LPHYS’26 @ Costa Rica

Disorder-promoted synchronization and coherence in coupled laser networks

Arthur Montanari

Center for Network Dynamics
Department of Physics and Astronomy
Northwestern University

July 9, 2026

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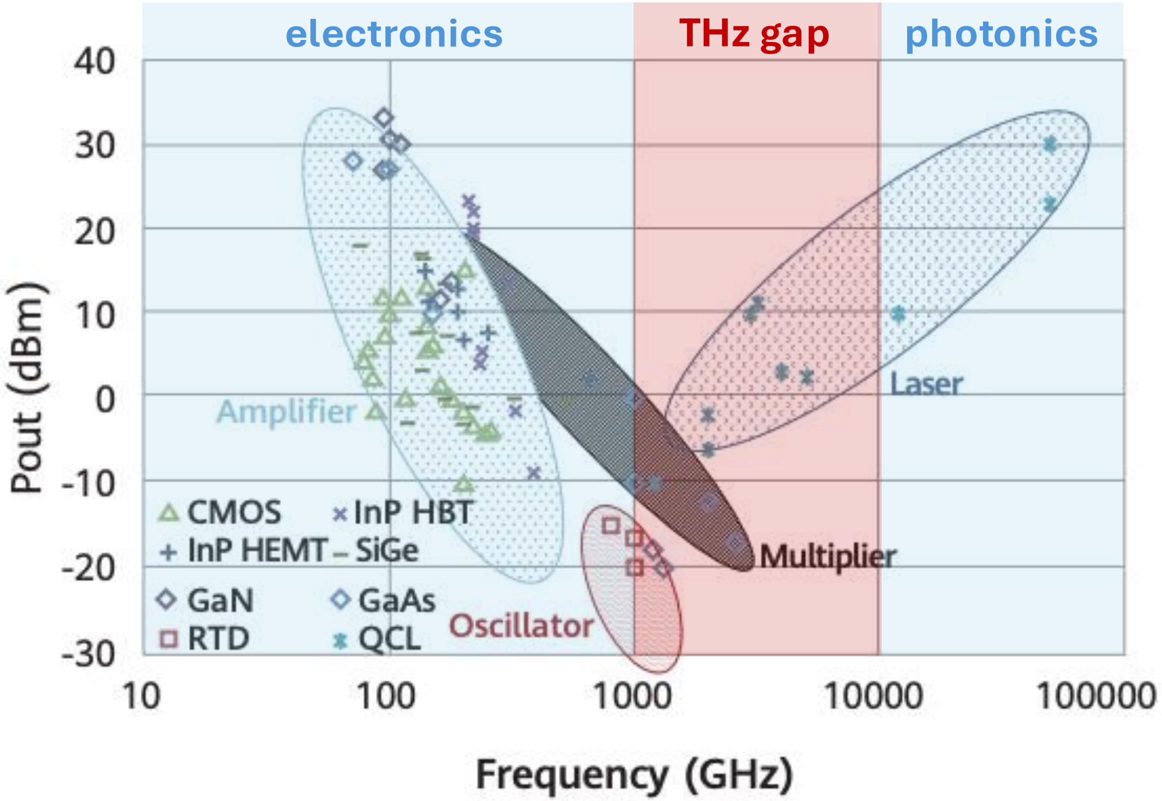
The THz Gap



Potential applications of THz gap:

- imaging, spectroscopy, sensing (ideal penetration)
- high-speed, free-space communication (wireless communication, LIDAR)
- THz computing (analog, neuromorphic computing, Ising machines)

Challenge: low power output ($P \approx 1$ mW per laser)

G Wang, et al. Terahertz sensing and communication towards future intelligence connected networks. *Huawei Research Report* (2022).

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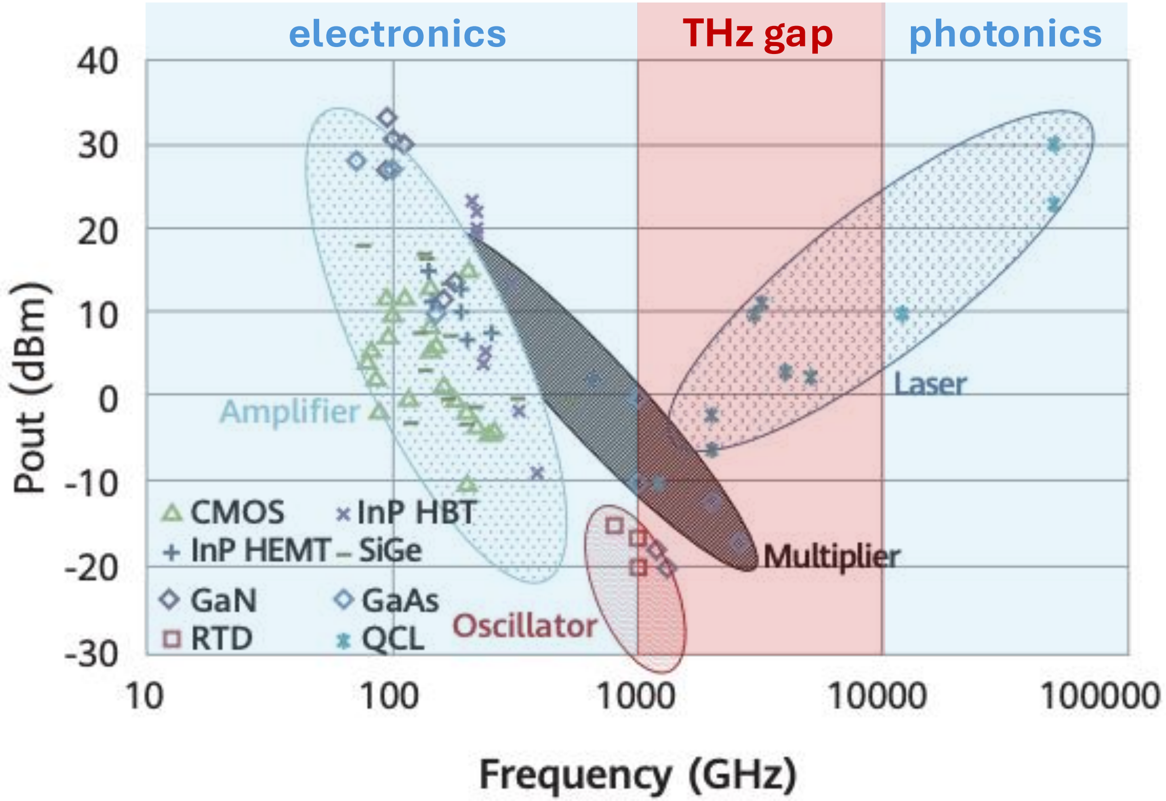

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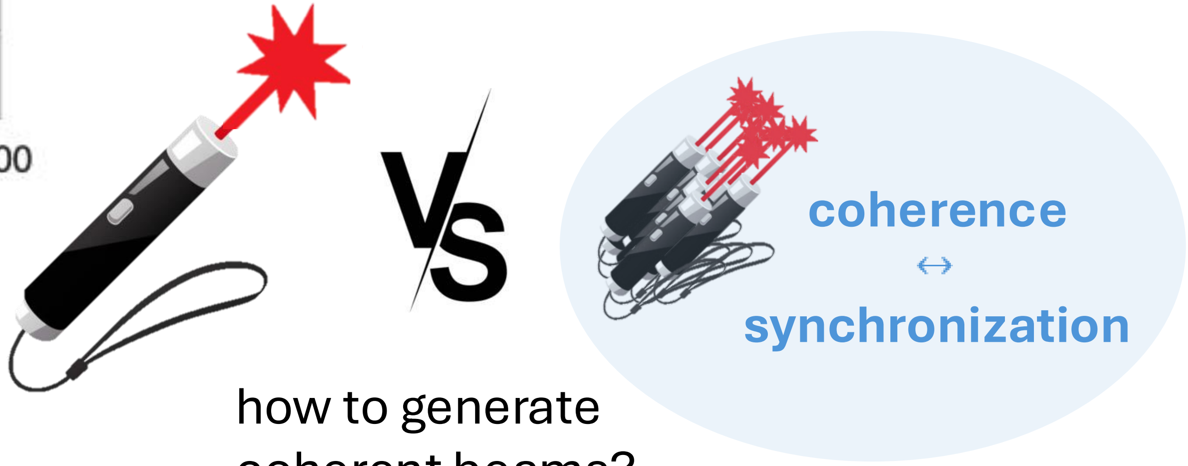
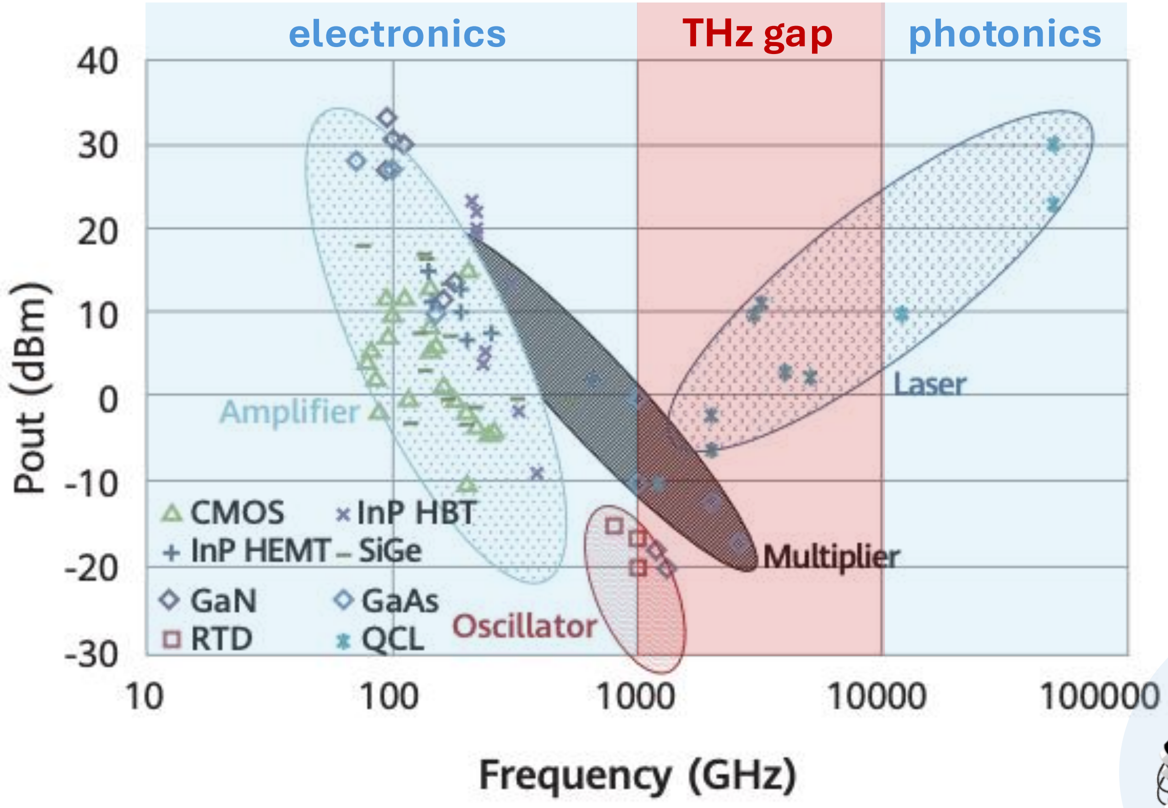


how to generate coherent beams?

The THz Gap



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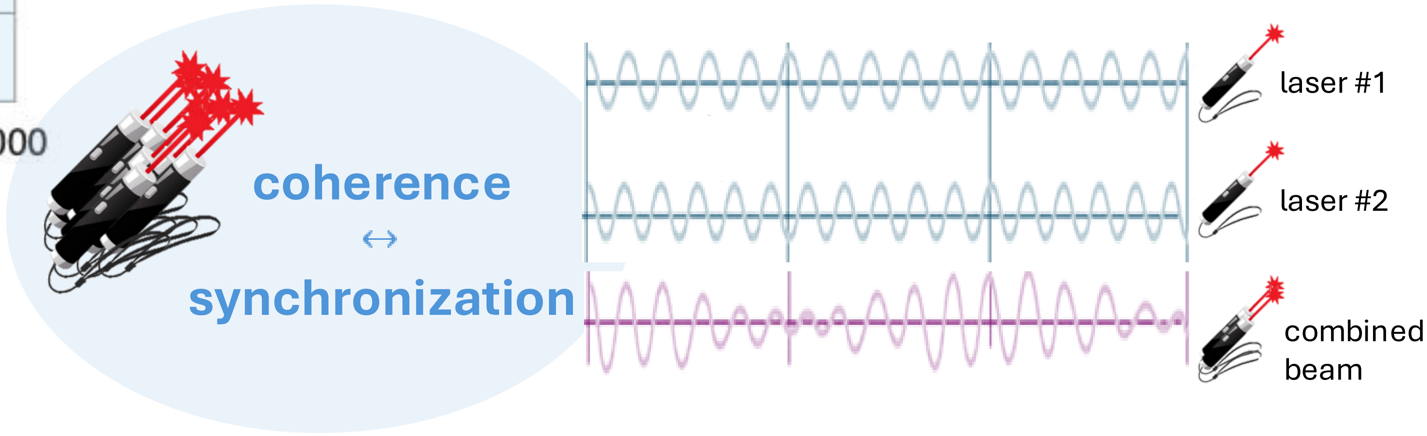

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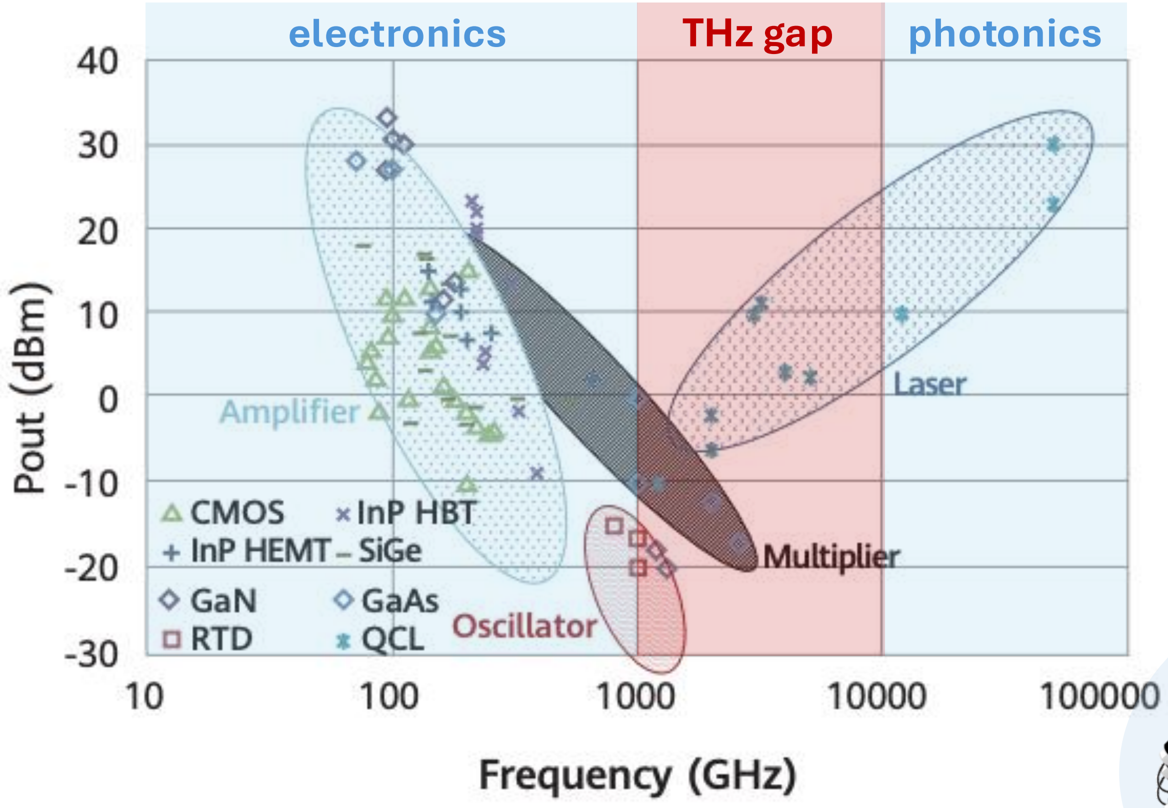

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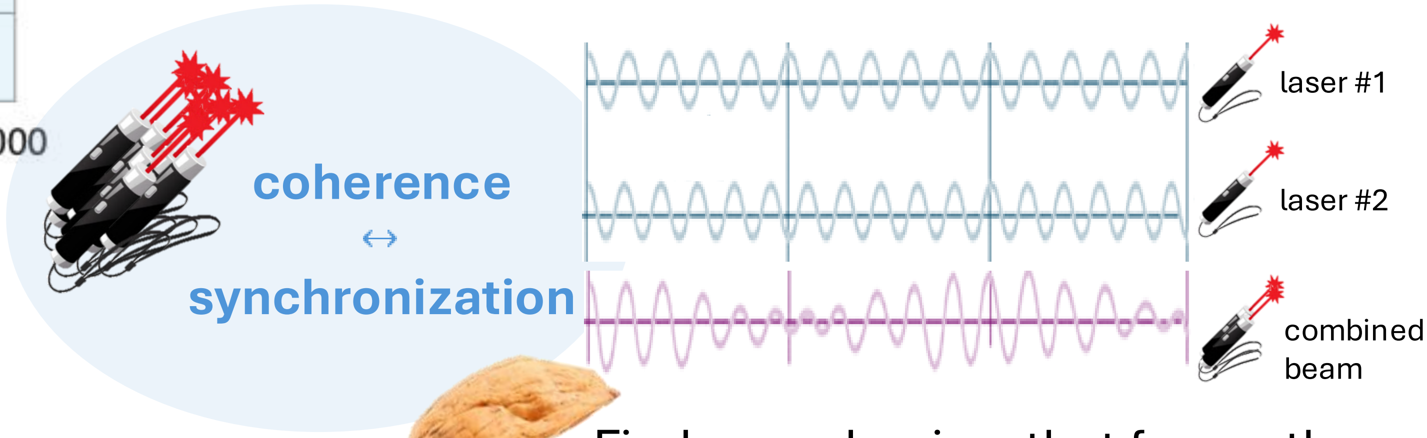
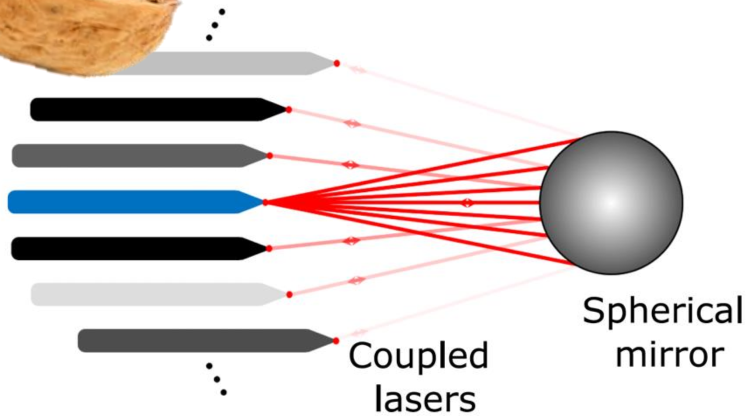
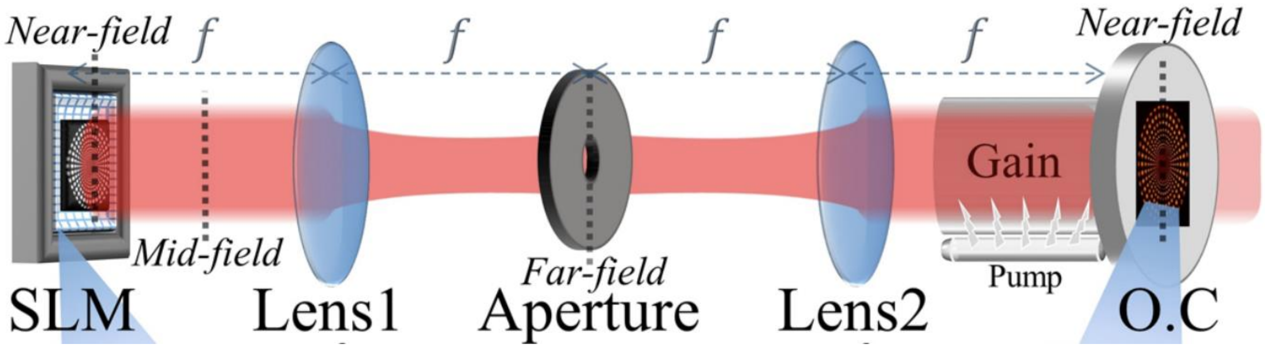


Find a mechanism that forces the laser beams to interact & sync

Laser sync

Find a mechanism that forces the laser beams to interact & sync





AED Barioni, **ANM**,
AE Motter. *PRL* (2025).

C Tradonsky, et al.
Optica (2021).

Laser sync

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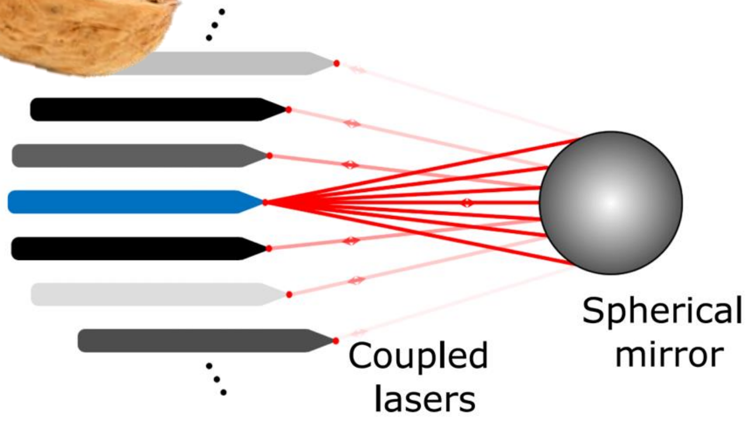
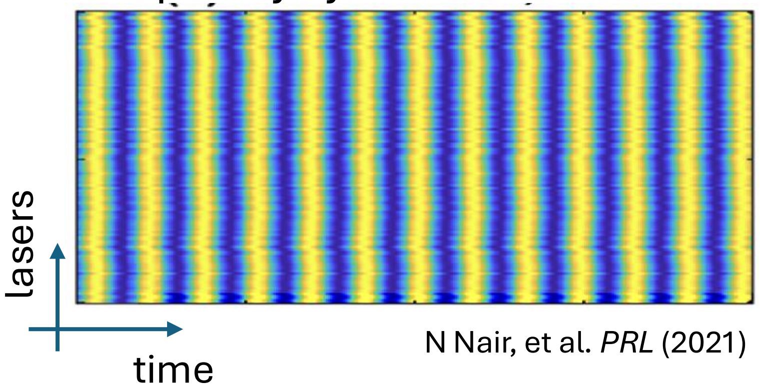


Frequency synchronization

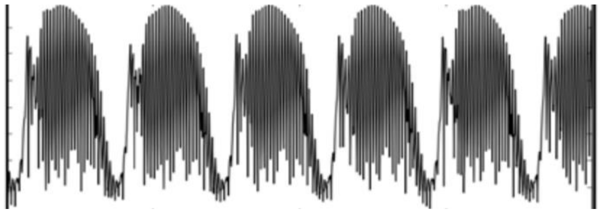


N Nair, et al. *PRL* (2021)

Low-frequency fluctuations

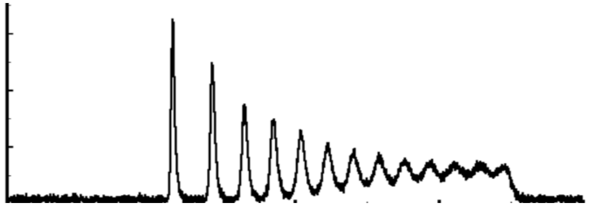
A Torcini, et al. *PRA* (2006)

Regular pulse packages



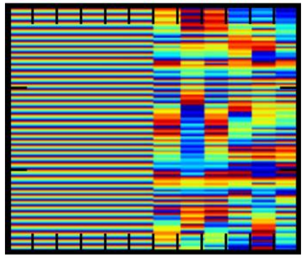
S Ruschel, S Yanchuk. *Chaos* (2017)

Relaxation oscillations



B Liu, et al. *Opt Express* (2021)

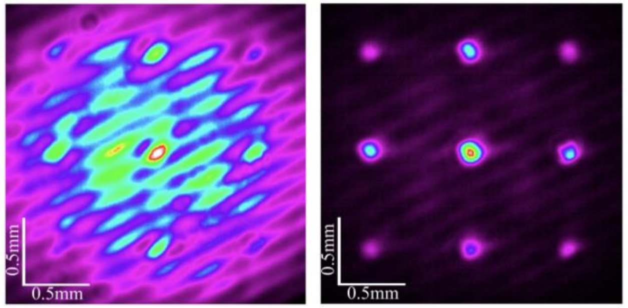
Chimera states



F Bohm, A Zakharova, E Scholl, K Ludge. *PRE* (2015).

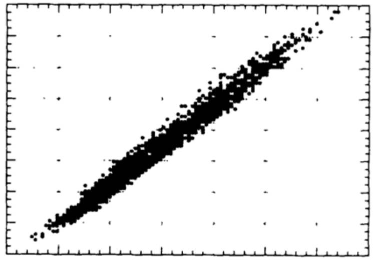
AM Hagerstrom, et al. *Nat Phys* (2012).

Crowd sync



S Mahler, AA Friesem, N Davidson. *PRR* (2020)

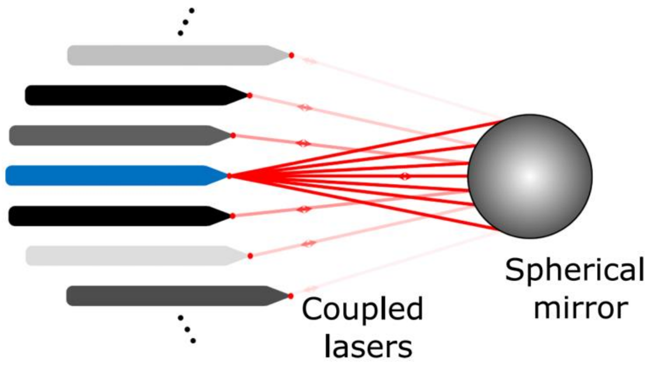
Chaos sync



R Roy, KS Thornburg. *PRL* (1994).

Lang-Kobayashi model

for single-mode semiconductor diode lasers (e.g., GaAs, InP, GaN)

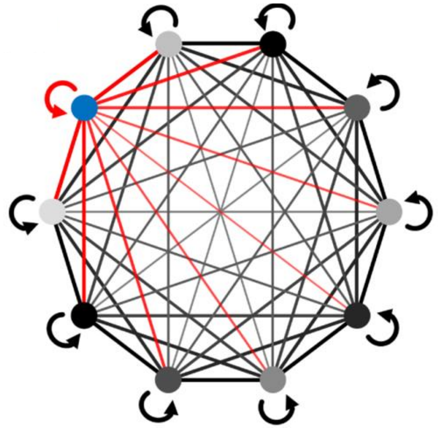


Electric field dynamics

$$\dot{E}_j(t) = \underbrace{\frac{1+i\alpha_j}{2}\left(G_j-\gamma\right)E_j(t)}_{\text{phase-amplitude coupling}} + \underbrace{i\omega_j E_j(t)}_{\text{natural frequency}} + \underbrace{\kappa_j \sum_{k=1}^{M} A_{jk} E_k(t-\tau_{jk})}_{\text{delayed-coupling (external cavity)}},$$

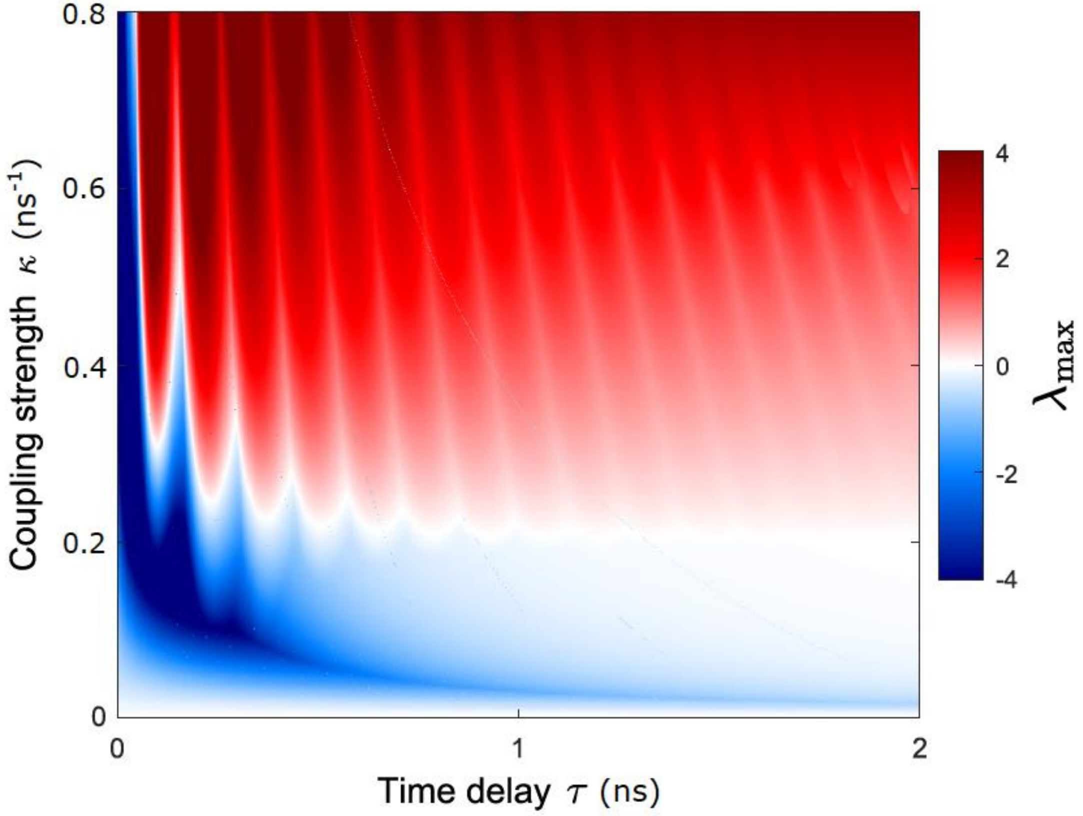
Carrier number dynamics

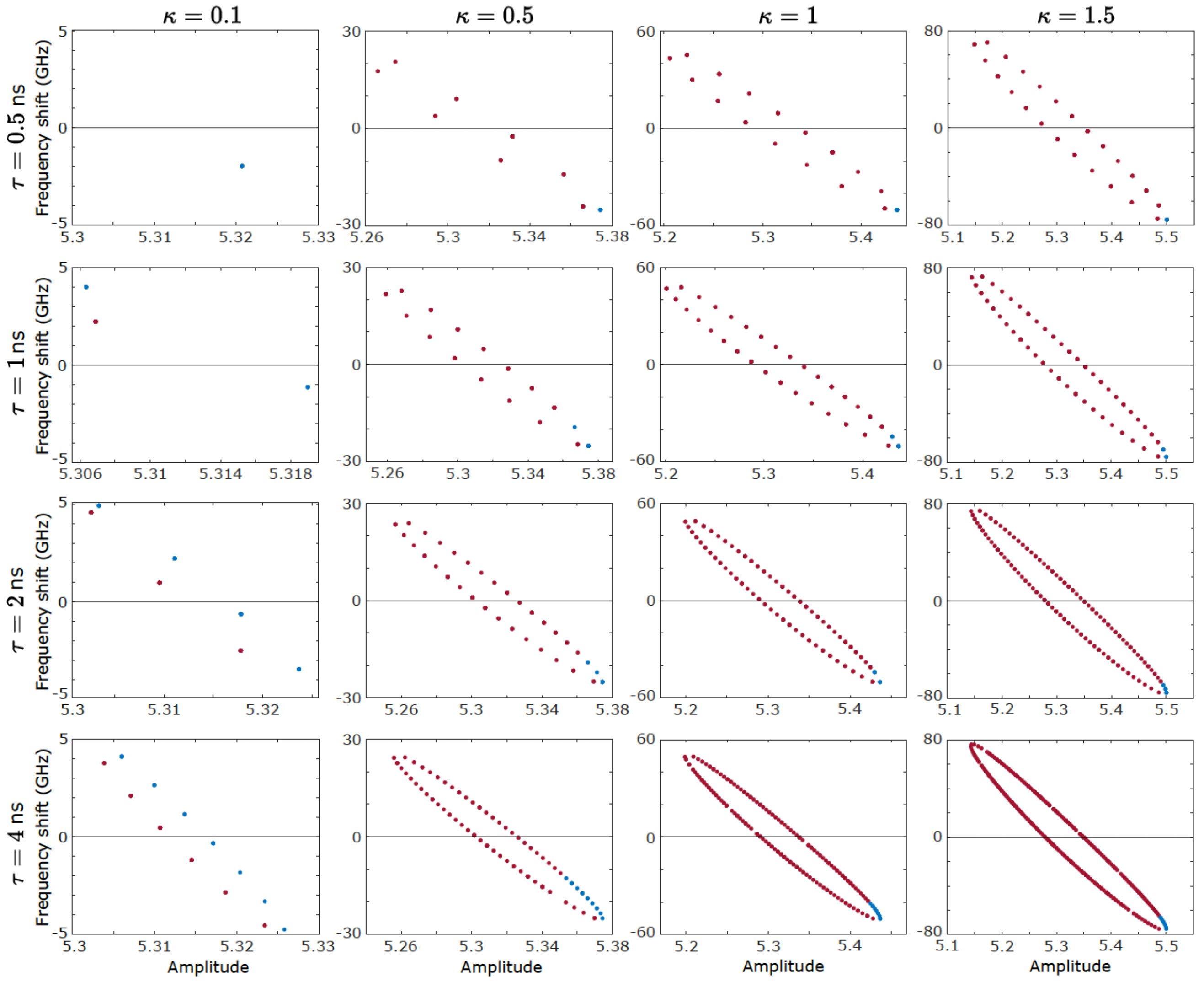
$$\dot{N}_j(t) = \underbrace{J_0}_{\text{current source}} - \underbrace{\gamma_n N_j(t)}_{\text{damping}} - \underbrace{G_j\left|E_j(t)\right|^2}_{\text{media gain}},$$



R Lang, K Kobayashi.
IEEE J. Quantum Electronics (1980).

Multistability of the LK model





Disorder-promoted sync

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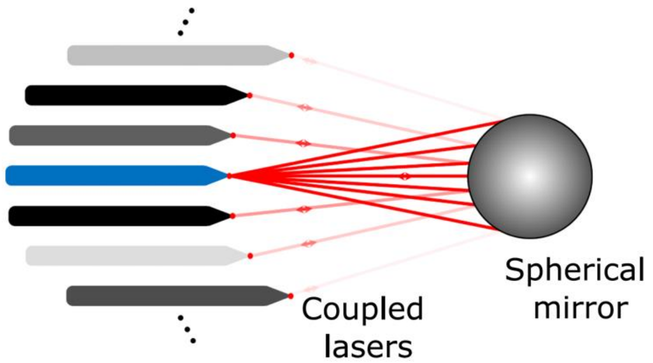

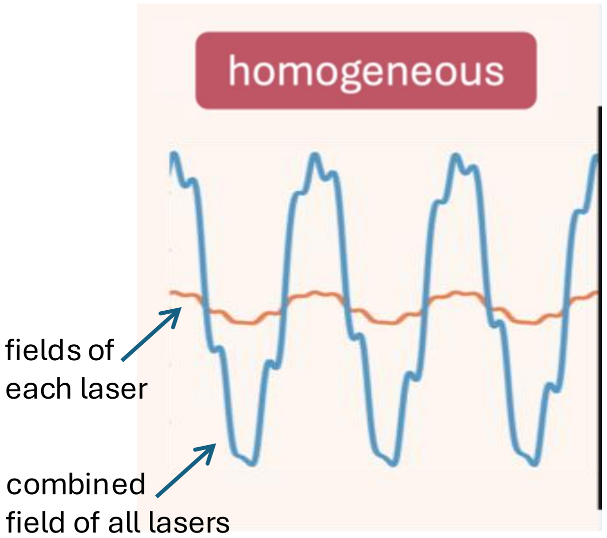

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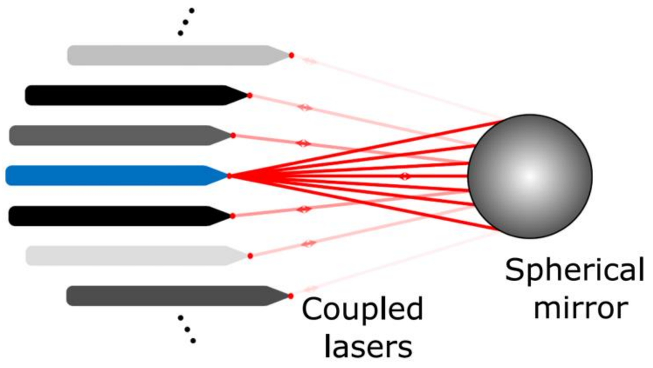

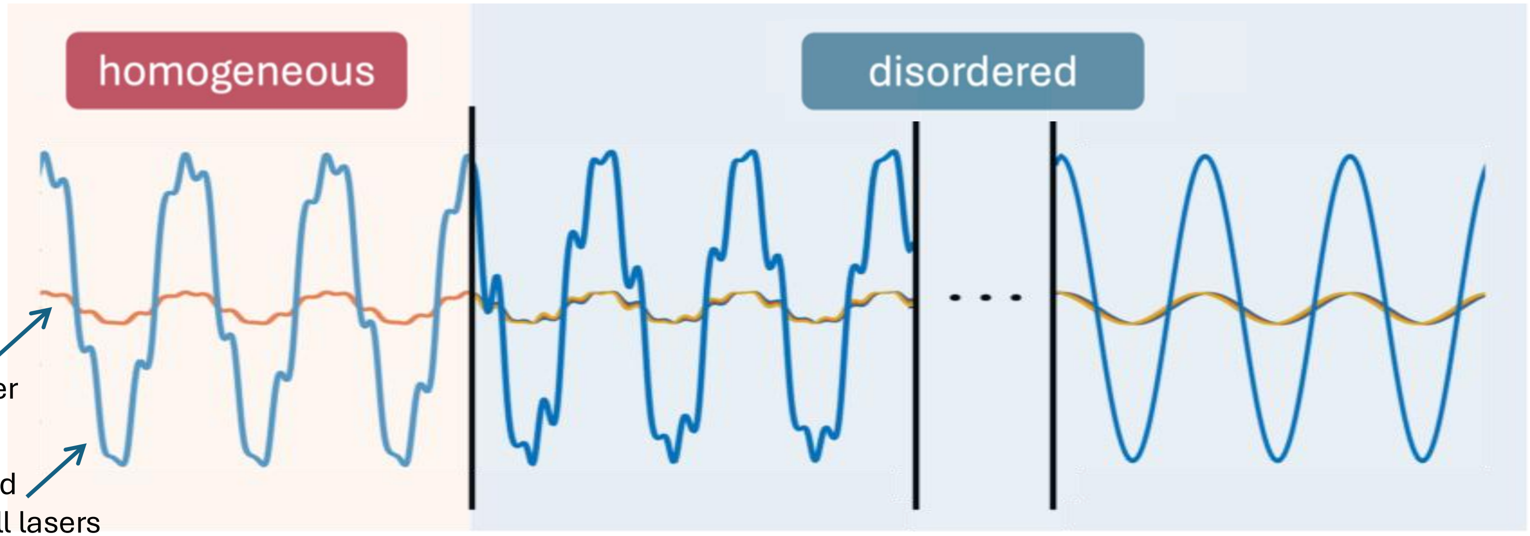

$\omega_j \sim \mathcal{N}(0,1)$

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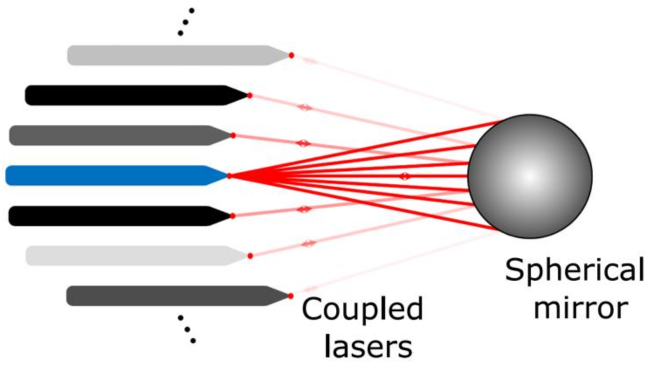
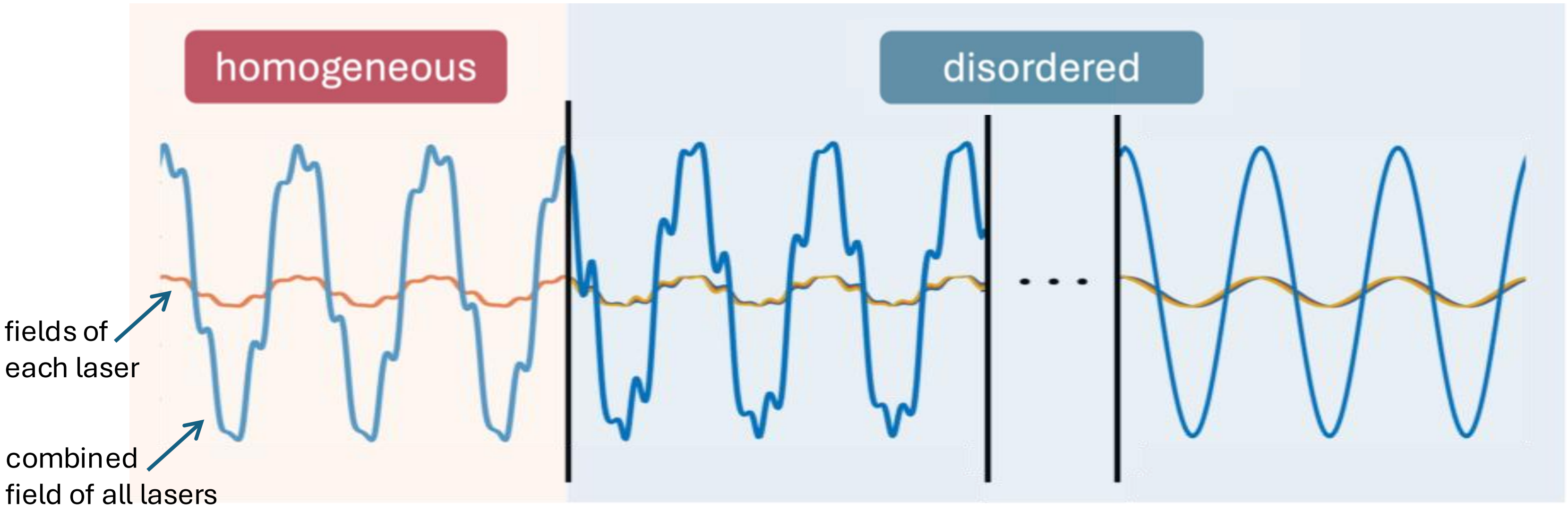
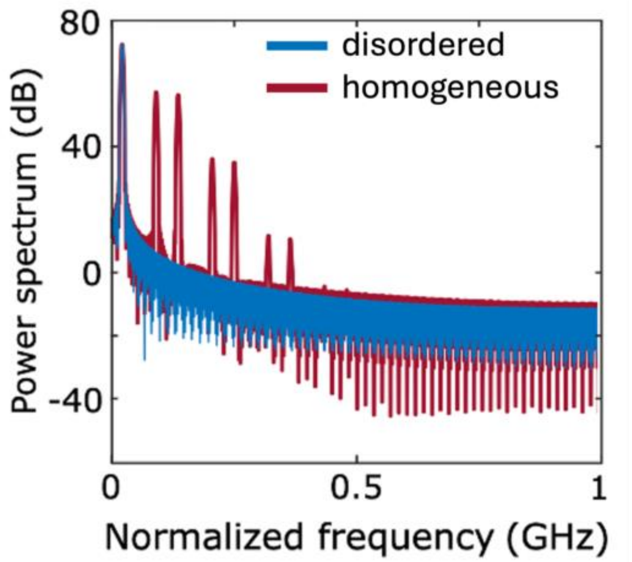






Some stability analysis to study it

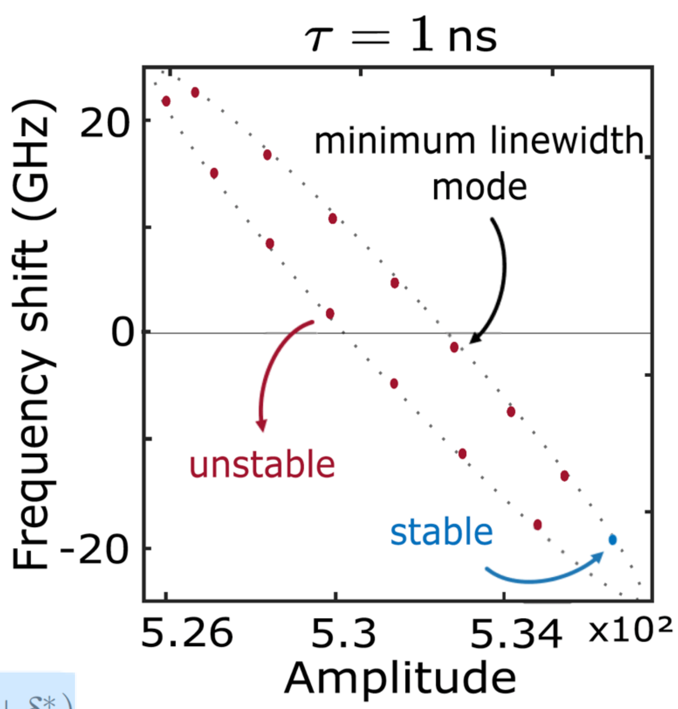
Nonlinear time-delay system (LK model)

$$\dot{\mathbf{x}}_j(t) = \underbrace{\mathbf{f}_j(\mathbf{x}_j(t))}_{\text{laser dynamics}} + \underbrace{\kappa_j \sum_{k=1}^{M} A_{jk}\mathbf{h}\big(\mathbf{x}_j(t), \mathbf{x}_k(t-\tau)\big)}_{\text{delayed coupling}}$$

Linearization around the desired synchronous state $E_j(t) = r_j^* e^{i(\Omega t + \delta_j^*)}$

$$\dot{\boldsymbol{\eta}}(t) = J_1 \boldsymbol{\eta}(t) + J_2 \boldsymbol{\eta}(t-\tau)$$

Stability analysis



ideal lasing frequency

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coherent if $\delta_j^* \approx 0,\ \forall j$

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Solve characteristic equation to find the (generalized) eigenvalues λ_ℓ

$$\det\left(J_1 + J_2 e^{-\lambda_\ell \tau} - \lambda_\ell I_{3M}\right) = 0$$

using MATLAB package DDE-BIFTOOL

Synchronous state is stable iff $\lambda_{\max} = \max \mathrm{Re}\{\lambda_\ell\} < 0$

Main result

Nonlinear time-delay system (LK model)

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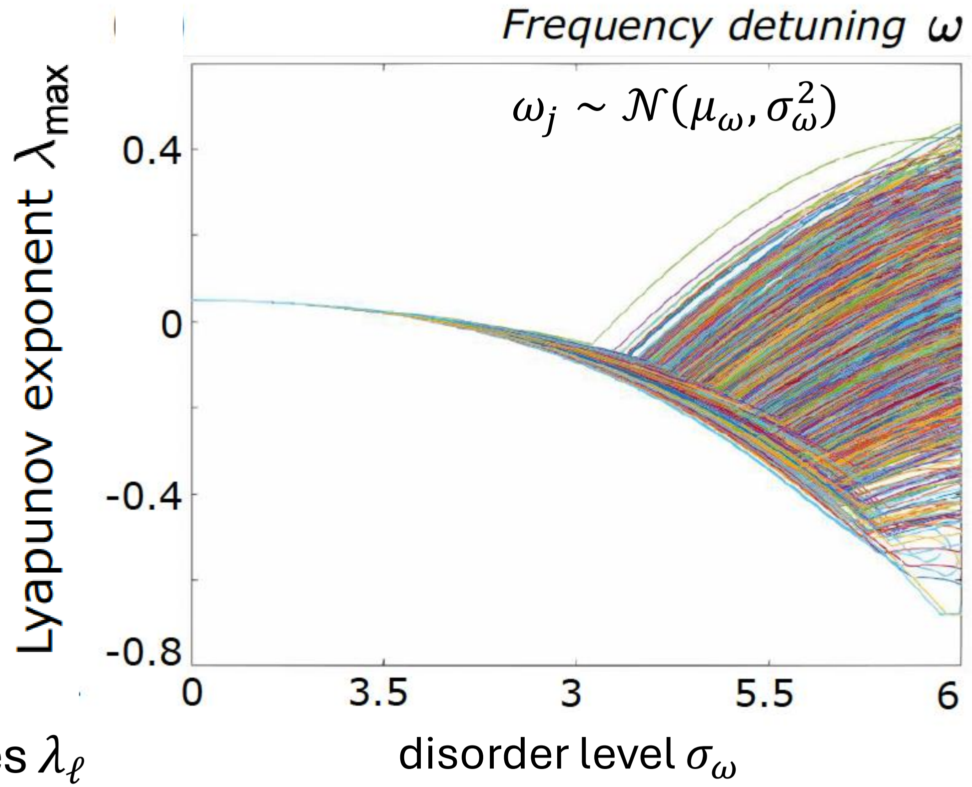

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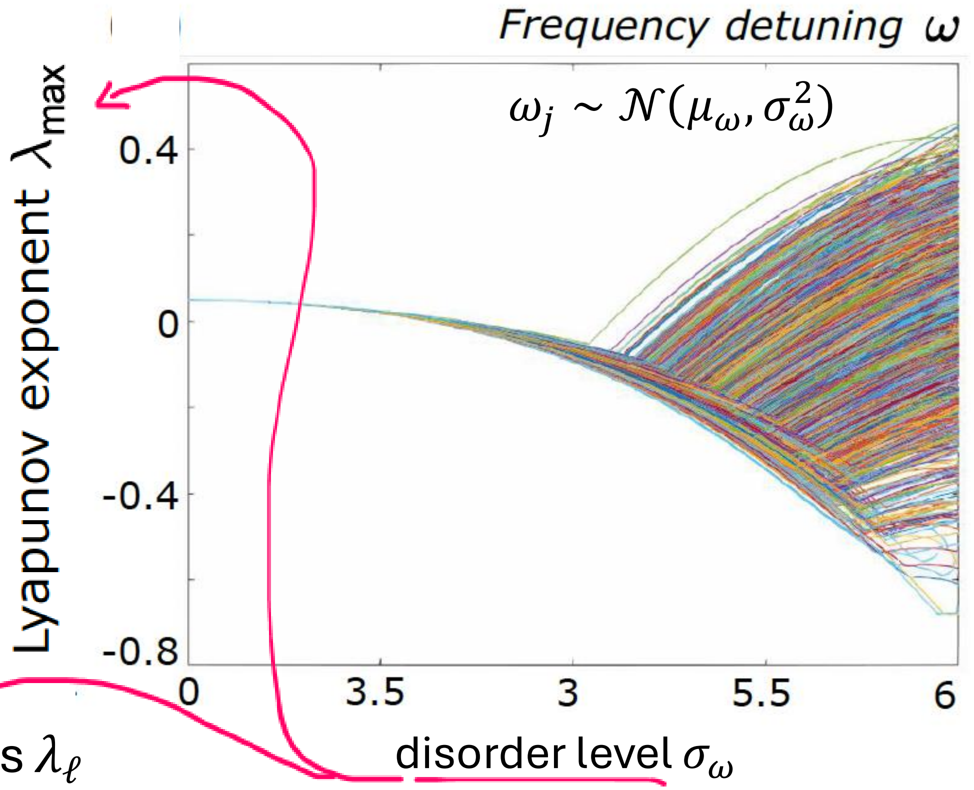

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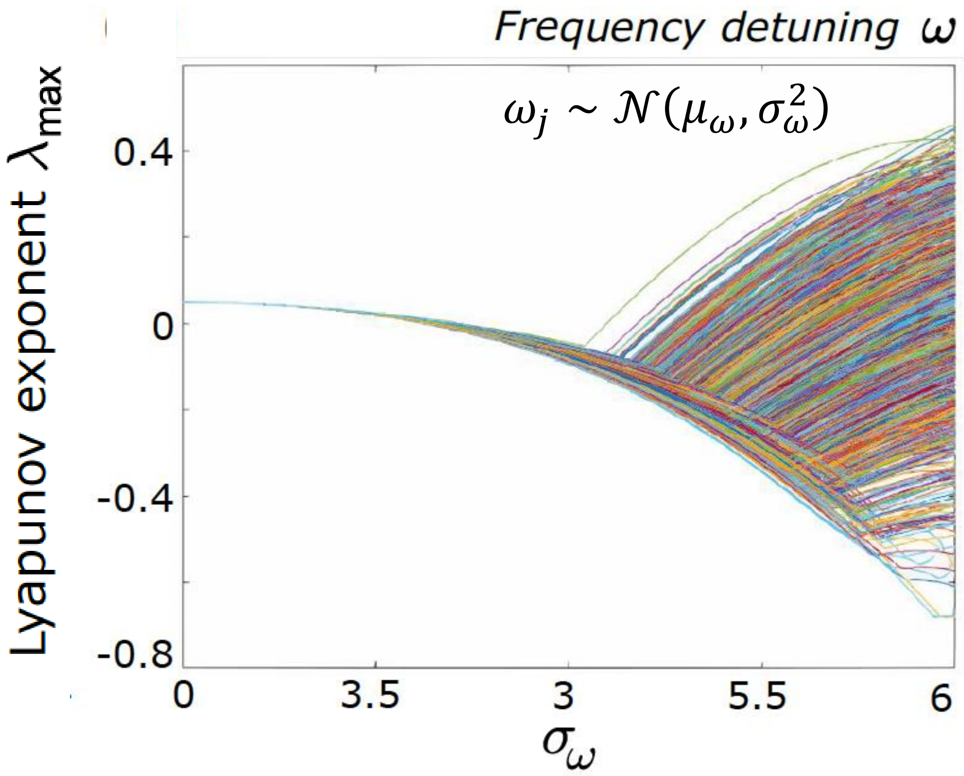
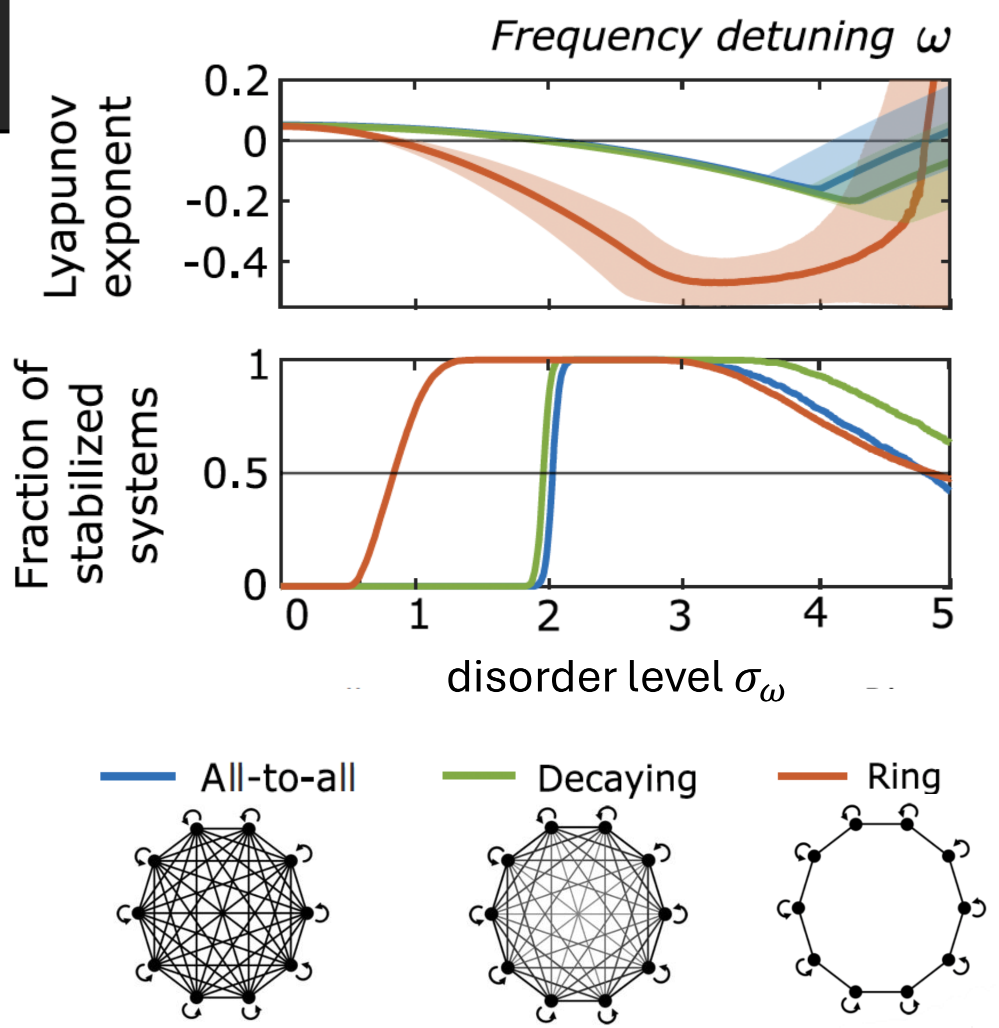
Disorder-promoted sync





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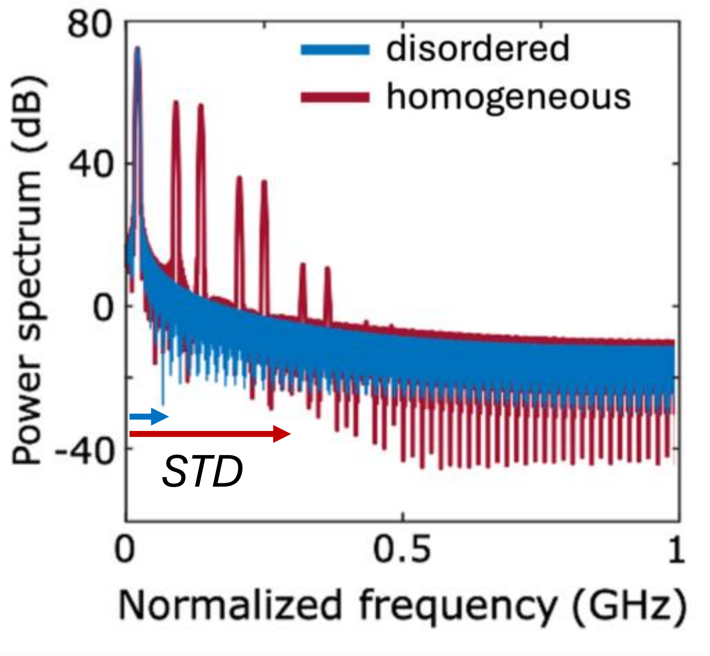
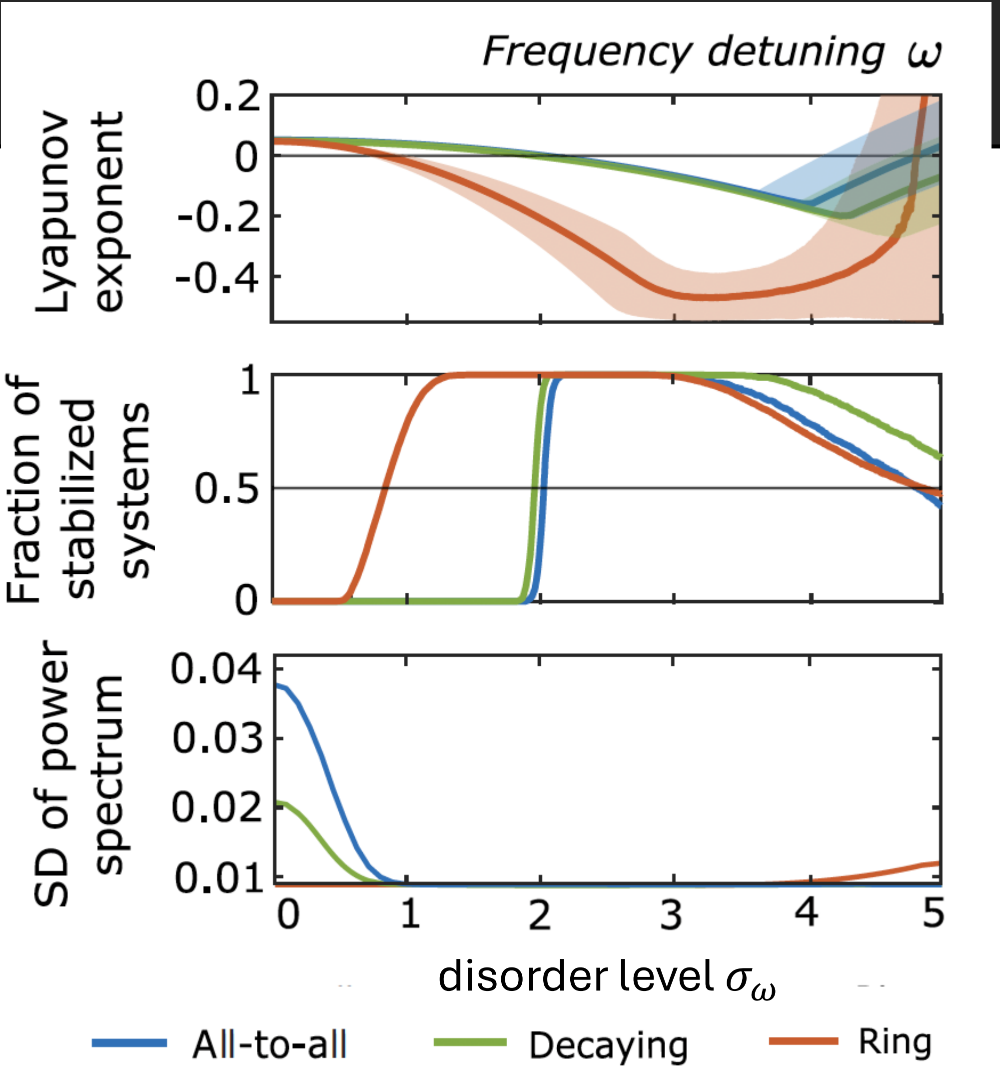


the synchronous state is stable

$\lambda_{\max} < 0$

the synchronous state is coherent

$\delta_j^* \approx 0, \ \forall j.$



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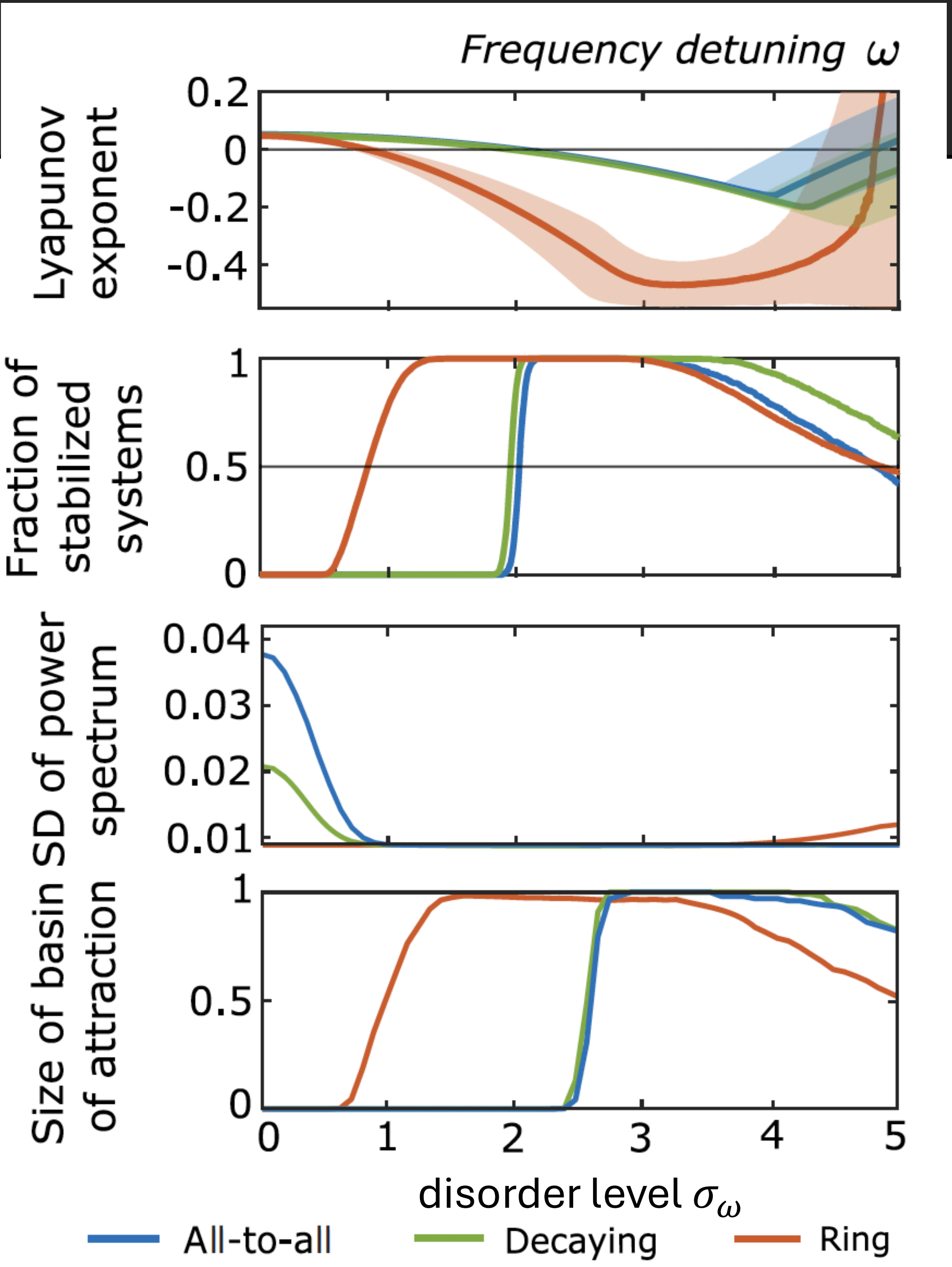

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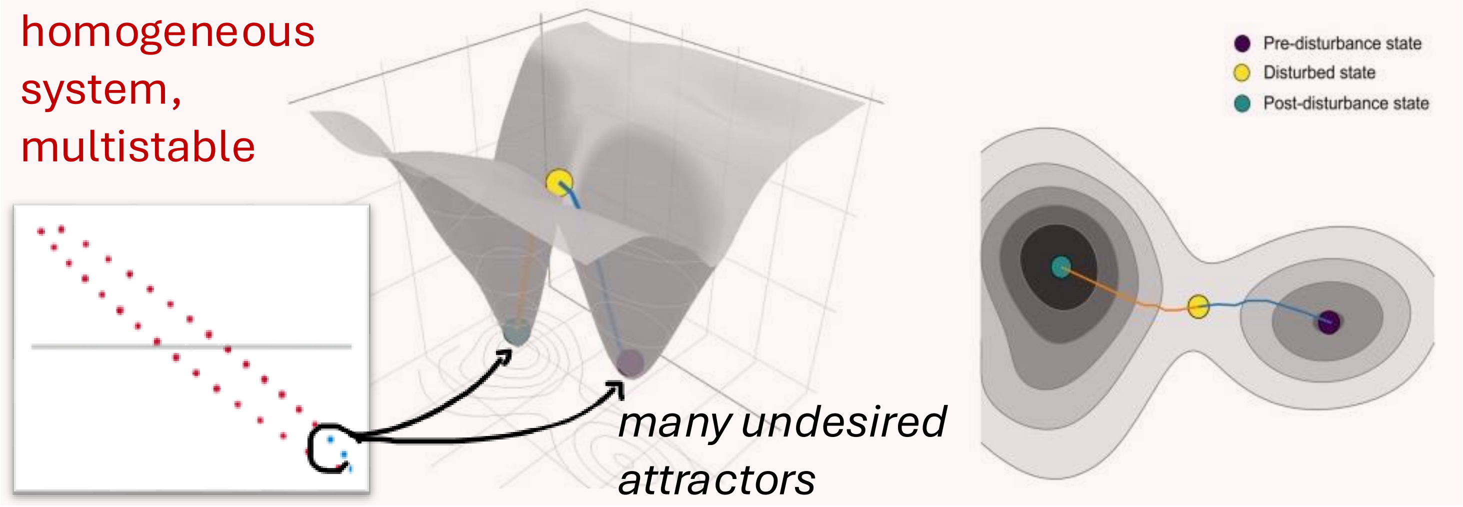

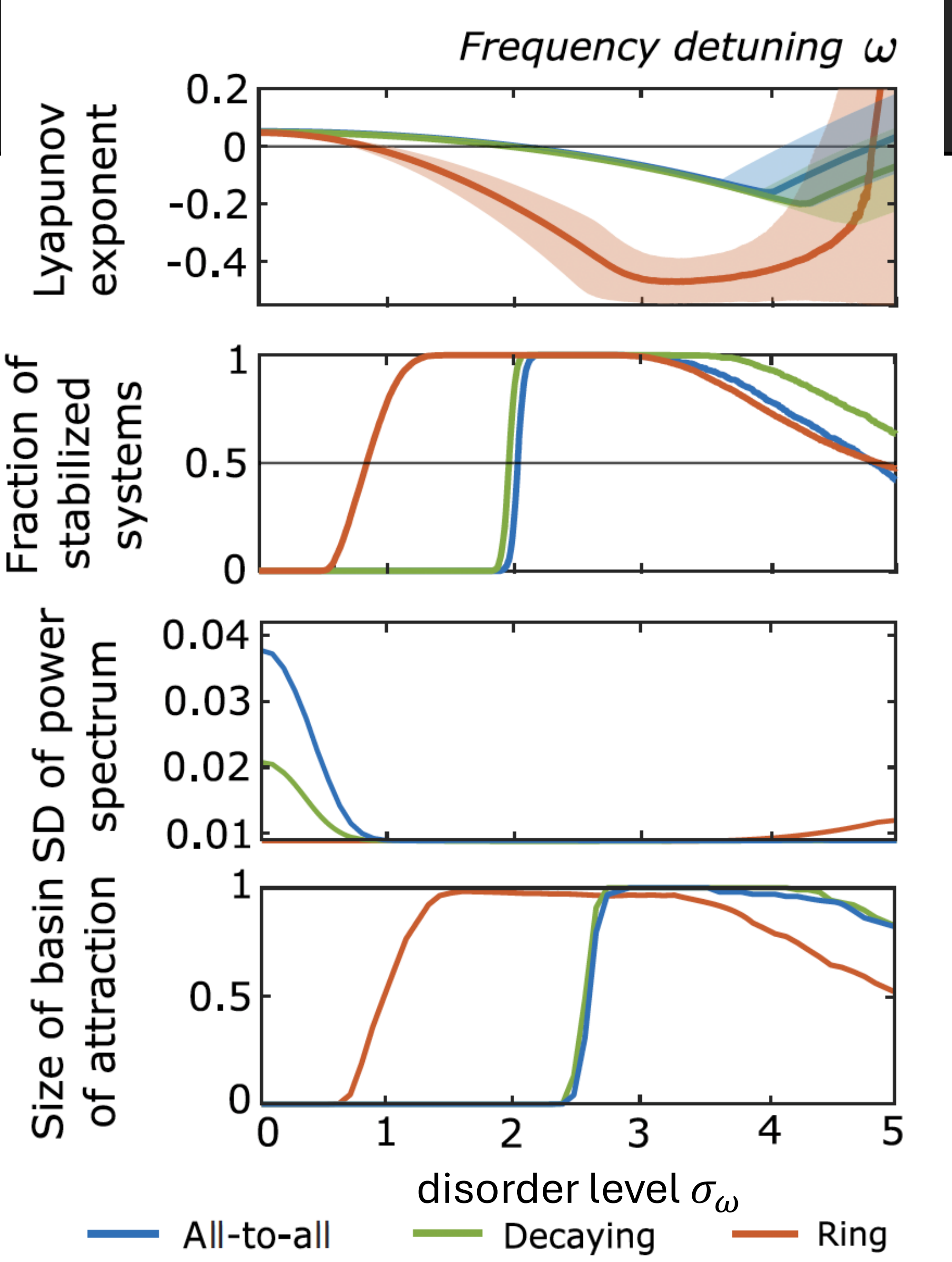

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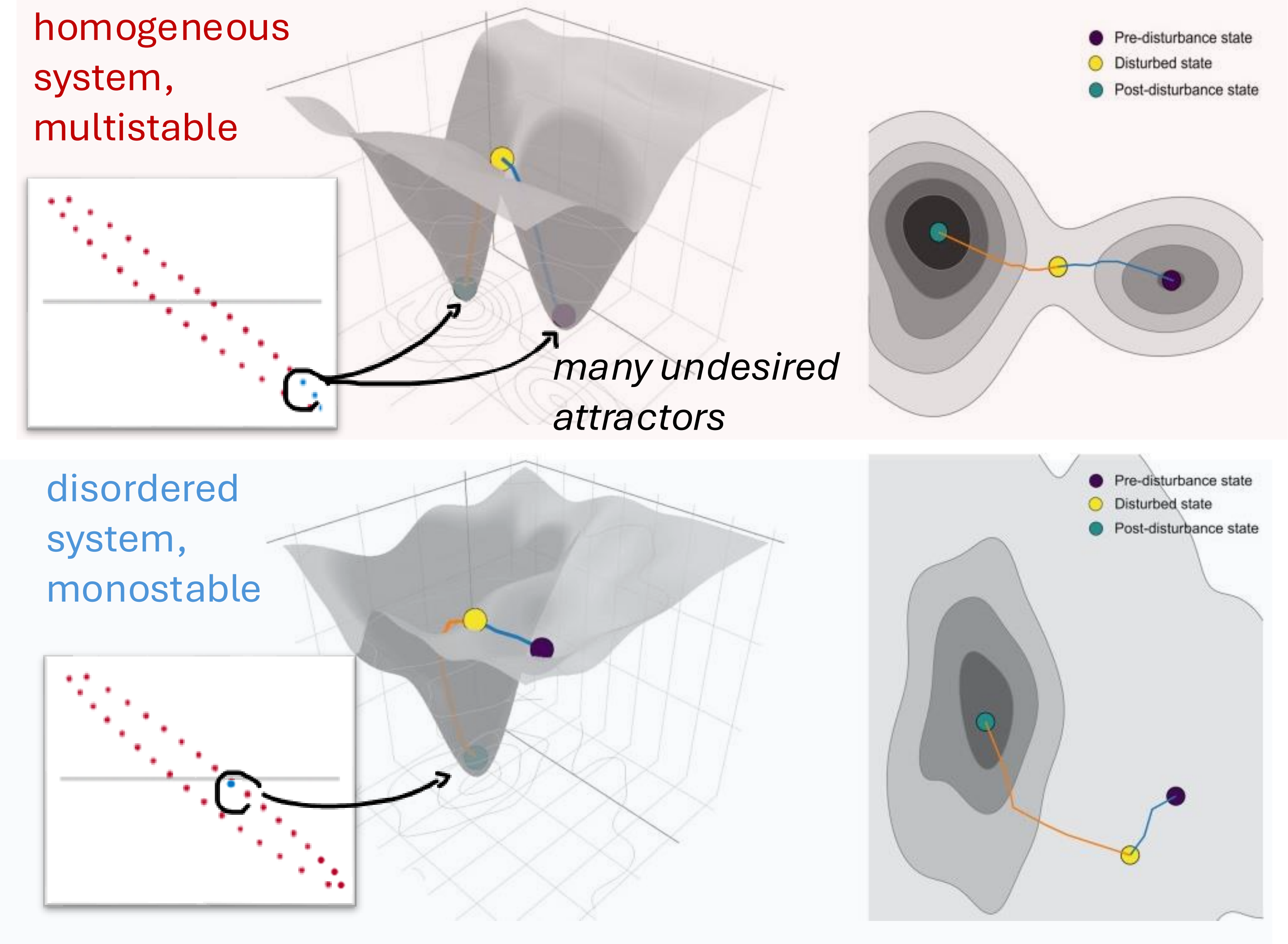
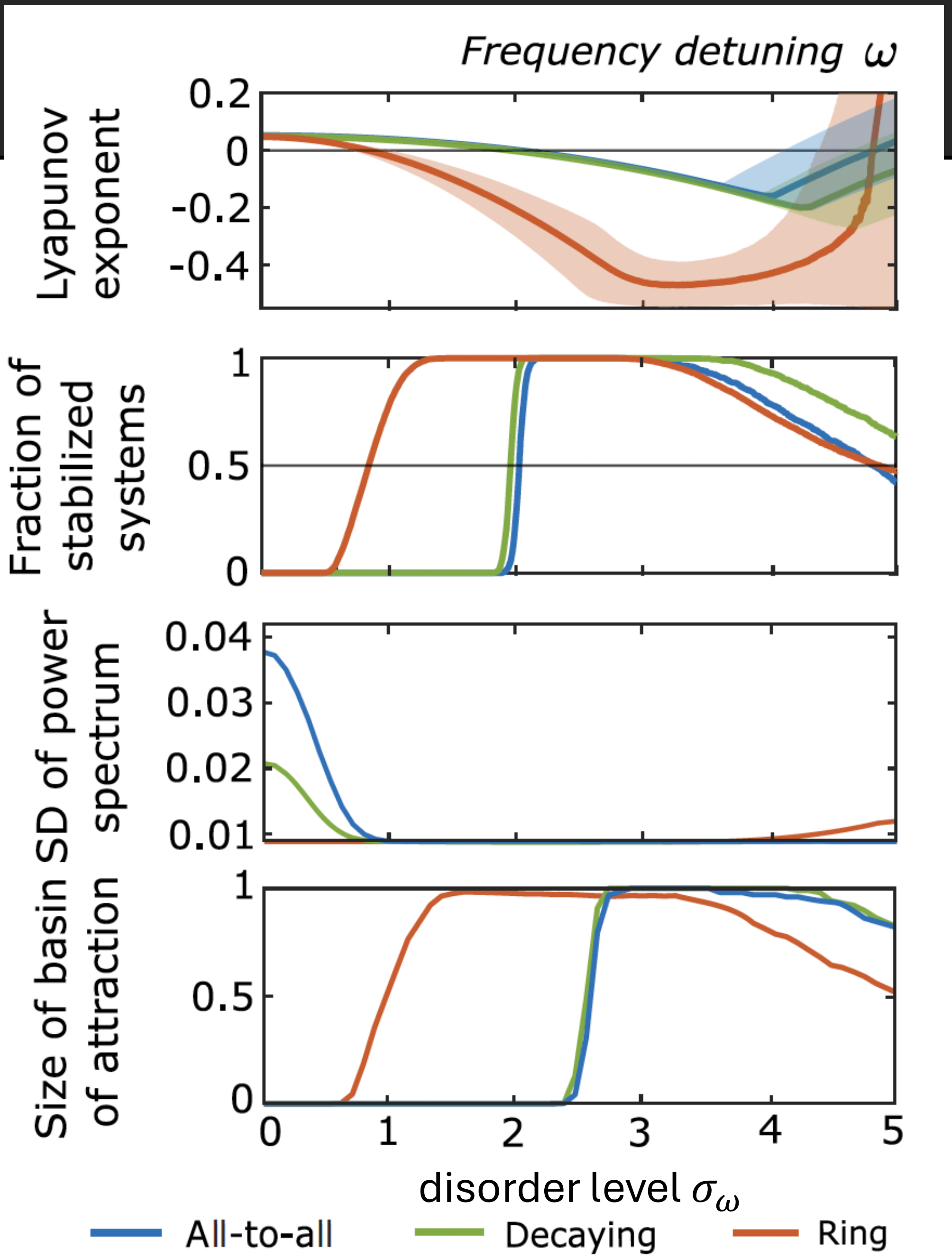
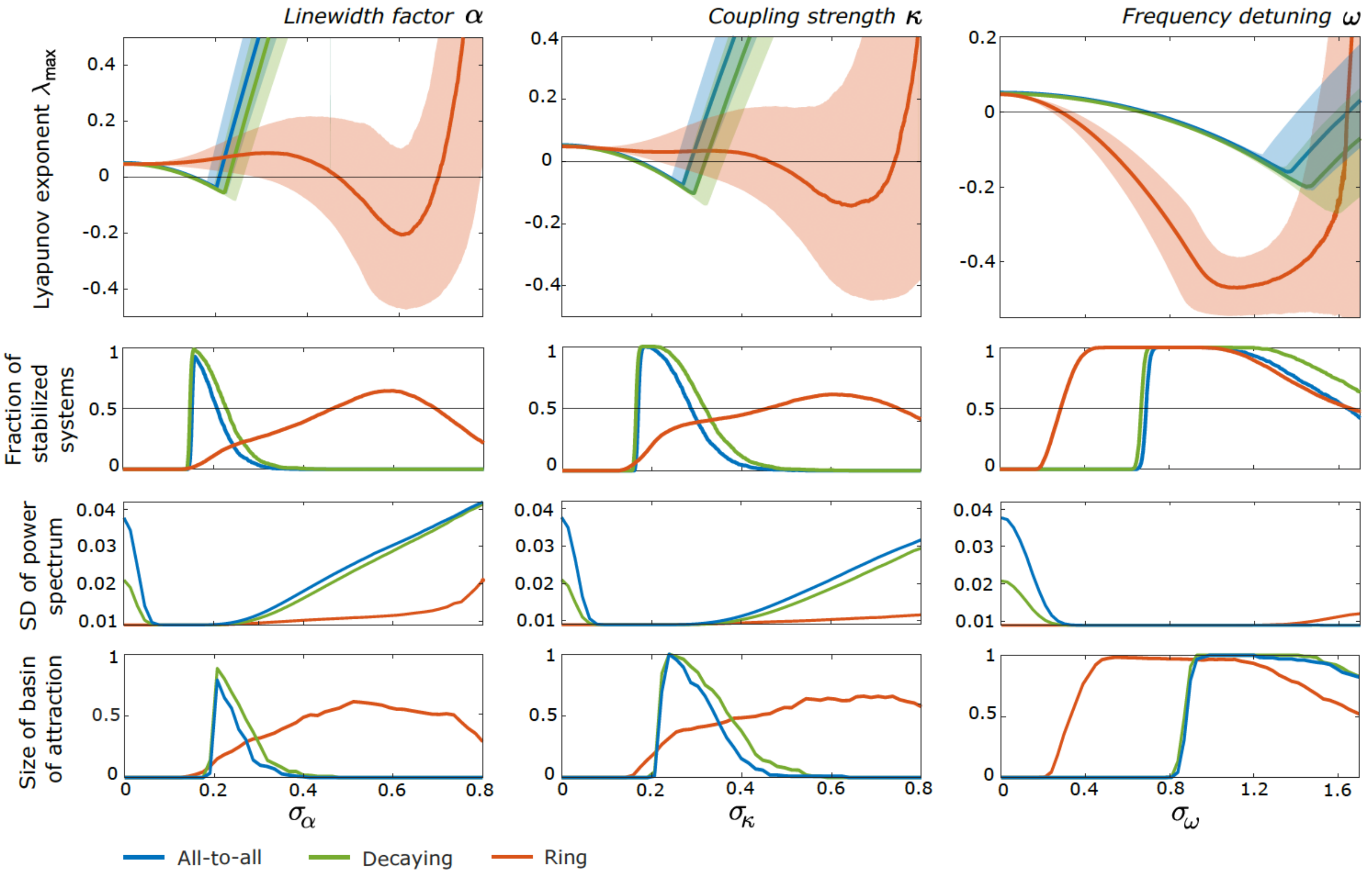




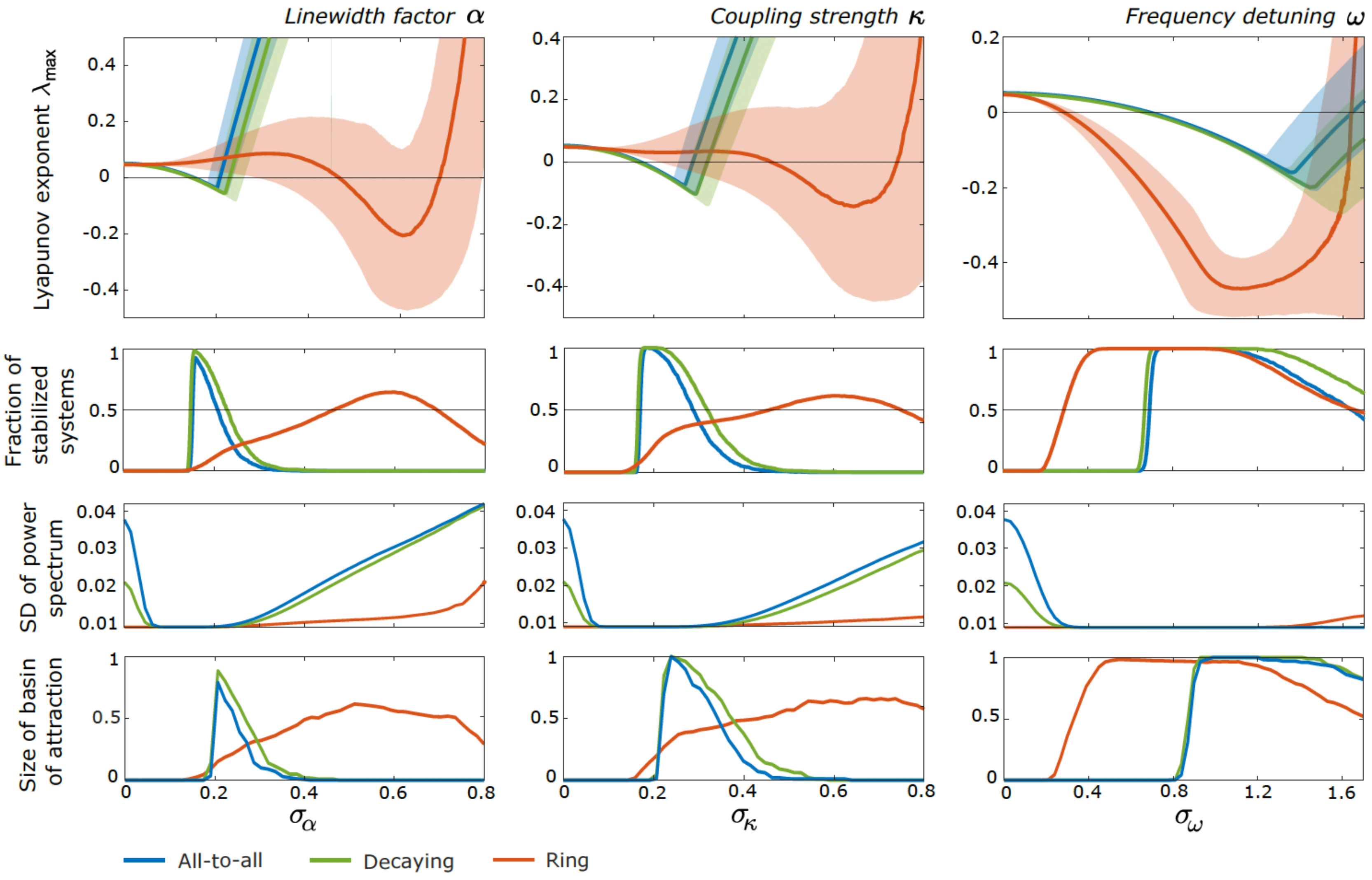
3 take-home messages

Take-home message #1

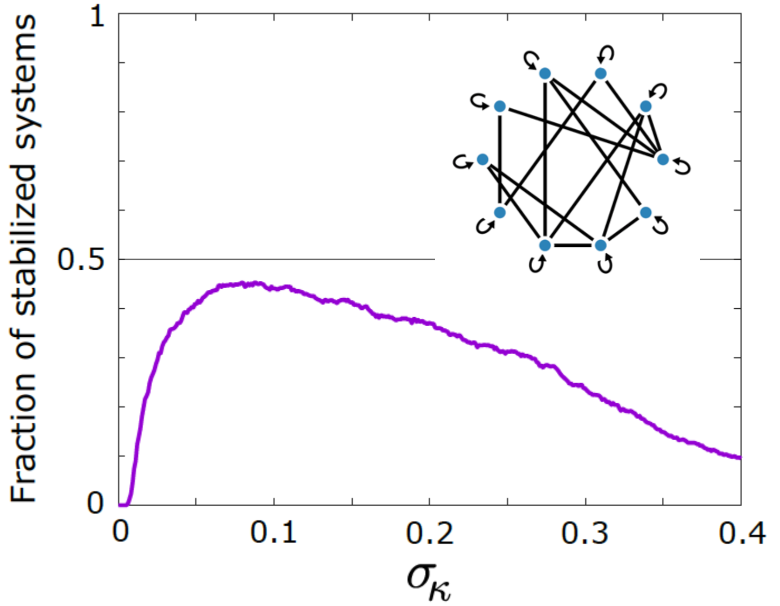


Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of ***parameter****, ...*

Take-home message #1



Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of parameter, ***network structure****, …*



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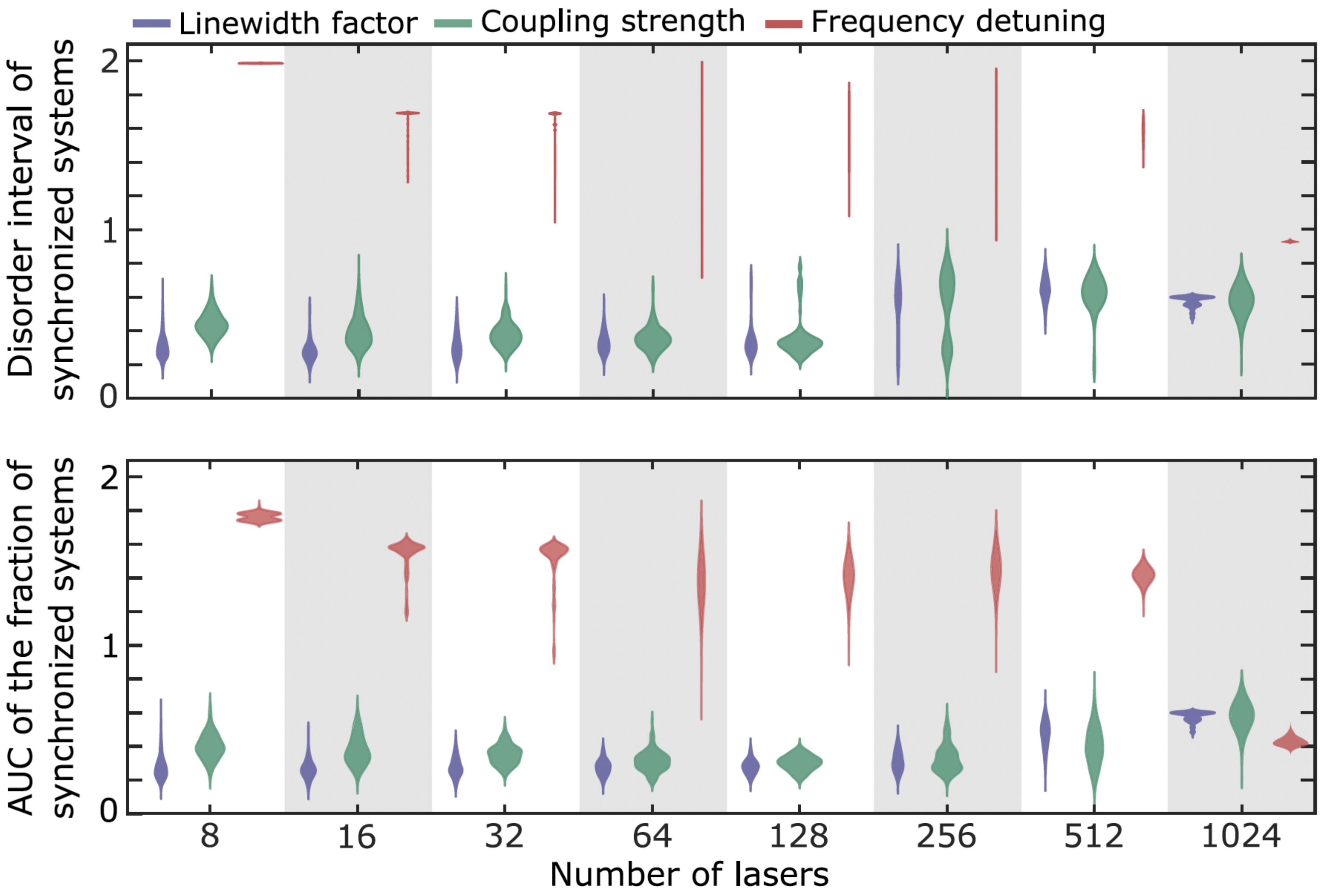
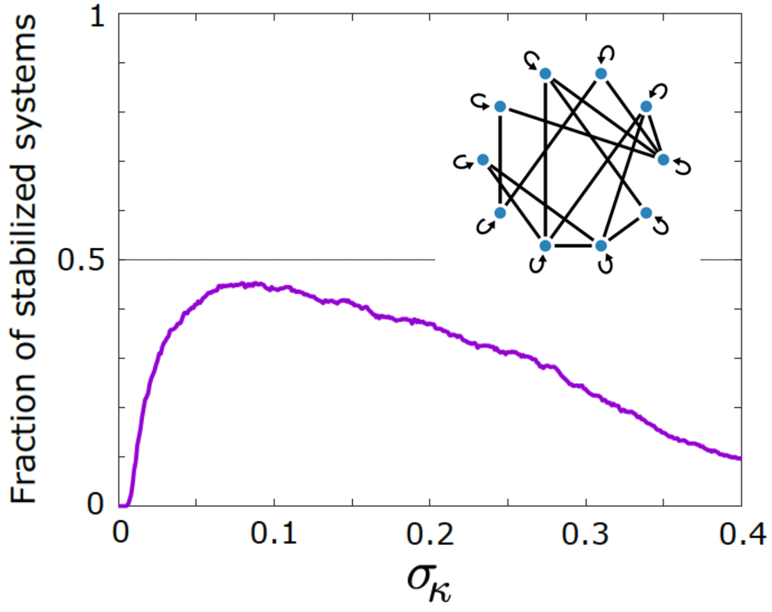
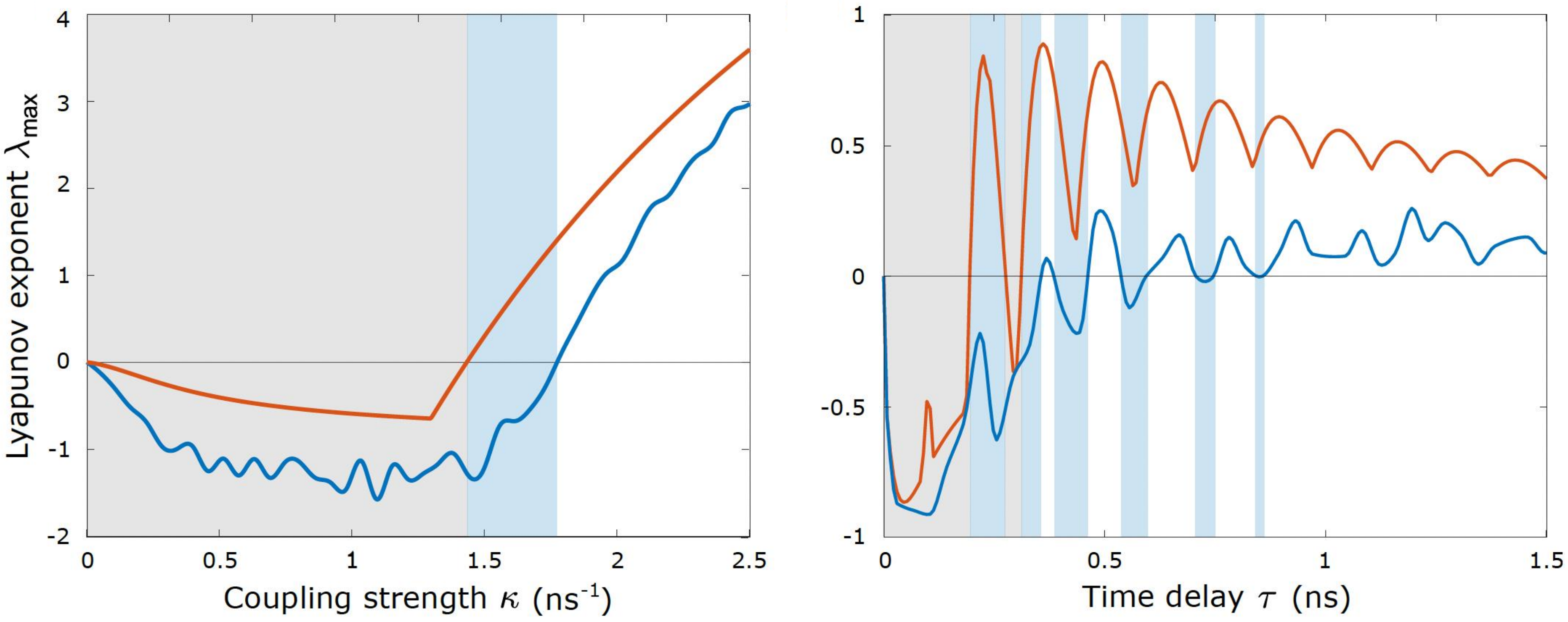


*Disorder provides a reliable mechanism for coherent beam generation, regardless of the choice of parameter, network structure, and **number of lasers**.*



Take-home message #2



Disordered systems exhibit larger stability margins, outperforming homogeneous ones

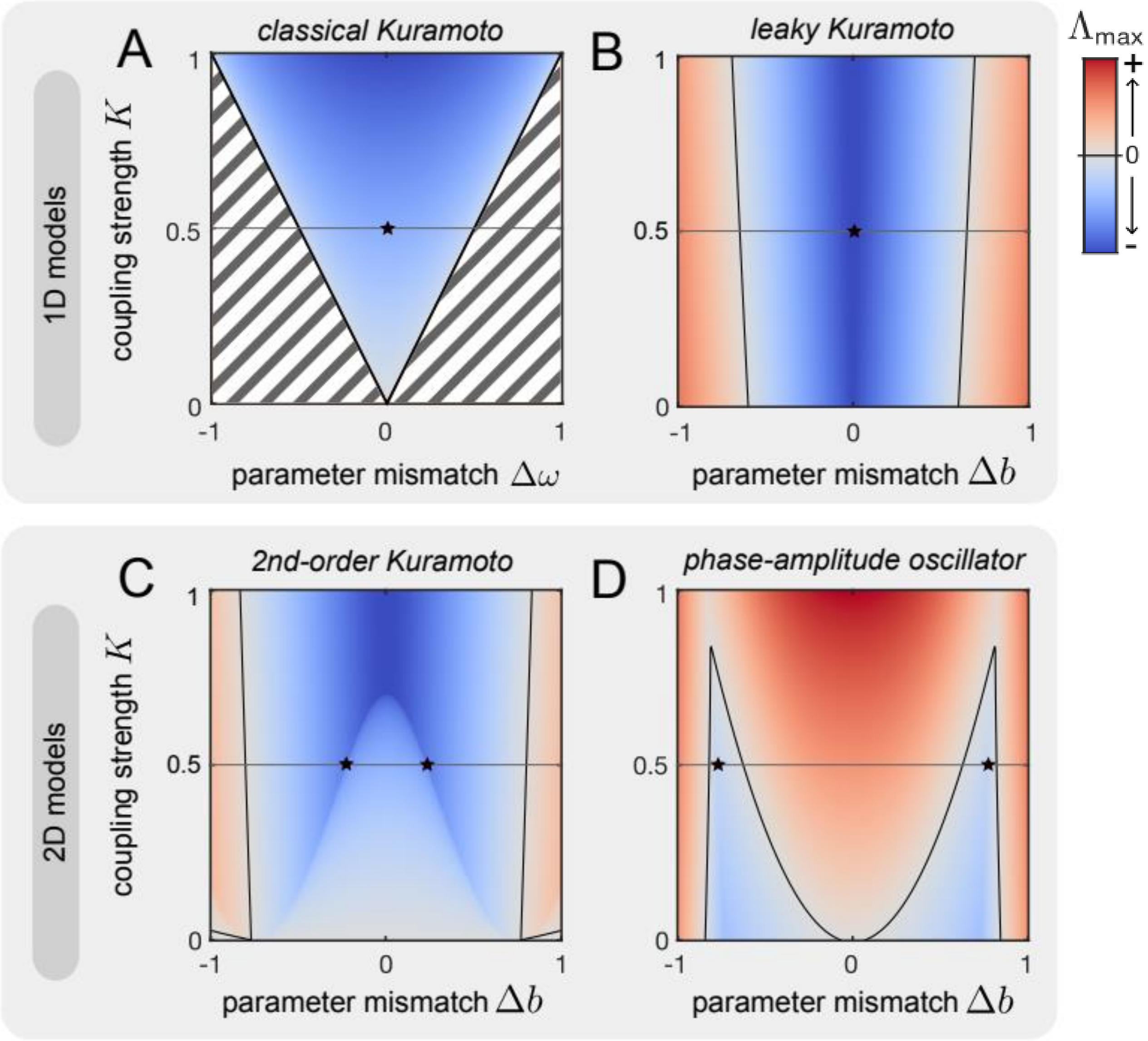
Take-home message #3

$$\dot{\boldsymbol{x}}_i = \boldsymbol{f}(\boldsymbol{x}_i; b_i) + \sum_{j=1}^{N} A_{ij}\boldsymbol{g}(\boldsymbol{x}_i, \boldsymbol{x}_j)$$

Given a network system, what is the optimal parameter assignment that maximizes stability ?

ANM, Zanin, Motter. Under review (2026).

Disorder-promoted stability: condition?



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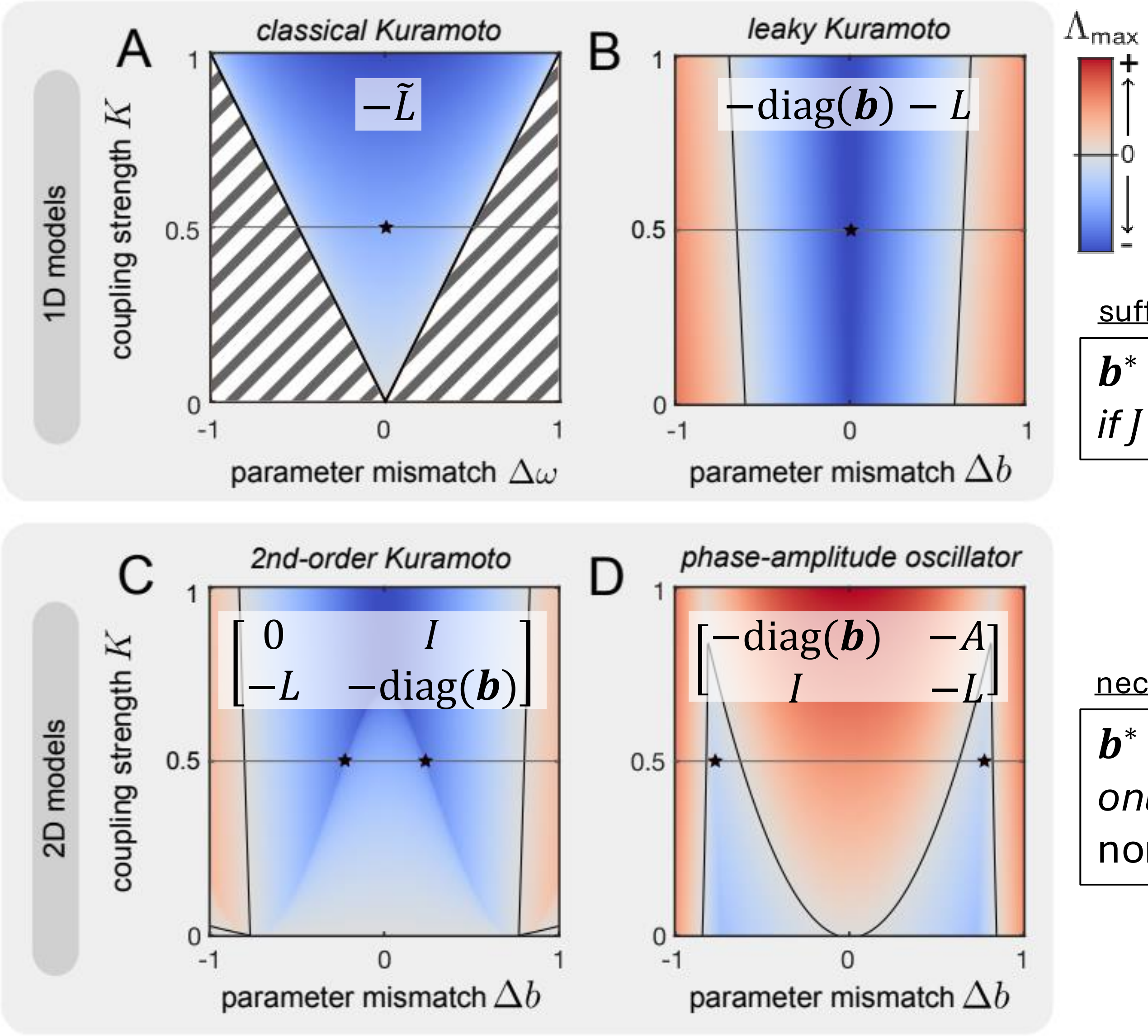

<u>sufficiency</u>

$\boldsymbol{b}^*$ is homogeneous *if* $J(\boldsymbol{b}; A)$ is Hermitian.

<u>necessity</u>

$\boldsymbol{b}^*$ is heterogeneous *only if* $J(\boldsymbol{b}; A)$ is non-Hermitian.

ANM, Zanin, Motter. Under review (2026).

Disorder: Good or Bad?

Nair, Hu, Berrill, Wiesenfeld, Braiman. *PRL* (2021)	→ Lang-Kobayashi model	misaligned time delays
Barioni, Montanari, Motter. *PRL* (2025)	→ Lang-Kobayashi model	frequency detuning
Pando, Gadasi, Bernstein, Stroev, Friesem, Davidson. *PRL* (2024)	→ experimental + LRE	frequency detuning

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Pando, Gadasi, Bernstein, Stroev, Friesem, Davidson. *PRL* (2024) → experimental + LRE frequency detuning

Laser class	**Laser rate equations** (weak phase-amplitude coupling) Bad	**Lang-Kobayashi model** (strong phase-amplitude coupling) Good
Class A (reduction)	$\dot{E}_j = \frac{1}{\tau_c}(G_j - \gamma)E_j + i\omega_j E_j + \frac{\kappa_j}{\tau_c}\sum_k A_{jk}E_k(t-\tau)$	$\dot{E}_j = \frac{1+i\alpha_j}{2}(G_j(t) - \gamma)E_j + i\omega_j E_j + k_j \sum_k A_{jk}E_k(t-\tau)$
Class B	$\dot{E}_j = \frac{1}{\tau_c}(G_j - \gamma)E_j + i\omega_j E_j + \frac{\kappa_j}{\tau_c}\sum_k A_{jk}E_k(t-\tau)$ $\dot{G}_j = \frac{1}{\tau_f}\left(J_0 - G_j\left(s\left\|E_j\right\|^2 + 1\right)\right)$	$\dot{E}_j = \frac{1+i\alpha_j}{2}(G_j(t) - \gamma)E_j + i\omega_j E_j + \kappa_j \sum_k A_{jk}E_k(t-\tau)$ $\dot{N}_j = J_0 - \gamma_n N_j - G_j(t)\left\|E_j\right\|^2$
Class C (extension)	$\dot{E}_j = -\frac{\gamma}{\tau_c}E_j + \frac{1}{\tau_c}P_j + i\omega_j E_j + \frac{\kappa}{\tau_c}\sum_k A_{jk}E_k(t-\tau)$ $\dot{P}_j = \gamma_\perp(-P_j + G_j E_j)$ $\dot{G}_j = \frac{1}{\tau_f}\left(J_0 - G_j\left(s\left\|E_j\right\|^2 + 1\right)\right)$	$\dot{E}_j = -\frac{\gamma}{2}E_j - P_j + i\omega_j E_j + \kappa_j \sum_k A_{jk}E_k(t-\tau)$ $\dot{P}_j = -(\gamma_\perp + i\Delta)P_j + G_j(t)E_j$ $\dot{N}_j = J_0 - \gamma_n N_j - G_j(t)\left\|E_j\right\|^2$

Disorder: Good or Bad?

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Highly Hermitian systems
(when coupling is small)

Highly non-Hermitian systems
(for all coupling)

Disorder: Good or Bad?

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Highly Hermitian systems (when coupling is small) $\xrightarrow{\text{complex coupling}}$ Highly non-Hermitian systems (for all coupling)

Acknowledgments

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Interpretable Disorder-Promoted Synchronization and Coherence in Coupled Laser Networks

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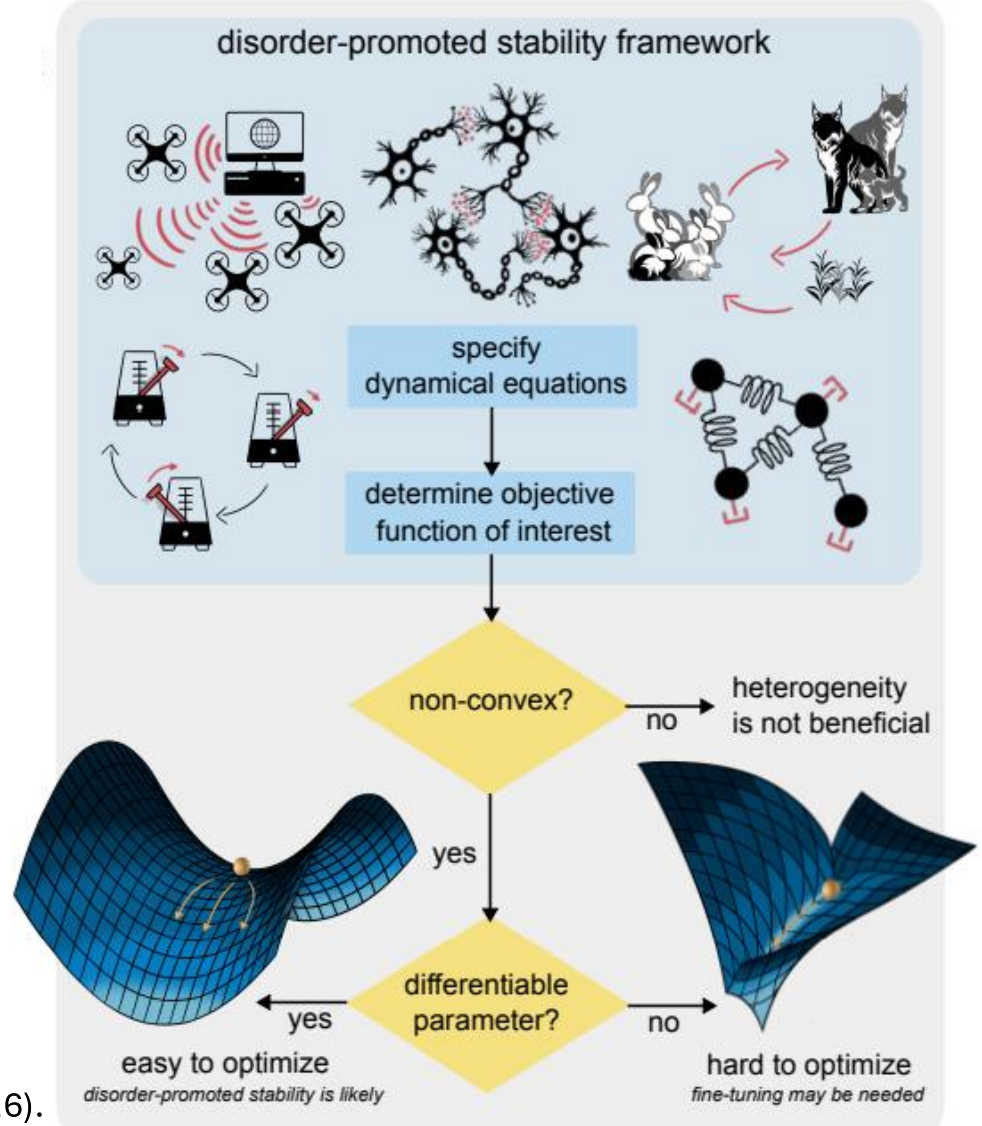

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