

[1]Department of Quantum Matter Physics, University of Geneva, Geneva CH-1211, Switzerland
[2]School of Engineering, University of Applied Sciences of Western Switzerland, Haute Ecole Spécialisée de Suisse occidentale (HES-SO), Sion CH-1950, Switzerland

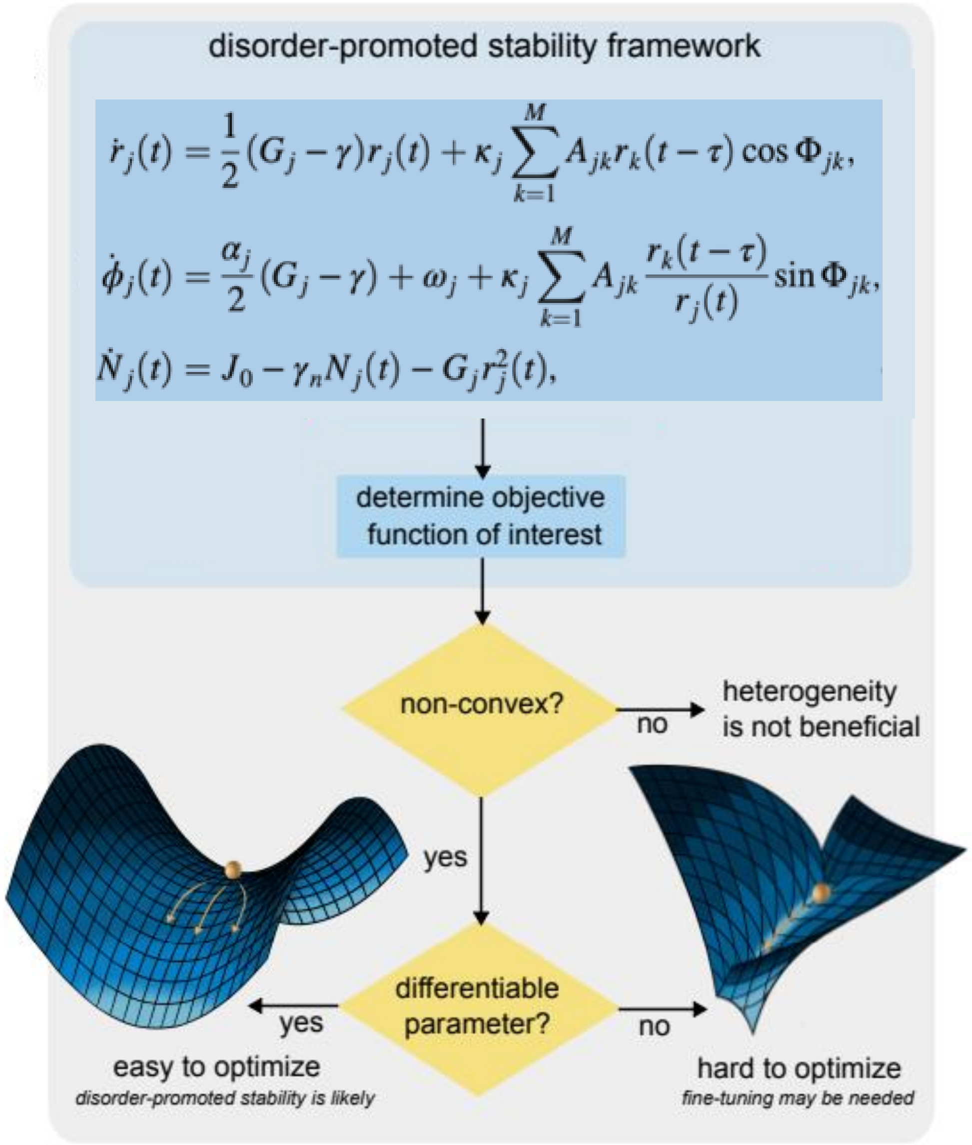
The framework: Coupled-laser arrays

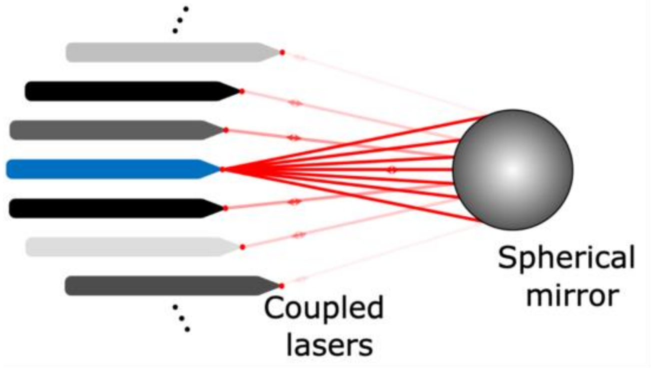


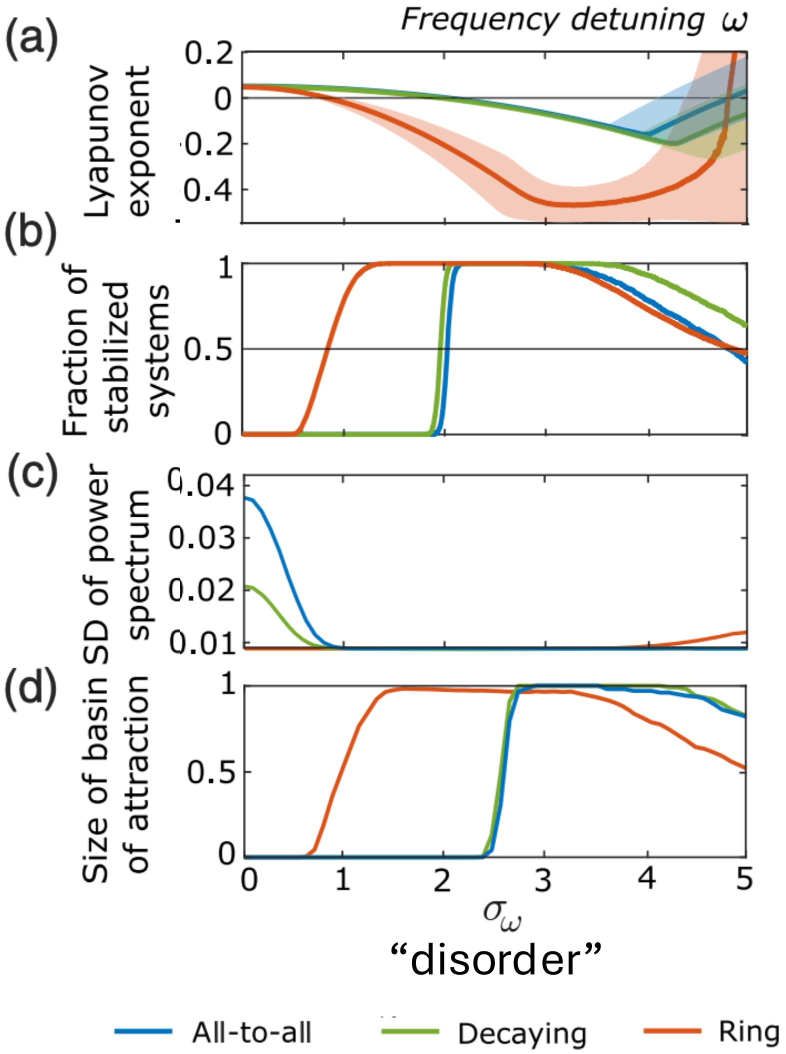


Barioni, **ANM**, Motter. *PRL* (2025).

The framework: Coupled-laser arrays

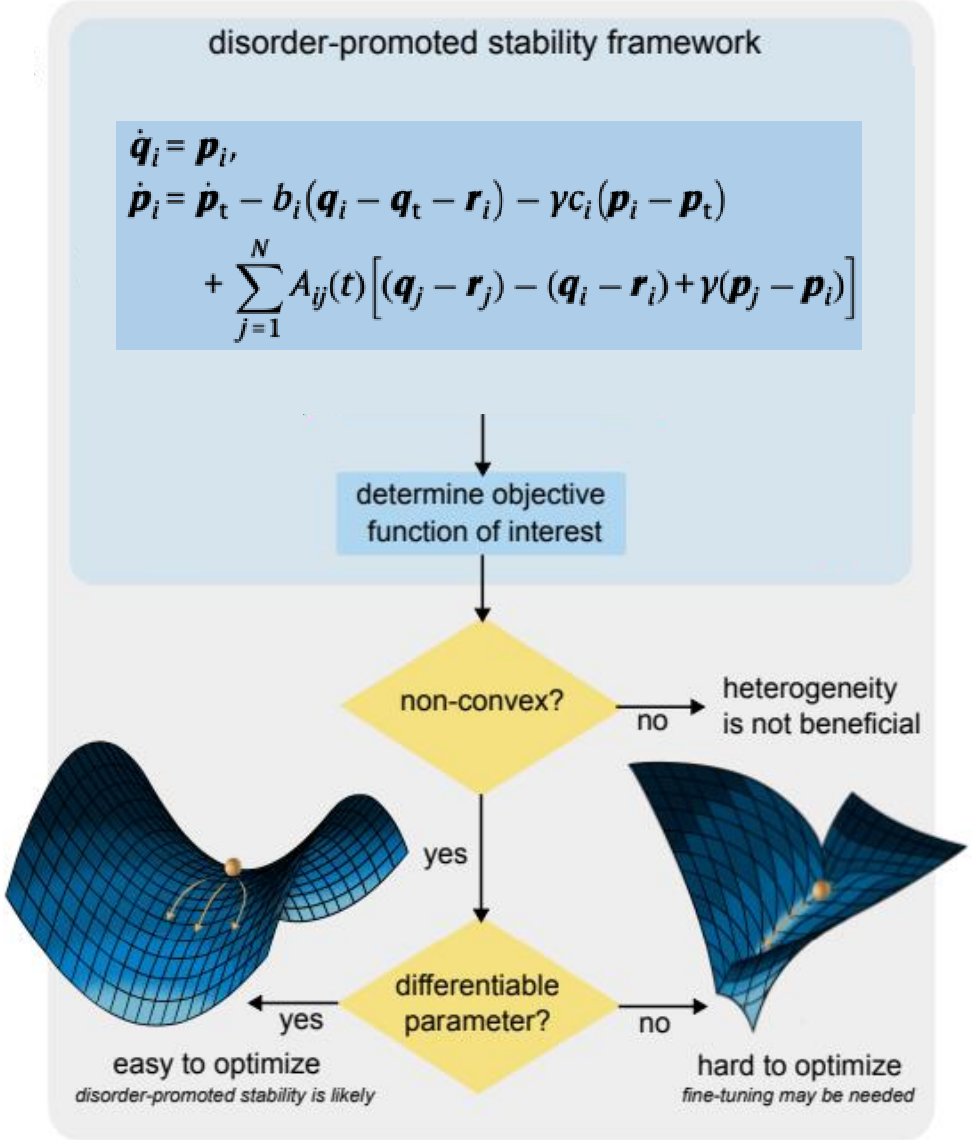


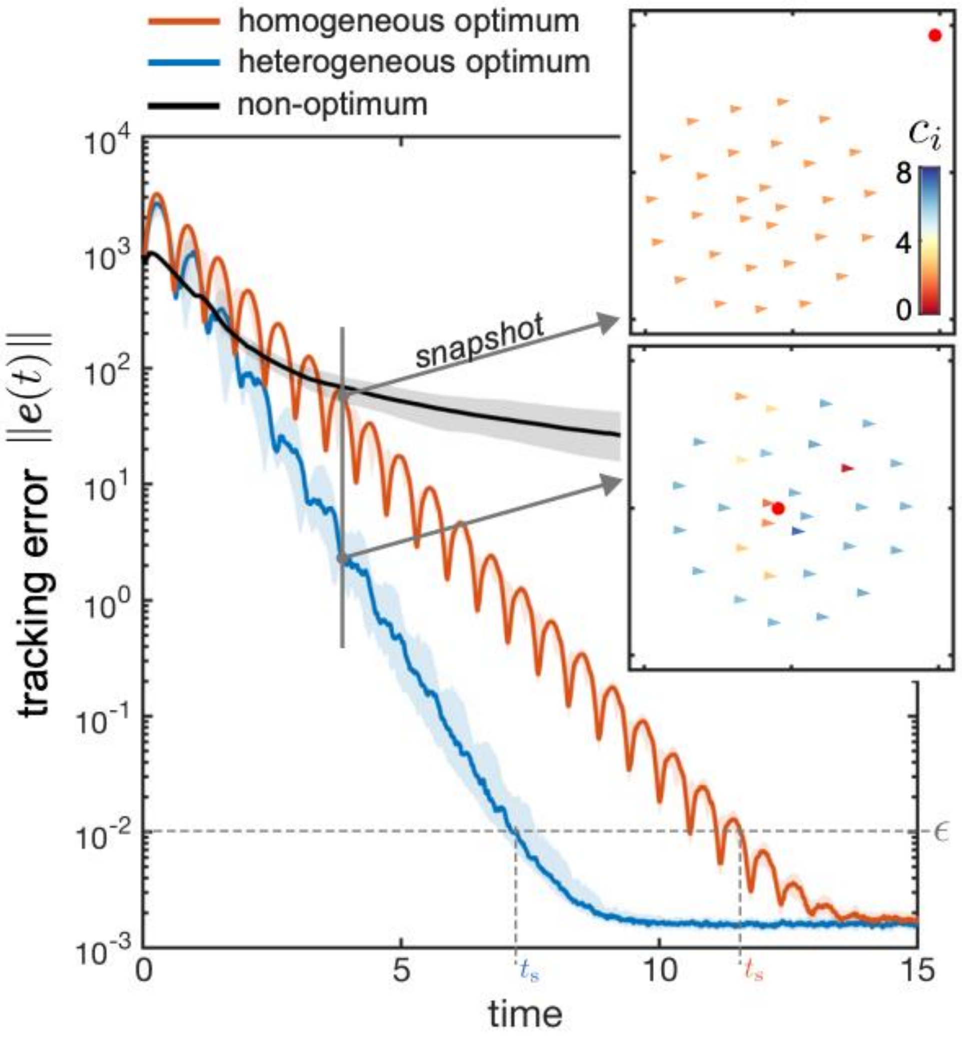




Barioni, **ANM**, Motter. *PRL* (2025).

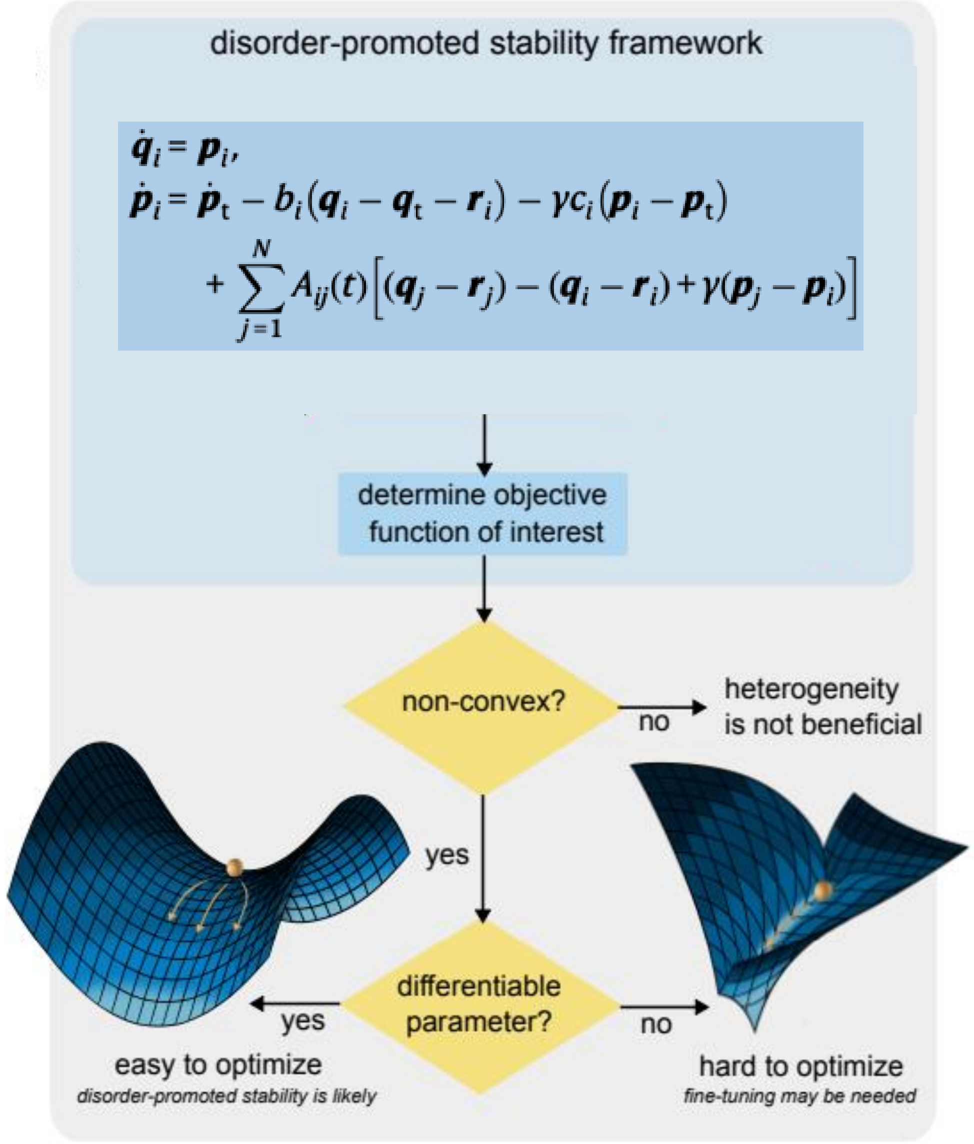
The framework: Optimal flocking





ANM, Barioni, Duan, Motter. *Nature Communications* (2025).

The framework: Optimal flocking



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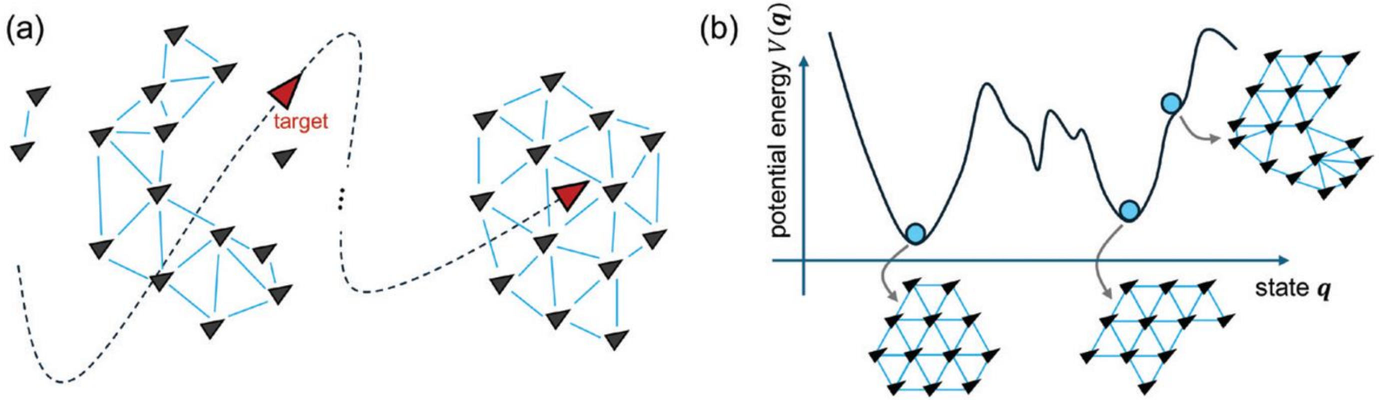
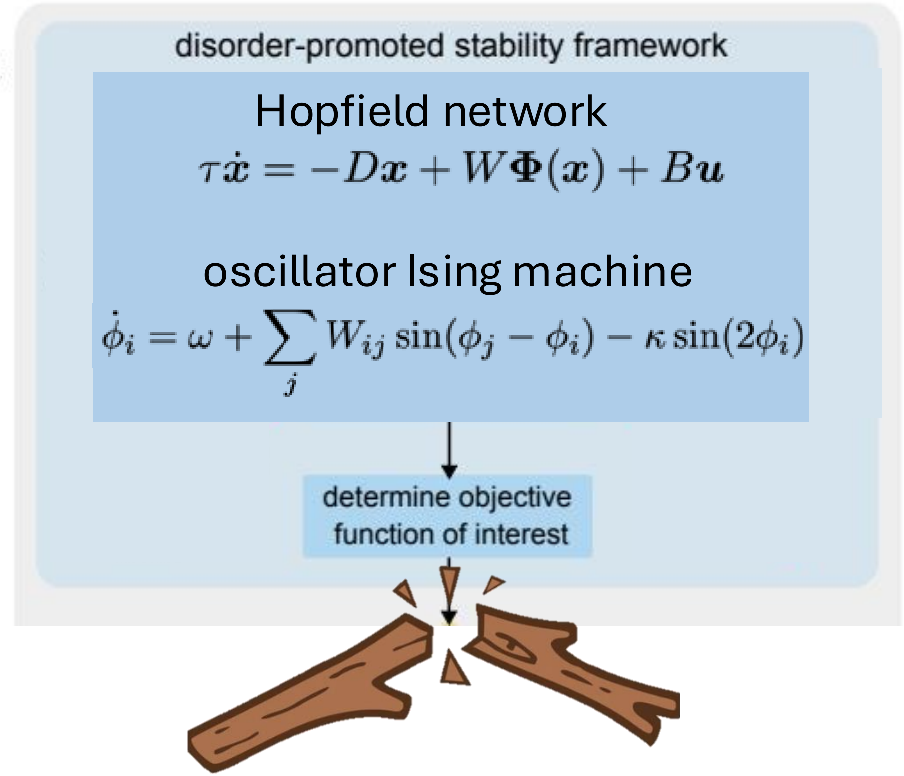
Heterogeneity for Flocking and Computation: From Biology to Mathematics

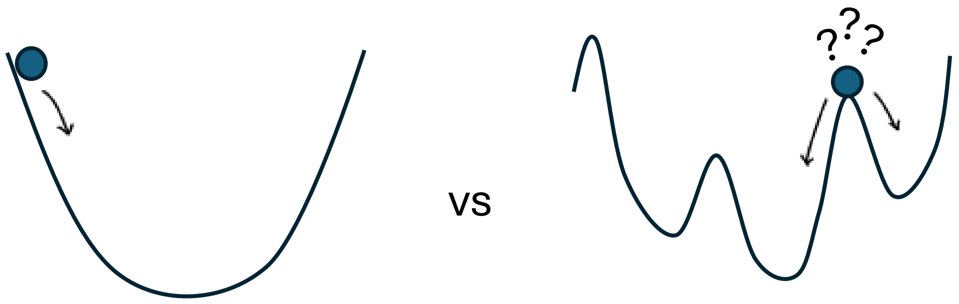
By Arthur N. Montanari, Ana Elisa D. Barioni, and Adilson E. Motter

Art by
Camila Montanari



The framework: Neurocomputation





stabilization
synchronization

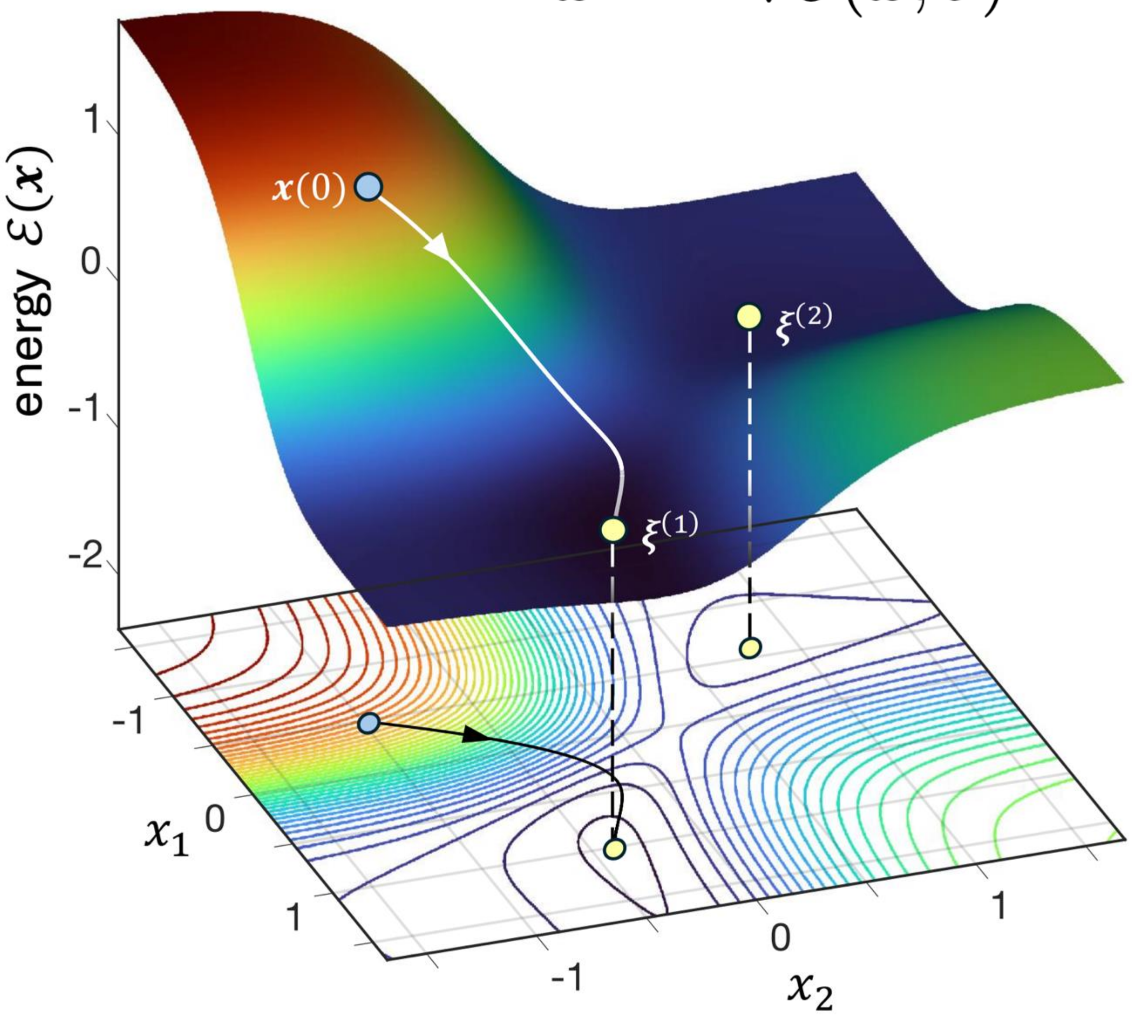
computation
decision-making

Part I: Disorder & Stability

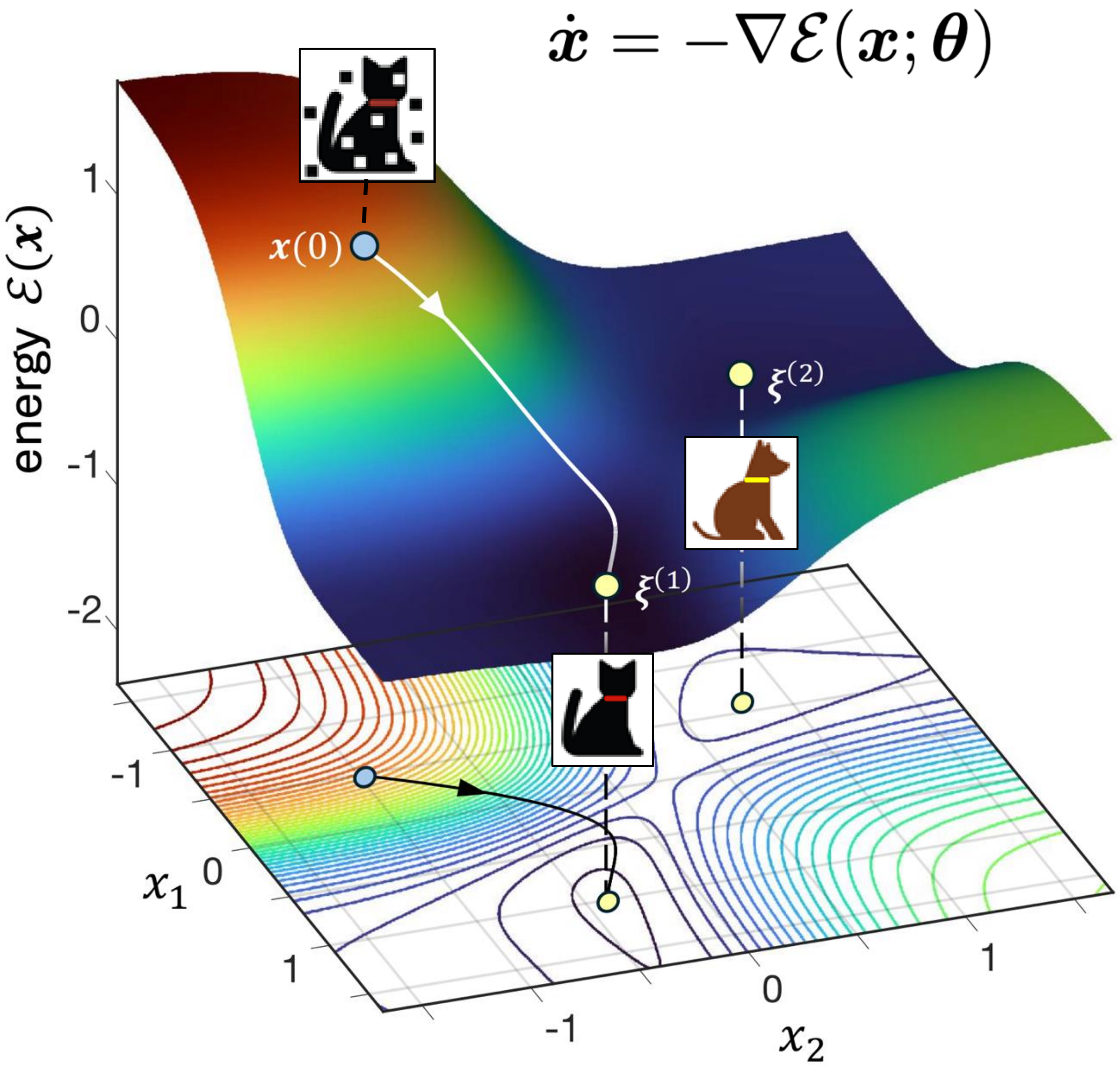
Part II: Disorder & Computation

Energy-based dynamical models

$$\dot{\boldsymbol{x}} = -\nabla\mathcal{E}(\boldsymbol{x};\boldsymbol{\theta})$$



Energy-based dynamical models



Hopfield network

$$\dot{\boldsymbol{x}} = -\nabla \mathcal{E}(\boldsymbol{x}; W)$$

$$\tau \dot{\boldsymbol{x}} = -D\boldsymbol{x} + W\boldsymbol{\Phi}(\boldsymbol{x}) + B\boldsymbol{u}$$

Ising machine

$\dot{\boldsymbol{\theta}} =$ gradient descent on the Ising Hamiltonian

ANM, Bullo, Krotov, Motter. *ACC* (2026).
Americal Control Conference (tutorial).
arXiv:2604.05042

Energy-based dynamical models

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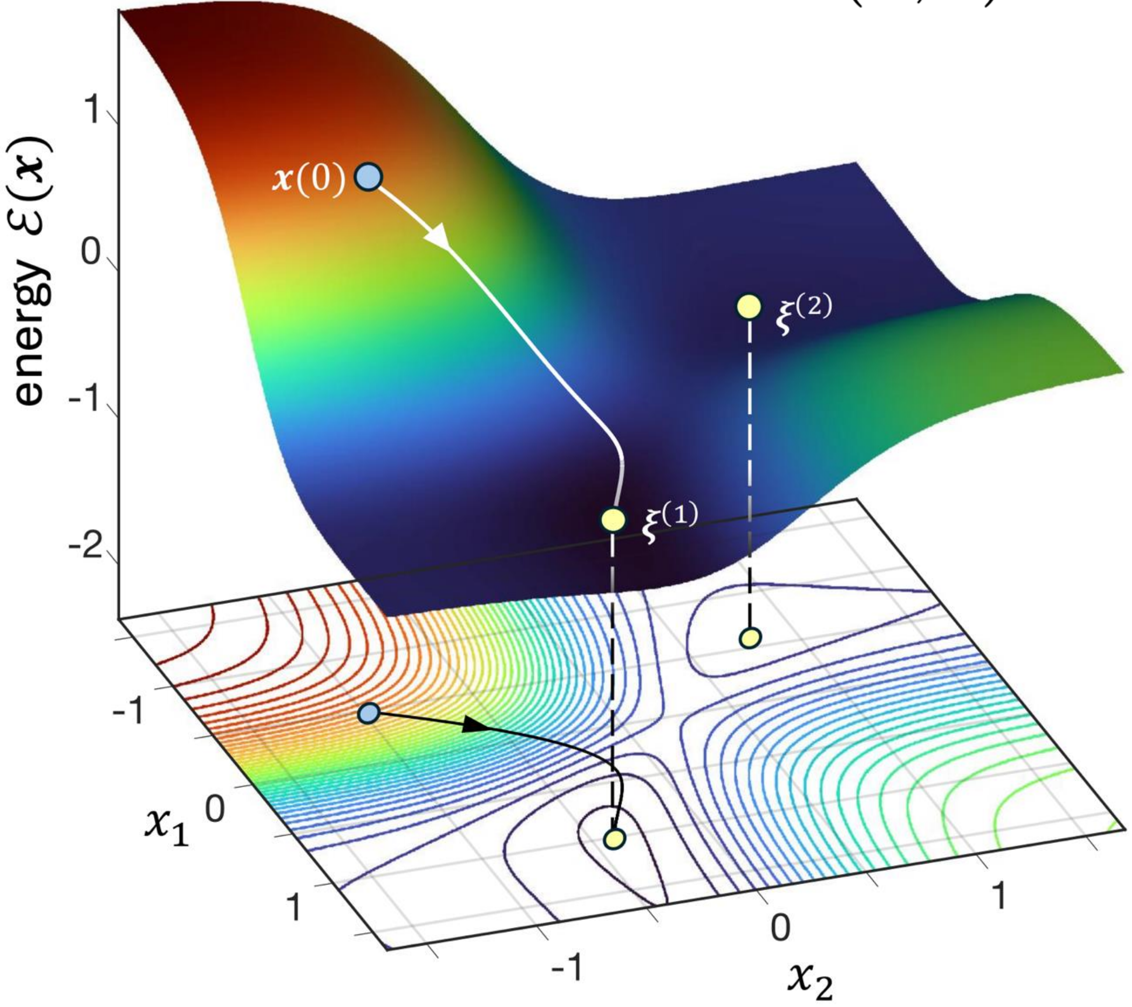

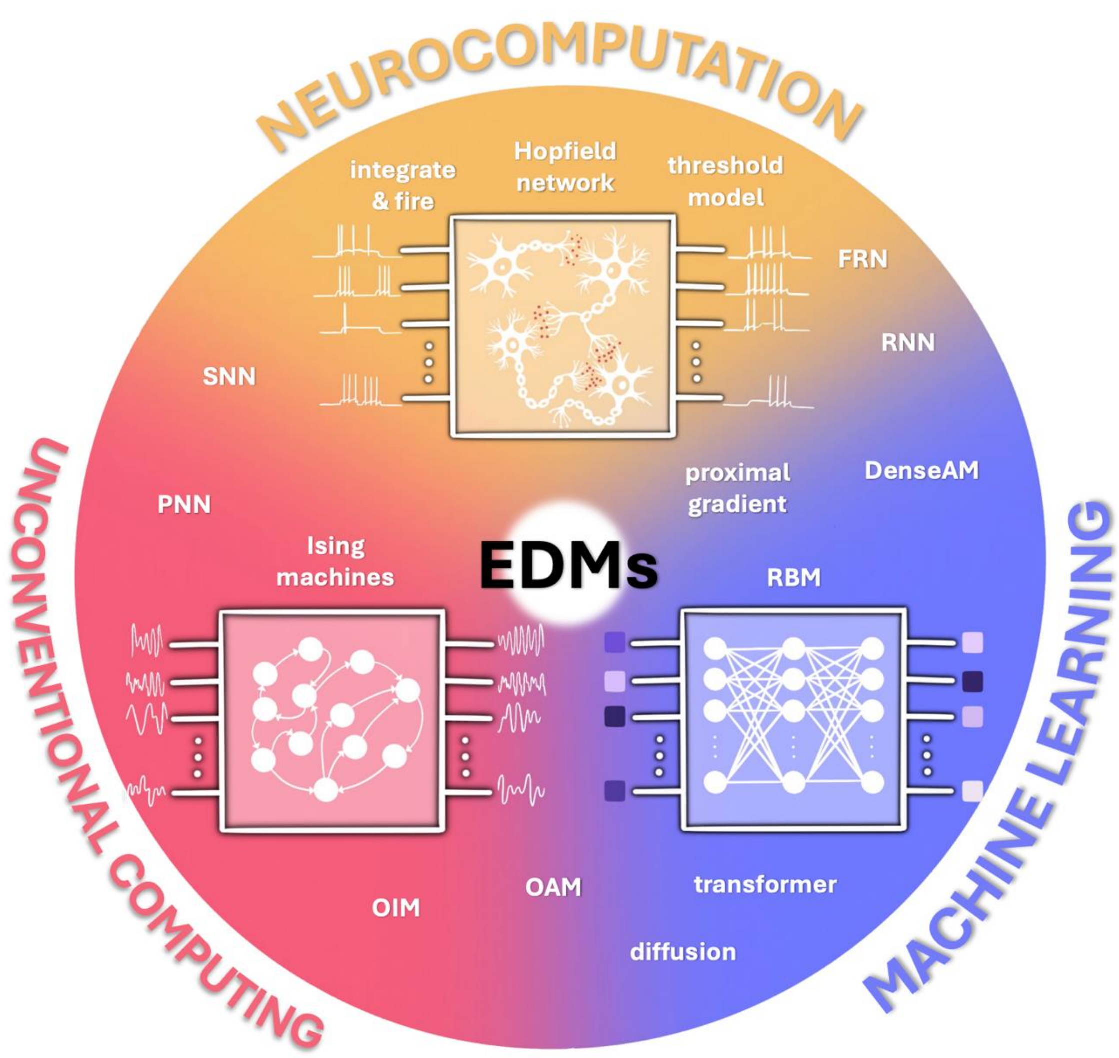

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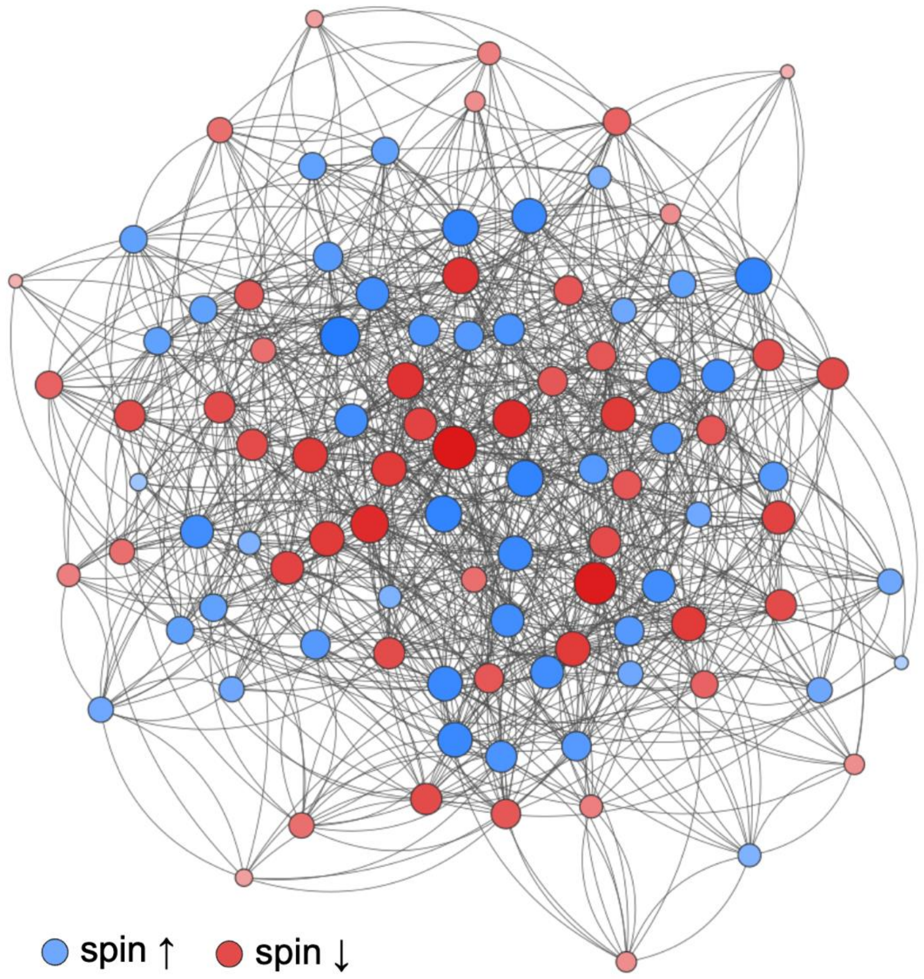
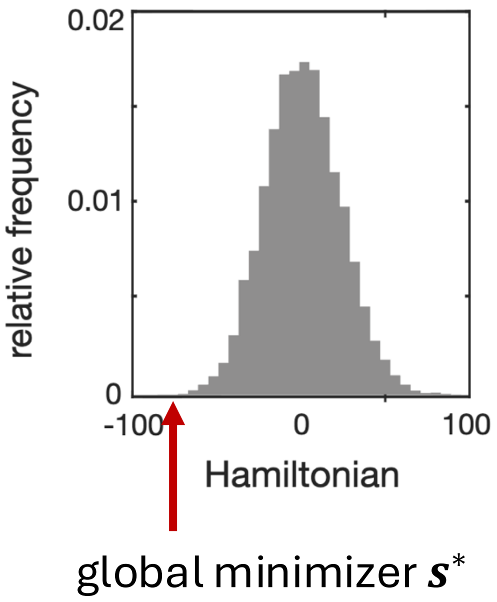
Ising optimization problem



Ising Hamiltonian: $H(s) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij} s_i s_j, \;\; s_i \in \{-1,+1\}$

Ising optimization problem: $\min_{\boldsymbol{s}} H(\boldsymbol{s})$



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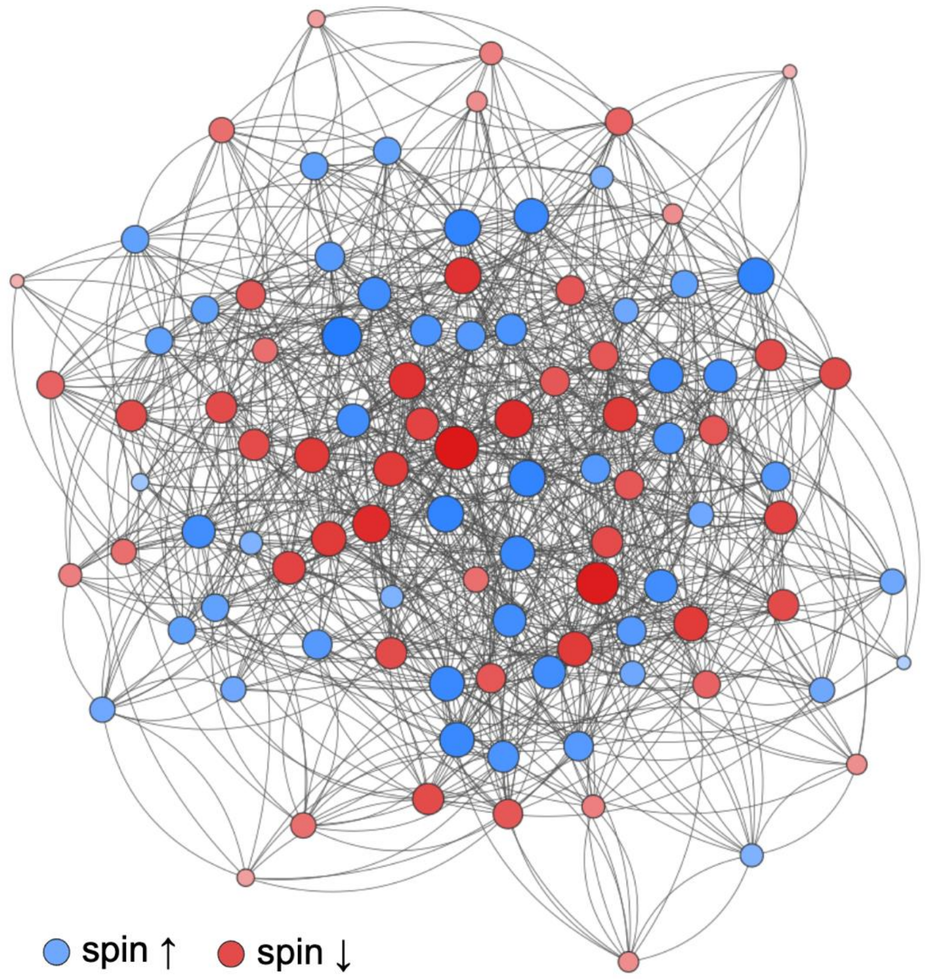
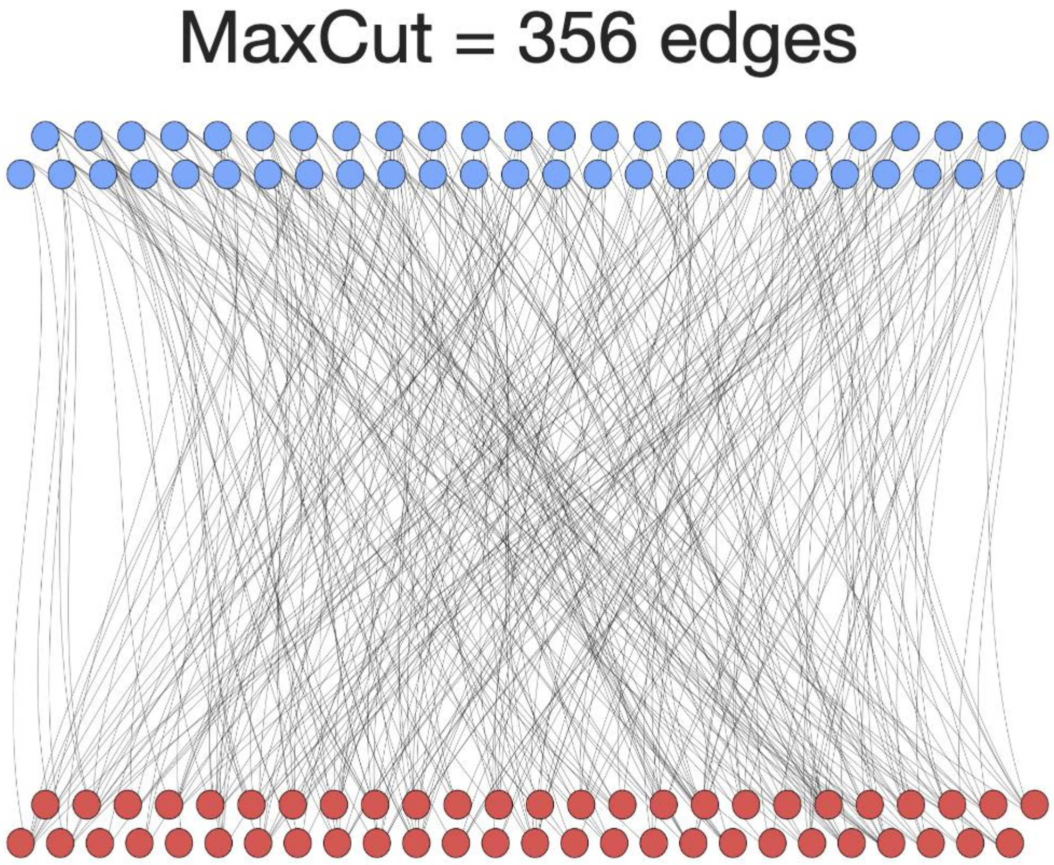
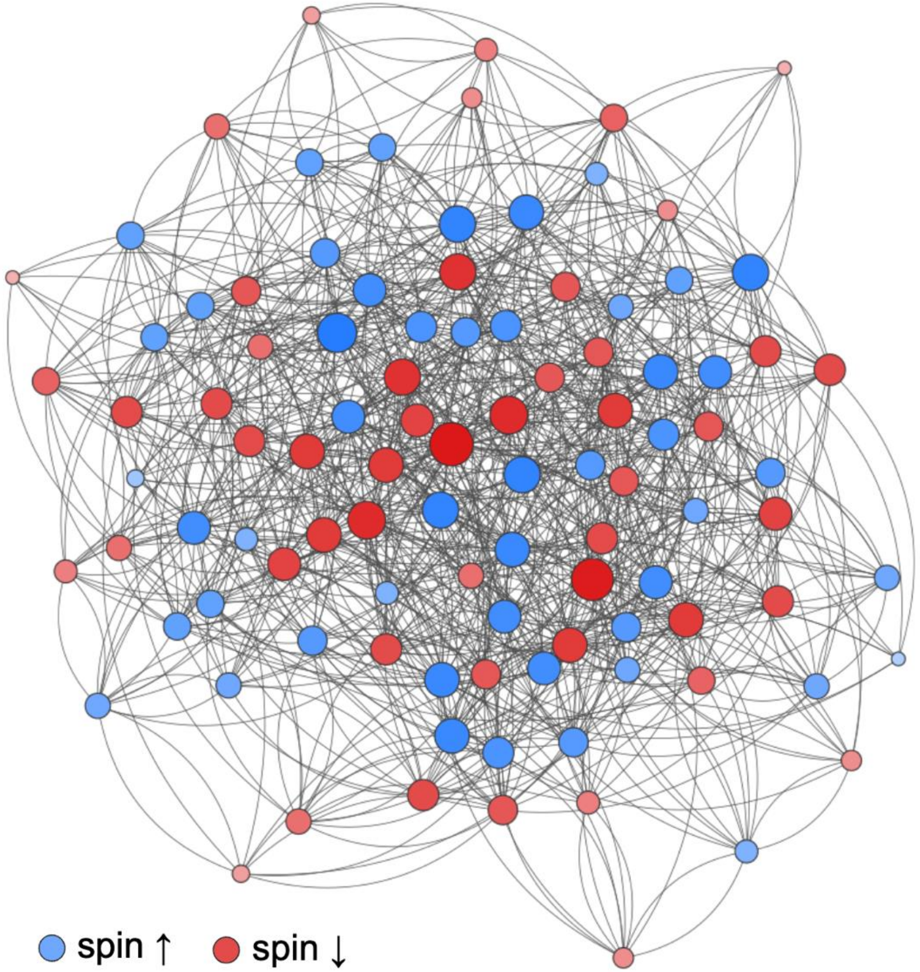


???

2^N possible solutions



Ising machines



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Ising machine (EDM): $\dot{\boldsymbol{\theta}} = -\nabla E(\boldsymbol{\theta})$

$$E(\boldsymbol{\theta}) = -\frac{1}{2}\sum_{i=1}^{N}\sum_{j=1}^{N} J_{ij}\cos(\theta_i - \theta_j) + \sum_{i=1}^{N} \mu_i \sin(\theta_i)^2$$

$$\theta_i^*(\boldsymbol{s}) = \begin{cases} 0, & \text{if } s_i = +1, \\ \pi, & \text{if } s_i = -1, \end{cases}$$

Oscillator Ising machines

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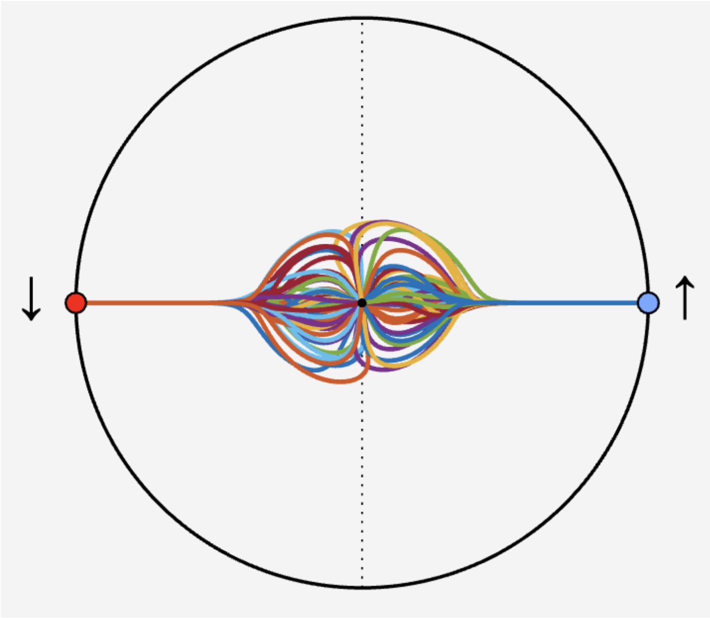
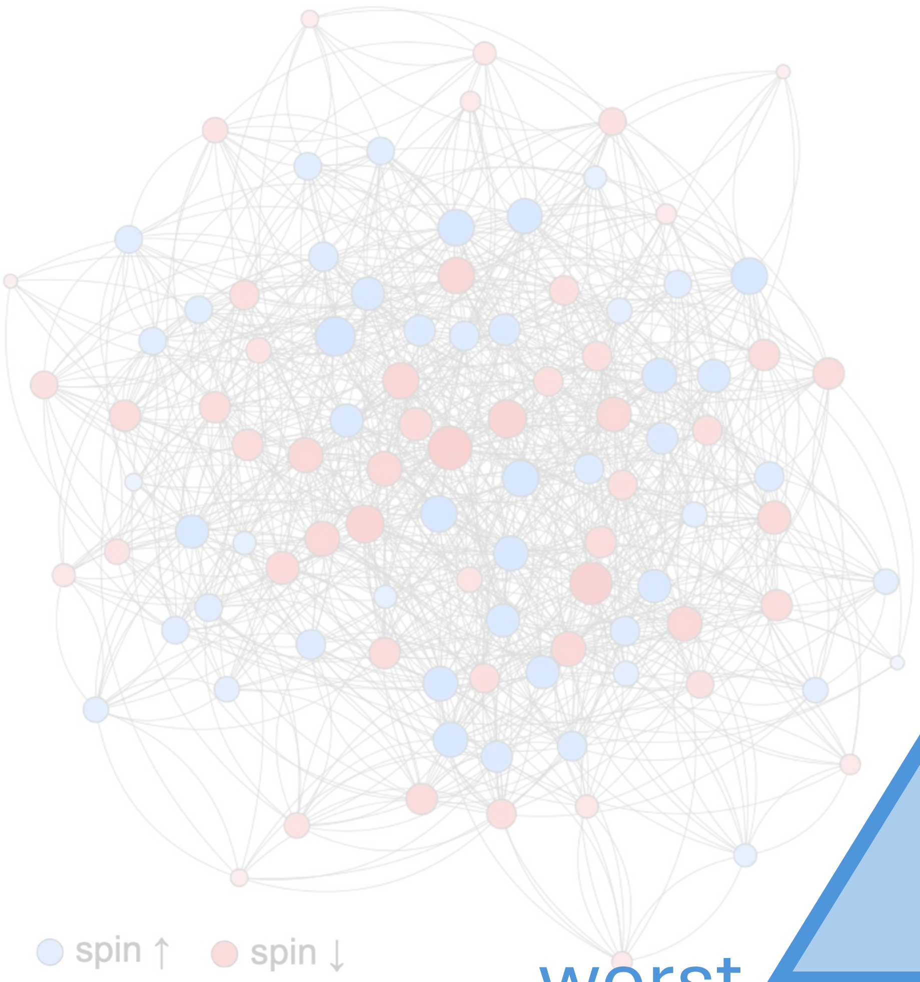


Oscillator Ising machine: (OIM) $\dot{\theta}_i = \sum_{j=1}^{N} J_{ij}\sin(\theta_j - \theta_i) - \mu_i \sin(2\theta_i)$

Wang, Roychowdhury. *UCNC* (2019). Cheng, Khairul, Shukla, Lin. *Chaos* (2024).

Oscillator Ising machines



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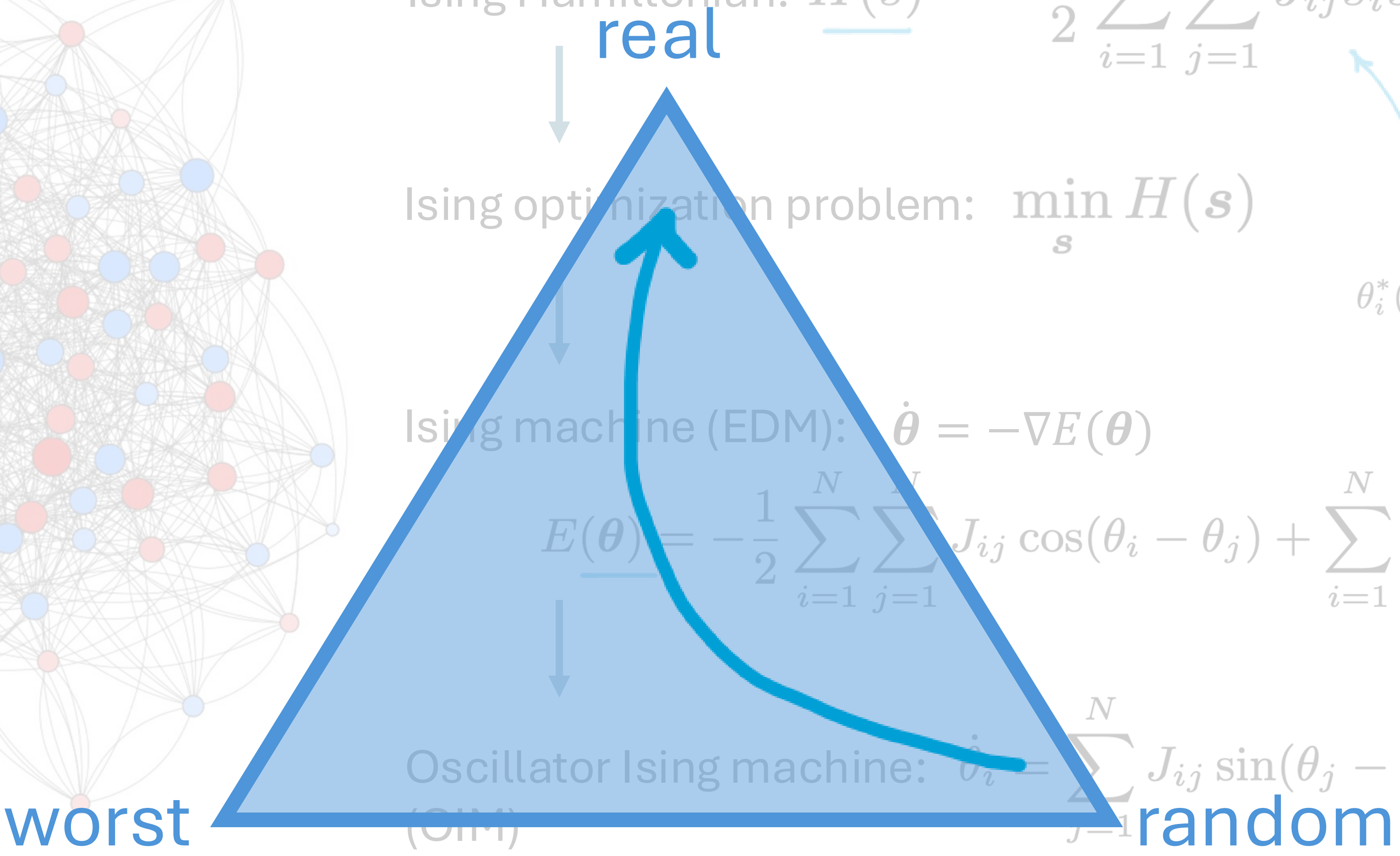

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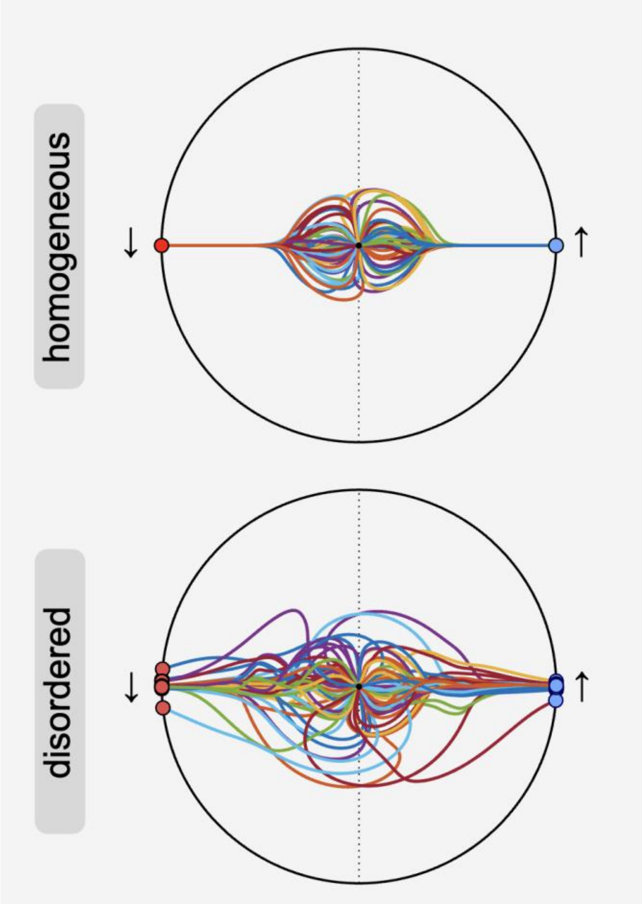
Disordered oscillator Ising machines



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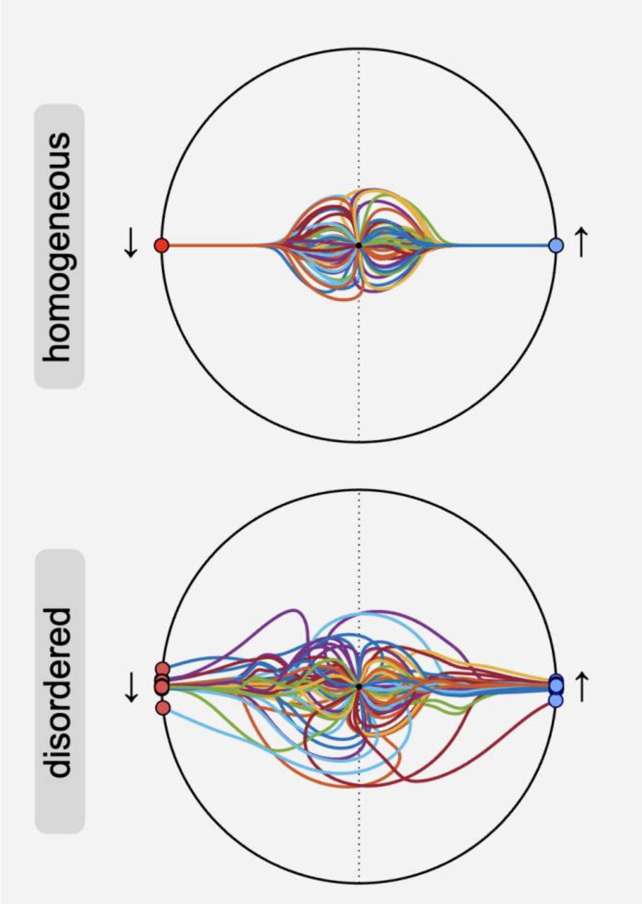
(OIM)

$\mu_i \sim \mathcal{N}(\mu, \sigma^2)$

Disordered oscillator Ising machines



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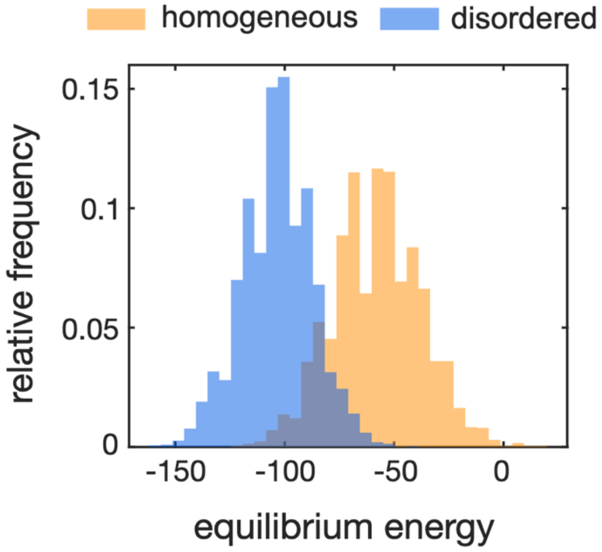

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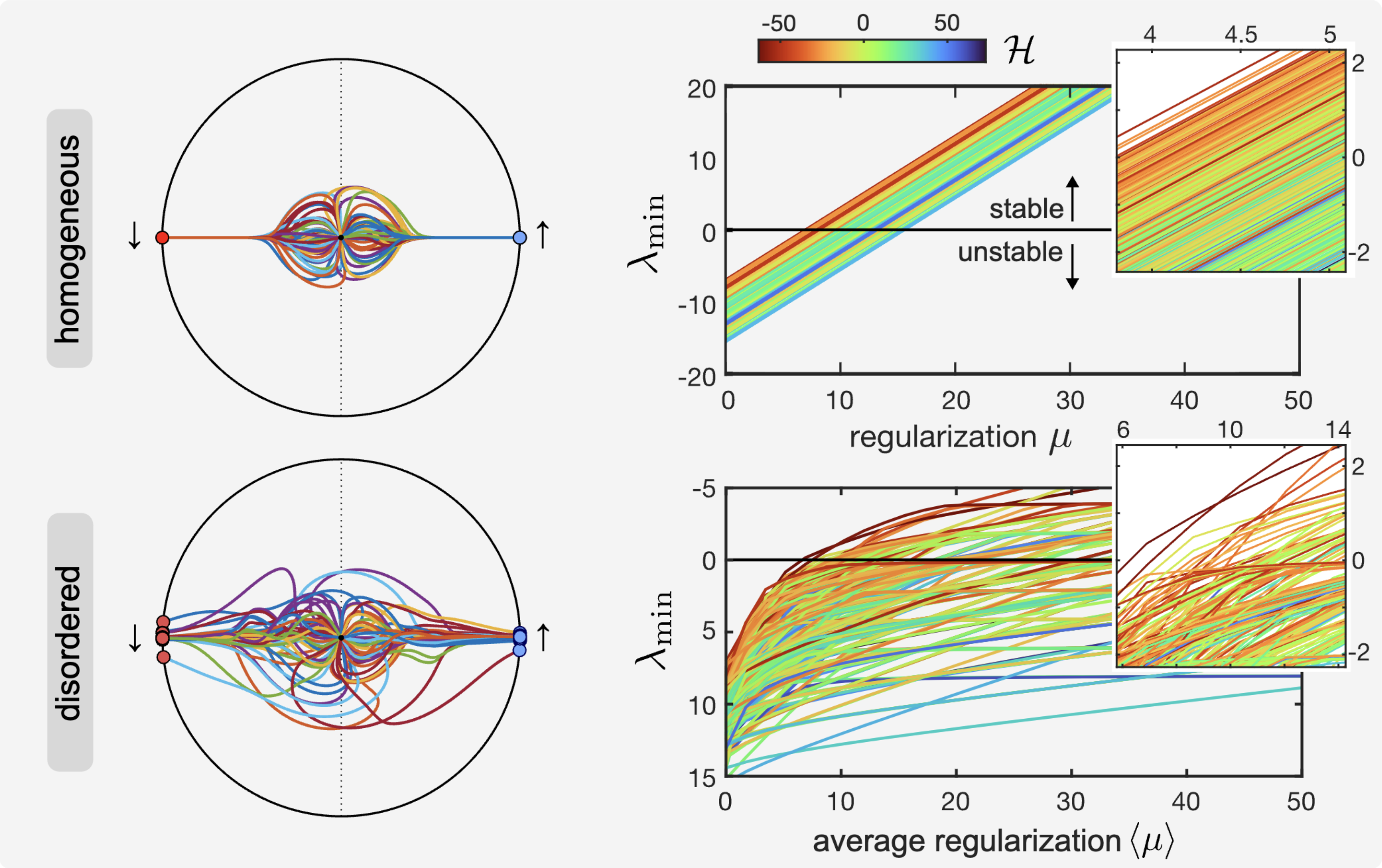
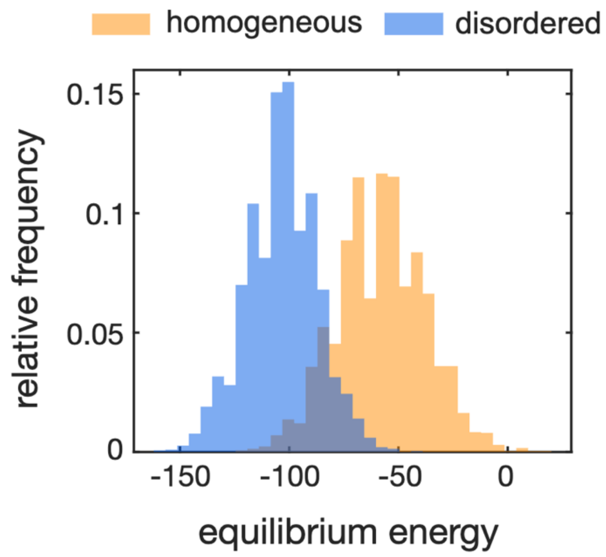
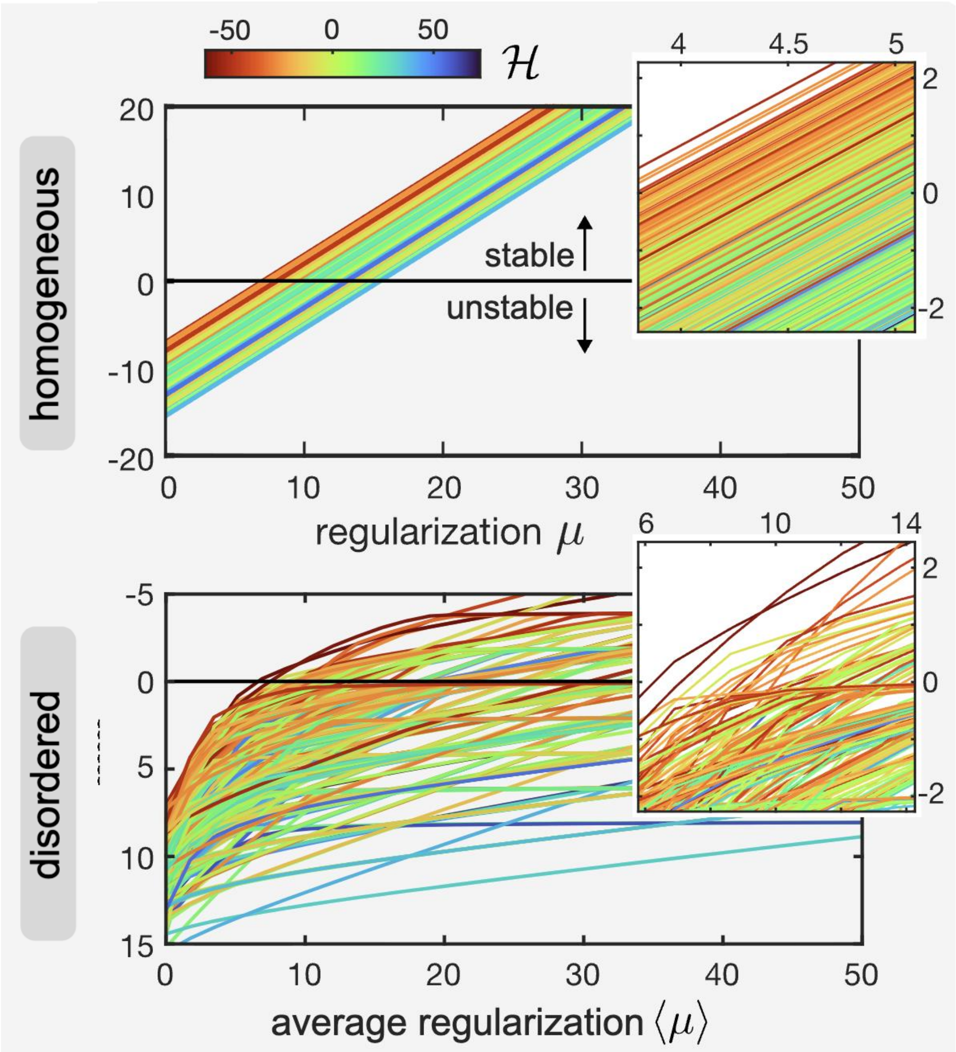
Disordered oscillator Ising machines





Allibhoy, **ANM**, Pasqualetti, Motter.
Proc. IEEE CDC (2025).

Design principles for biased OIMs



$\mathbb{E}[\lambda_{\min} \mid H = h] \approx$ Function of:

- graph statistical properties (N and p)
- regularization statistical properties ($\mathbb{E}[\mu]$ and $\mathrm{Var}[\mu]$)

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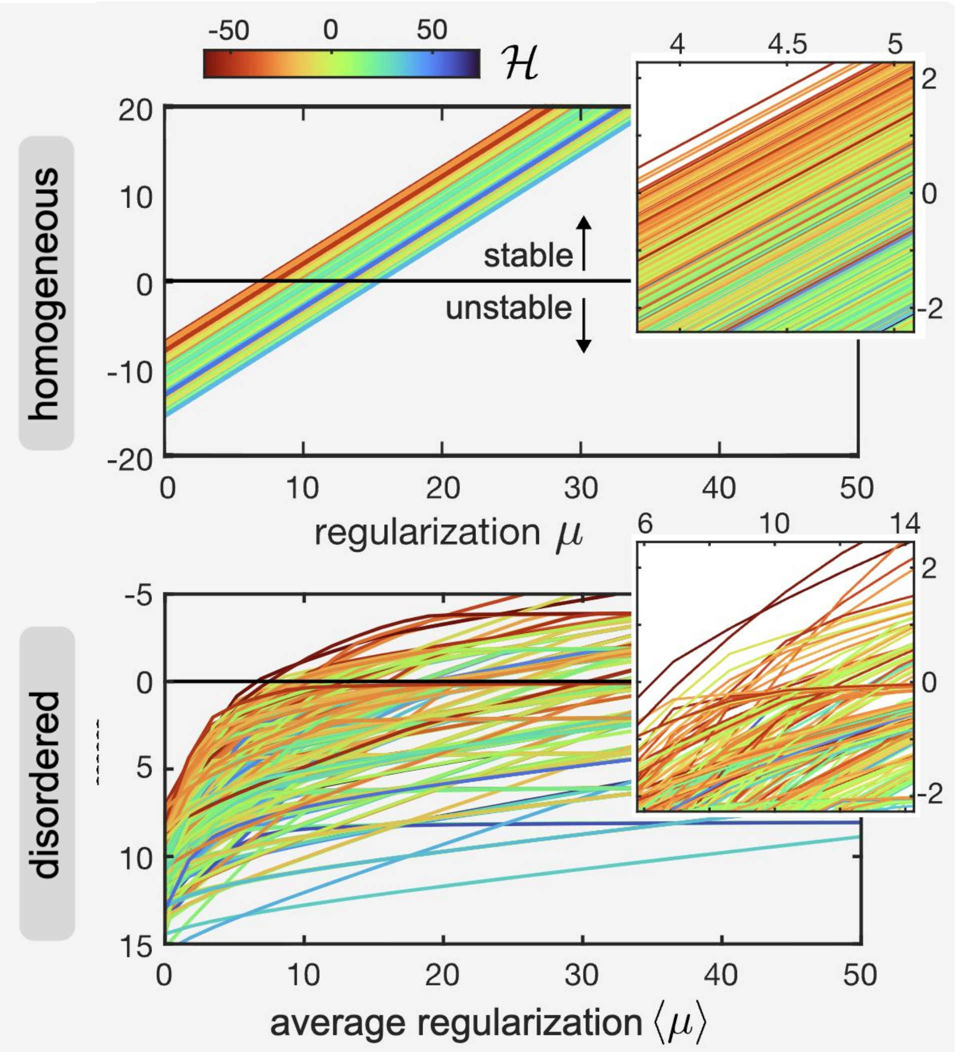

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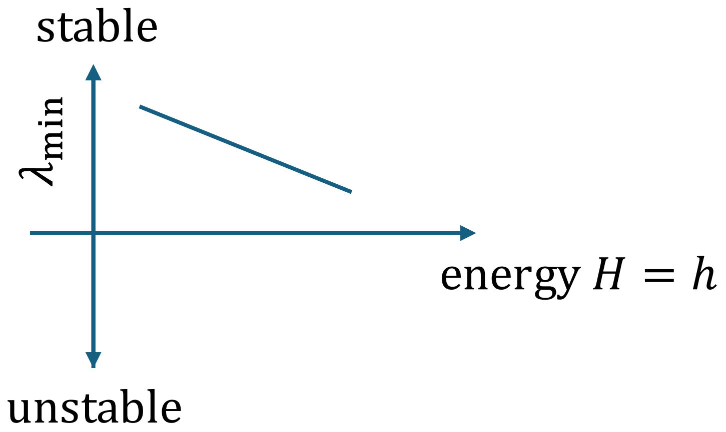

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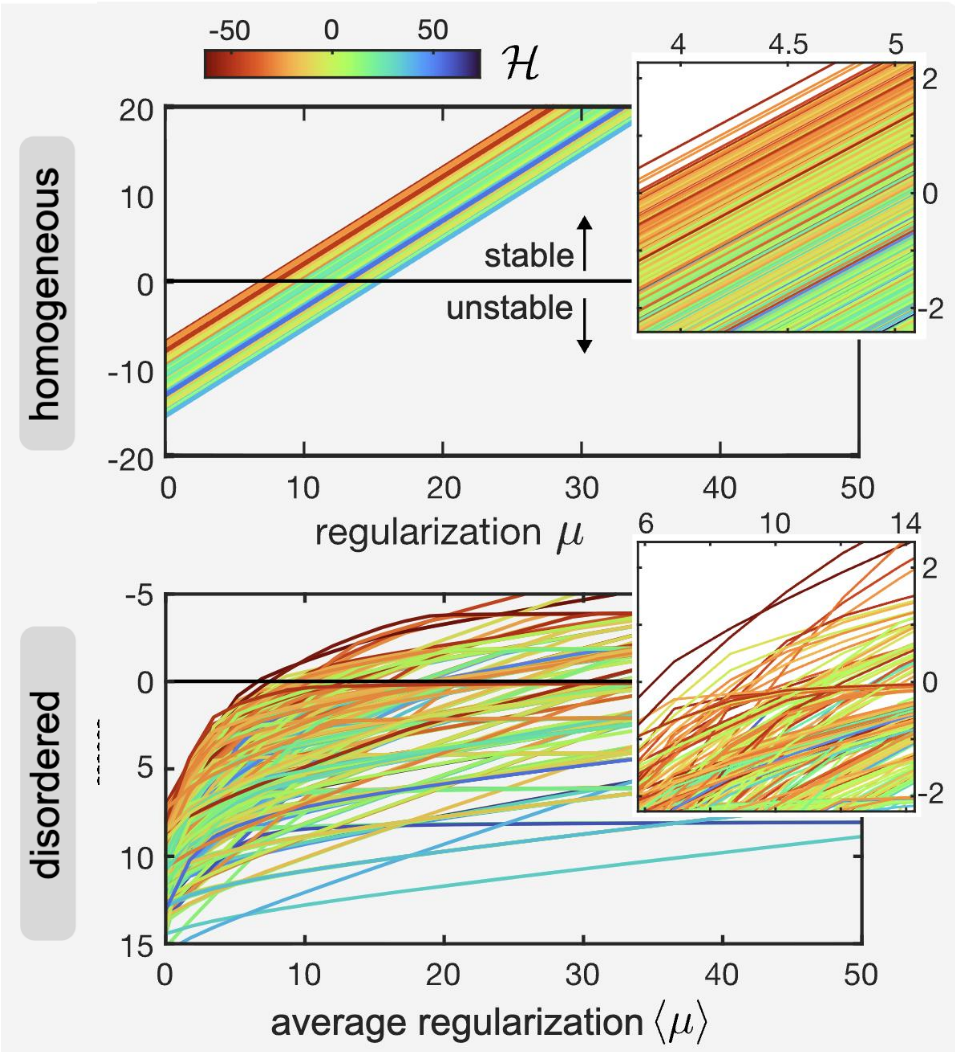

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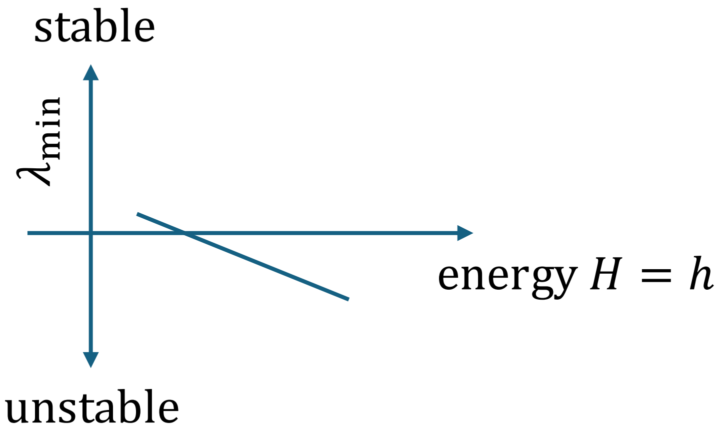

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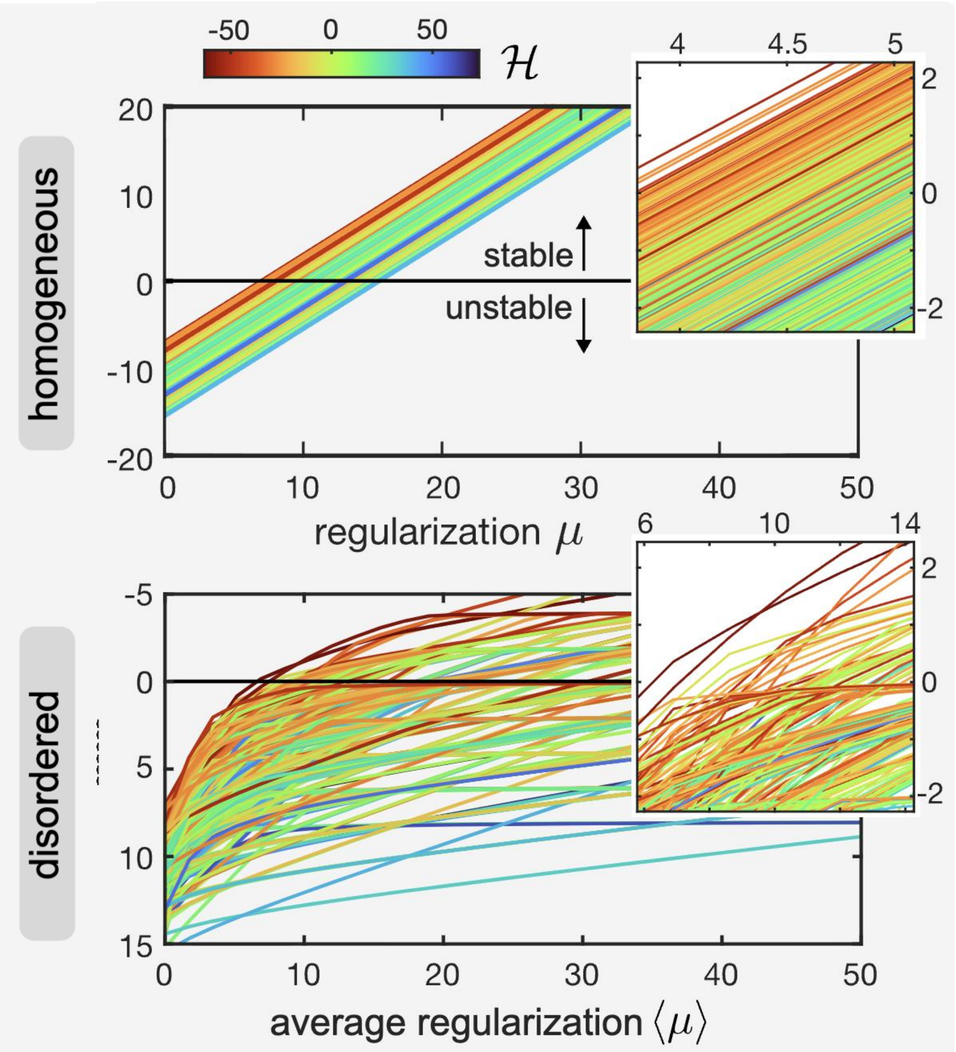

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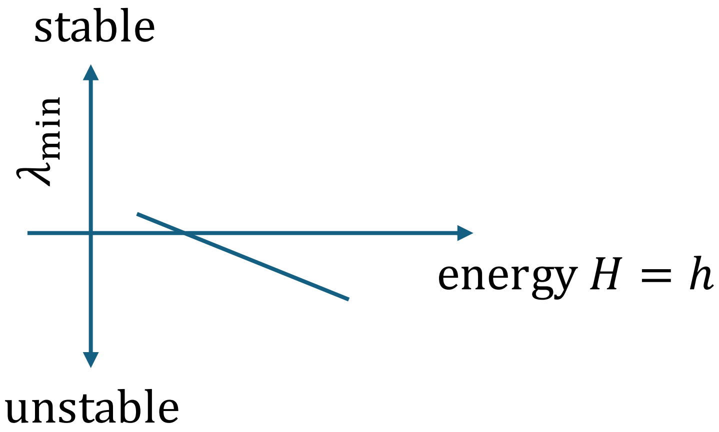

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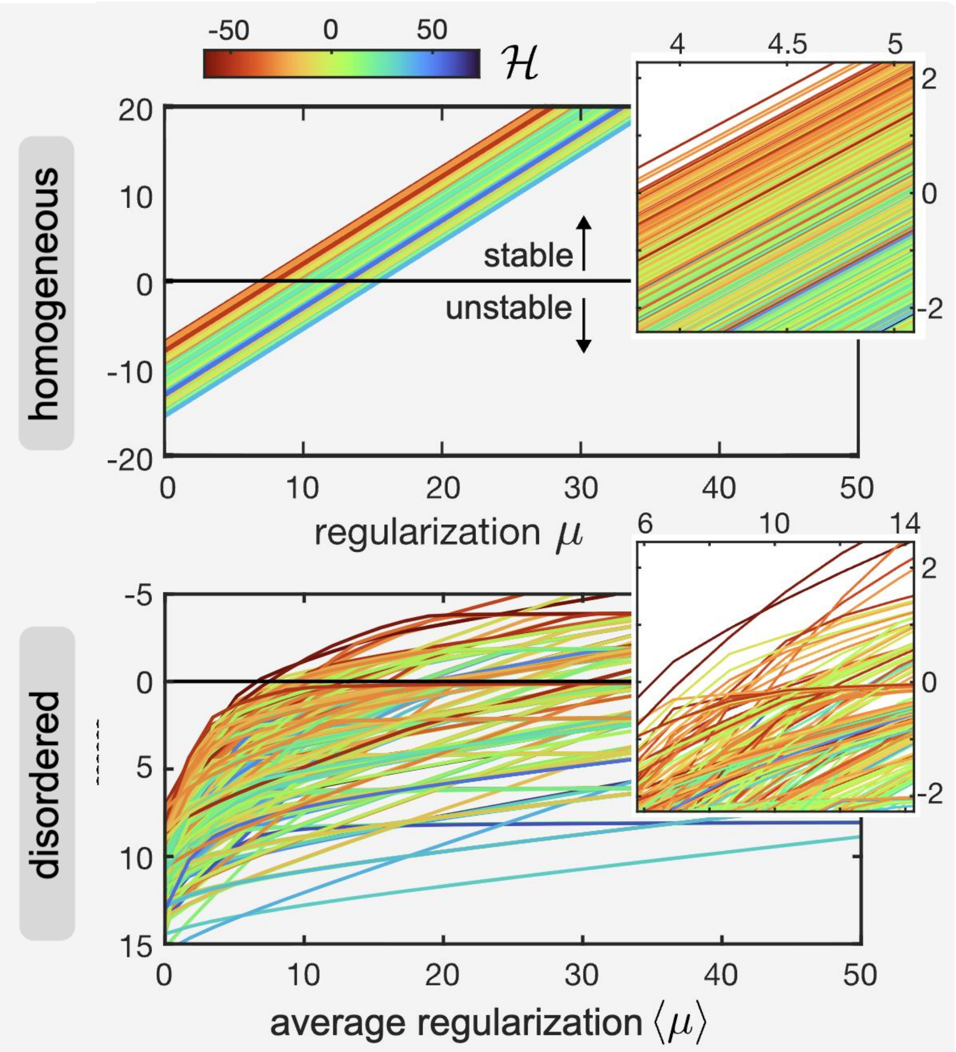

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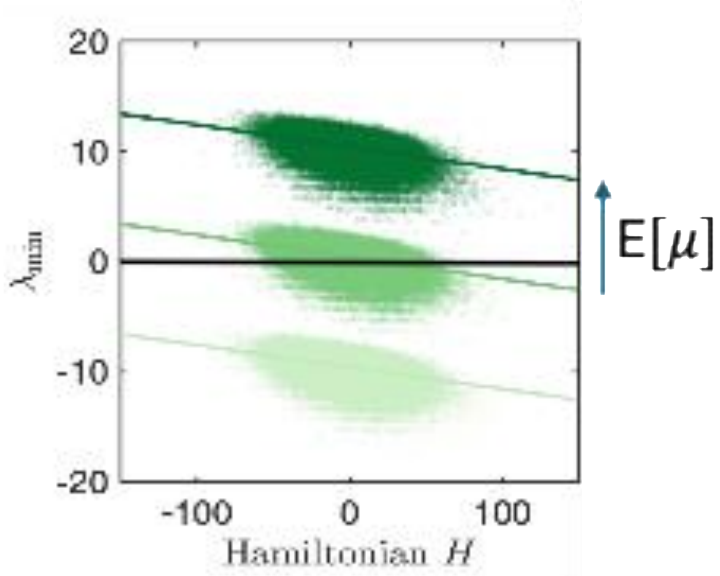

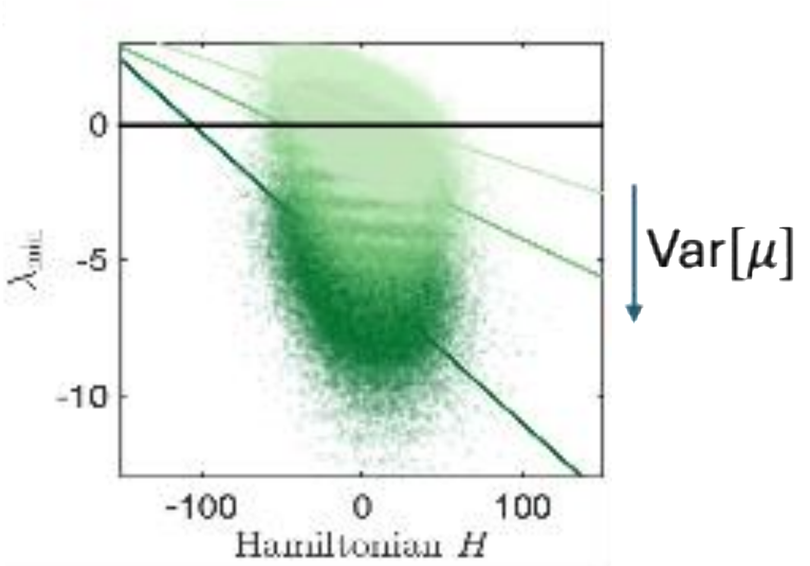

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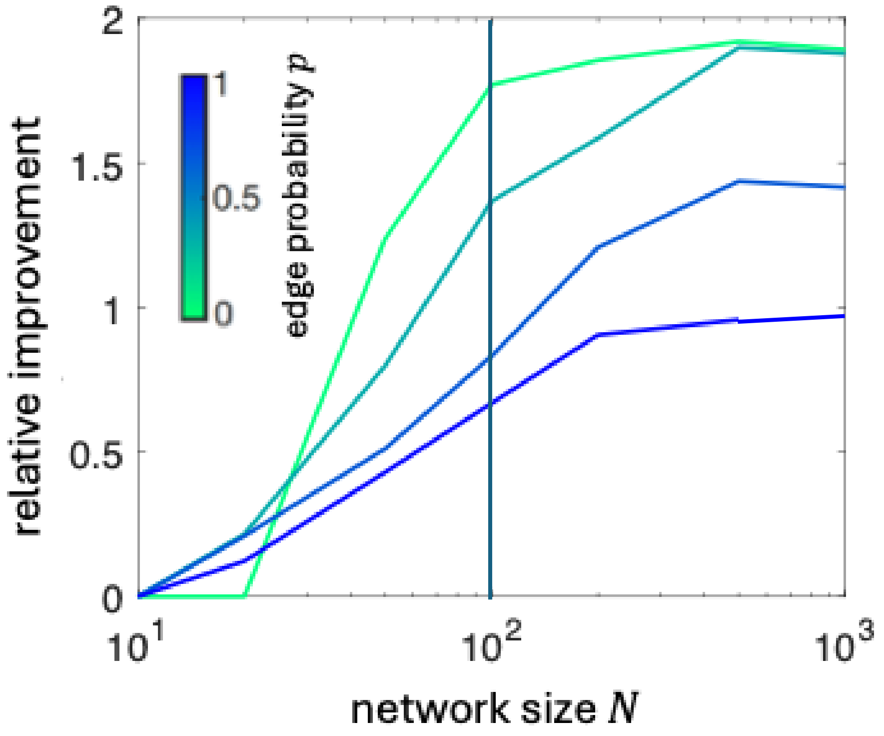
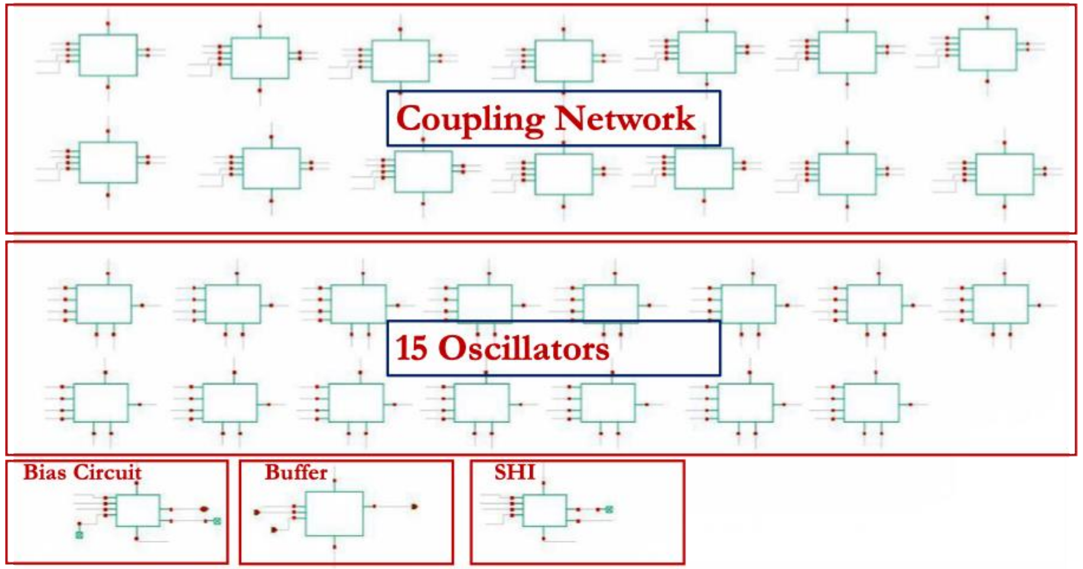
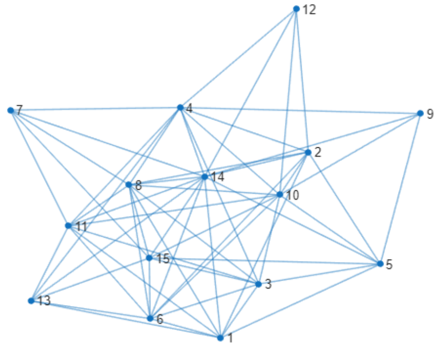
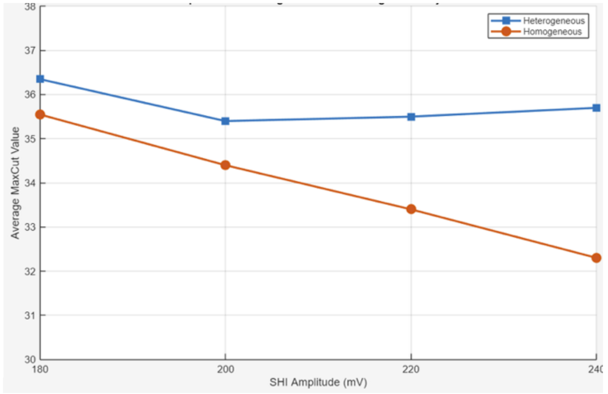
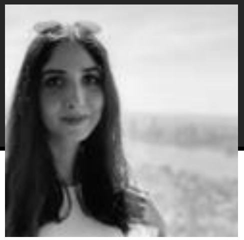
Ongoing simulations and experiments











Malihe Farasat

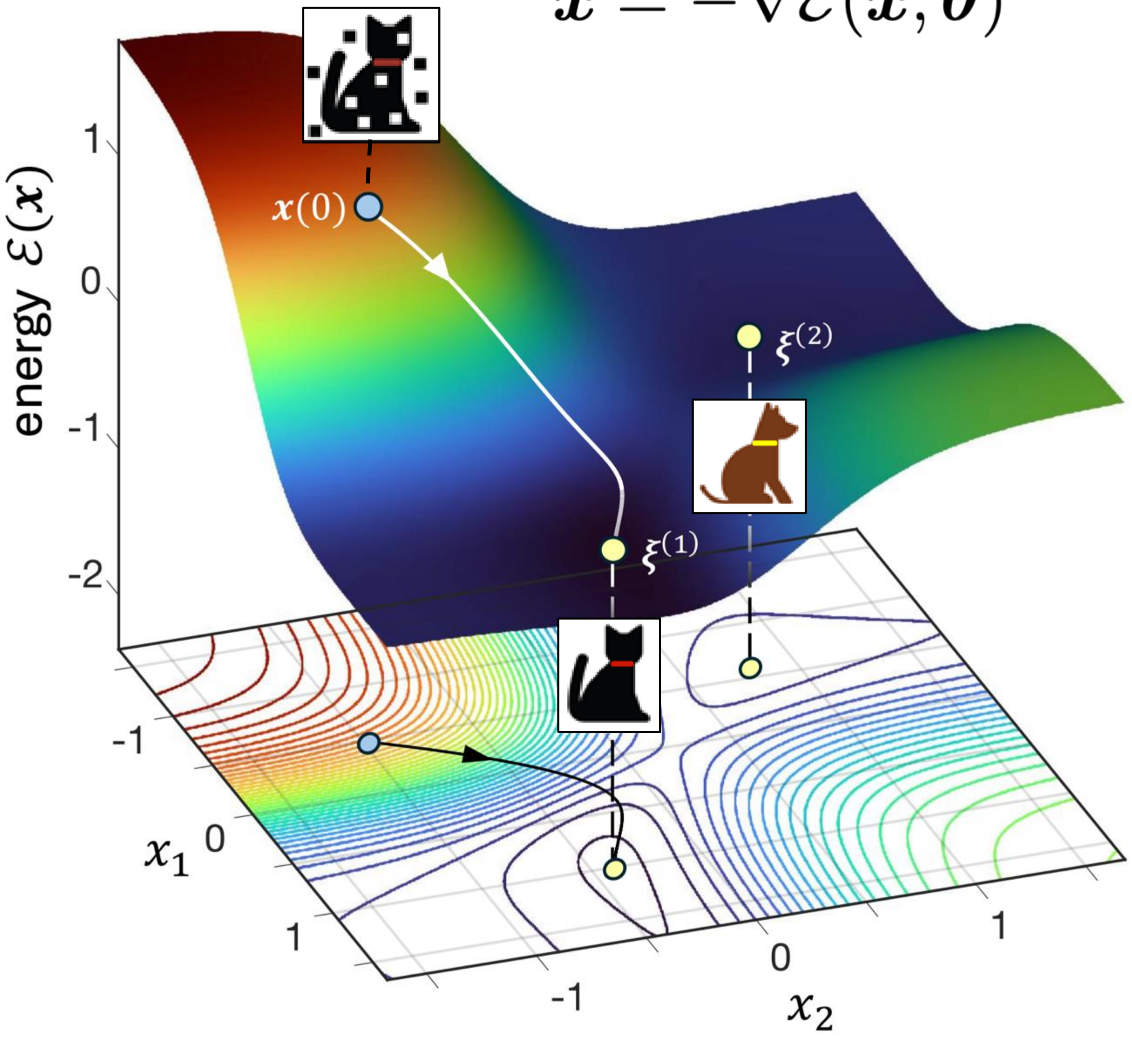
Sajib Bain

Abir Hasan

Nikhil Shukla
UVA

Future research direction

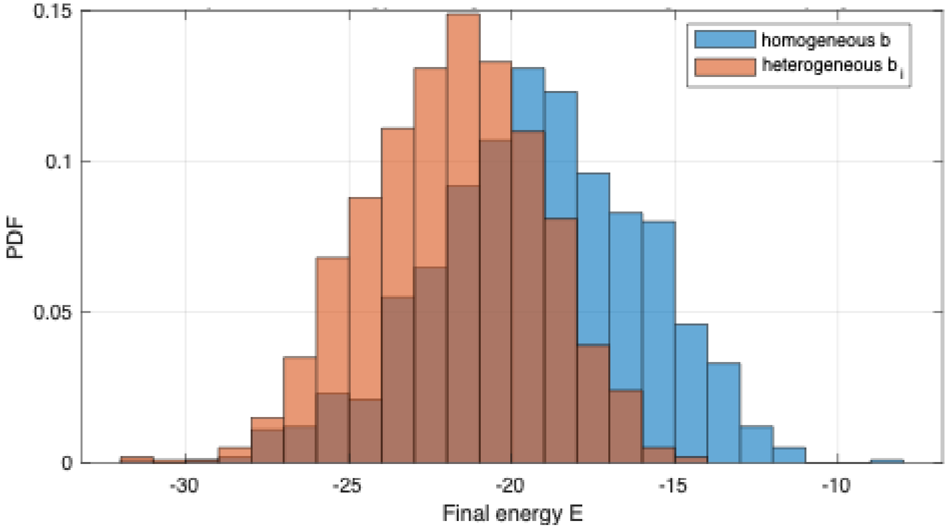
$$\dot{\boldsymbol{x}} = -\nabla\mathcal{E}(\boldsymbol{x};\boldsymbol{\theta})$$



Hopfield network

$$\dot{\boldsymbol{x}} = -D\boldsymbol{x} + W\boldsymbol{\Phi}(\boldsymbol{x}) + B\boldsymbol{u}$$

$$D = \begin{bmatrix} d_1 & & & \\ & d_2 & & \\ & & \ddots & \\ & & & d_N \end{bmatrix}$$



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Cats!

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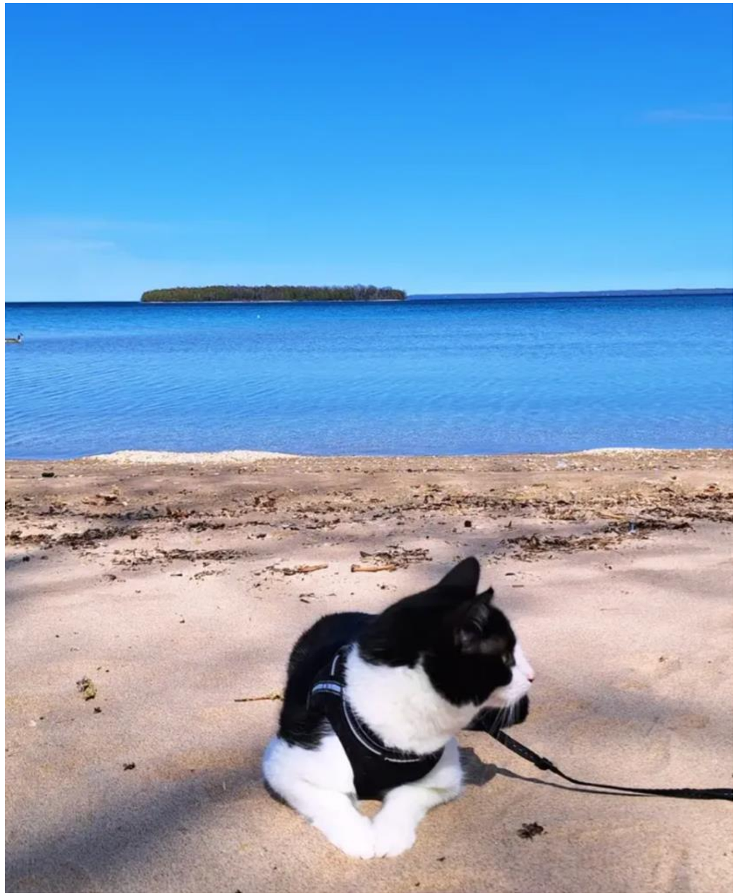
Pollo



Sushi

Tofu